Unit 10: Multiplication and division of whole numbers by a fraction

5.3I, 5.3J, 5.3L

From cooking to construction, multiplying and dividing fractions are used in a wide range of real world situations. Mastering them opens up a world of possibilities!

- Multiply fractions and whole numbers using models
- Divide unit fractions by whole numbers
- Divide whole numbers by unit fractions

TEKS standards

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<th>TEKS standards</th>
<th>Common misconceptions</th>
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<td>5.3I: Represent and solve multiplication of a whole number and a fraction that refers to the same whole number using objects and pictorial models, including area models</td>
<td>“Multiplying makes things bigger”</td>
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<td>5.3J: Represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $\frac{1}{3} \div 7$ and $7 \div \frac{1}{3}$ using objects and pictorial models, including area models</td>
<td>“Fractions are smaller than whole numbers”</td>
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How to help:

- For “Multiplying makes things bigger”, use visual models with students to show why multiplying a whole number by a fraction results in a smaller number - because we are adding a number smaller than 1 multiple times.

- For “Fractions are smaller than whole numbers”, spend time placing a variety of fractions on a number line to ensure students understand that some fractions are larger than 1. Do examples of multiplying whole numbers with fractions smaller than 1 and larger than 1 so students can compare results and see that just because they are multiplying by a fraction, the product can still be larger than the whole number if the fraction is greater than 1.
“The rules for multiplying fractions are different from multiplying whole numbers” | Sometimes seeing fractions can confuse students and they may think that they need to do something differently than they normally would with two whole numbers, but the process is the same. We’re still counting equal groups, it’s just that the groups are now fractional parts instead of whole numbers.

**How to help:** Rely on the visual models to help students conceptualize the process and vocalize what they are doing. For example, if they are doing $3 \times \frac{1}{2}$, talk about it as having 3 groups of $\frac{1}{2}$, just like it would be if they were doing $3 \times 2$ and had 3 groups of 2.

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\[3 \times 2 = 6\]
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\[3 \times \frac{1}{2} = \frac{3}{2}\]
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Misunderstanding the concept of division | Some students might think of division only as “sharing” or “grouping.” But, when it comes to fractions, it’s helpful to think of it as “how many times does the divisor fit into the dividend?” This can reinforce students’ understanding of why the quotient gets larger when dividing by a fraction less than 1.

**How to help:** As you work through examples and the visual models with students for division, be sure to verbalize what you are doing to help students build their conceptual understanding. For a problem like $6 \div 2$, we can talk about dividing 6 into 2 equal groups and finding out how many are in each group, dividing 6 into groups of 2 and finding how many groups we will have, or how many times 2 fits into 6. When we have a problem like $6 \div \frac{1}{2}$, it might be confusing to think about dividing 6 into groups of size $\frac{1}{2}$ or into $\frac{1}{2}$ equal groups. It is more clear to think about how many times $\frac{1}{2}$ goes into 6, and that can be represented with a visual model.

“Dividing makes things smaller” | This is true when dividing whole numbers, but not when we divide a whole number by a unit fraction. To simplify $4 \div \frac{1}{6}$, it’s like saying, “How many one-sixths are in 4?” and there are a lot more one-sixths in 4 than there are ones.

**How to help:** Rely on the visual models to show why this is true and verbalize the problems as “how many times the divisor can go into the dividend”. Discuss why the result is smaller than the whole number and how that is seen in each model.
Unit resources

- Students can use these templates when working with number lines, strip diagrams, or area models to solve problems in this unit.
- Print this Number line/strip diagram/area model graphic organizer and put it in a sheet protector for students to use with dry erase markers.
- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

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| Lesson 1: Multiplying fractions and whole numbers | Students will be able to multiply fractions and whole numbers using number lines and area diagrams. | - Warm up activity: Give students addition and subtraction problems with fractions, as they did in the previous unit (Unit 9). Encourage them to use number lines or area models in their answers.  
- Encourage students to use visual models such as repeated addition, number lines, and area models as they solve the problems to solidify their conceptual understanding. They should know how to use all of them but they can choose their favorite!  
- The algorithmic method of multiplying the numerators and denominators should not be presented in this unit (numerator × numerator and denominator × denominator). The goal here is to build conceptual knowledge of multiplying with fractions. If they notice the pattern themselves, discuss it together and make sure they understand why it works based on the visual models before allowing them to use it. Students will see the algorithm in future math courses. |
| Lesson 2: Dividing unit fractions by whole numbers | Students will be able to divide unit fractions by whole numbers using a number line, strip diagram, or area model. | - Students will be presented with visual models for dividing a fraction by a whole number, just as they were in the previous lesson. Again, the focus here is on building conceptual understanding.  
- Division can be challenging to do, especially when fractions are involved. Encourage students to take... |
Lesson 3: Dividing whole numbers by unit fractions

Students will be able to divide whole numbers by unit fractions using number lines, strip diagrams, and area models.

- Dividing whole numbers by unit fractions is less intuitive than the operations in the previous lessons. It may be challenging for students to conceptualize dividing a whole number by a fraction, so encourage them to think about how many times the divisor goes into the dividend. Continue to use the same visual models for continuity and be sure to discuss how to interpret these problems. See “Best practices” for more.
Best practices

Dividing whole numbers and unit fractions
Let’s take a deeper look at the visual models (number lines, strip diagrams, and area models) presented, as well as verbal descriptions, for dividing whole numbers and unit fractions. The area models will look different for the two different types of division problems.

Dividing unit fractions by whole numbers
We’ll look at the expression $\frac{1}{3} \div 5$ using all three visual models. Help students to see the similarities and differences between the methods. They should know how to use each one but can choose to work most often with their favorite.

Words: $\frac{1}{3} \div 5$

- If I divide $\frac{1}{3}$ into 5 equal pieces, how big will each piece be?
- Another way to think about it is “How many times does 5 go into $\frac{1}{3}$?” but that doesn’t make much intuitive sense since 5 is bigger than $\frac{1}{3}$. While this isn’t helpful here, we will use this thinking when we divide a whole number by a unit fraction.

Area model: $\frac{1}{3} \div 5$

First, divide the rectangle that represents 1 whole into thirds (purple). Then, divide each third into five pieces, or fifths.

The striped region represents the size of a piece when $\frac{1}{3}$ is divided into 5 equal parts.

The resulting piece is $\frac{1}{15}$ of the whole, so $\frac{1}{3} \div 5 = \frac{1}{15}$.

Number line: $\frac{1}{3} \div 5$

First, divide a number line from 0 to 1 into thirds (green). Then, divide each third into 5 equal parts.

The dark green region represents the size of a piece when $\frac{1}{3}$ is divided into 5 equal parts.

The resulting piece is $\frac{1}{15}$ of the whole, so $\frac{1}{3} \div 5 = \frac{1}{15}$.

Strip diagram: $\frac{1}{3} \div 5$

First, divide a strip that represents 1 whole into thirds (pink). Then, divide each third into 5 equal parts.

The dark pink region represents the size of a piece when $\frac{1}{3}$ is divided into 5 equal parts.

The resulting piece is $\frac{1}{15}$ of the whole, so $\frac{1}{3} \div 5 = \frac{1}{15}$.
Dividing whole numbers by unit fractions

We'll look at the expression $5 \div \frac{1}{3}$ using all three visual models. Note that the area model in this type of problem is different than in the previous section.

**Words: $5 \div \frac{1}{3}$**

- Using similar wording as for $\frac{1}{3} \div 5$, we would have: If I divide 5 into $\frac{1}{3}$ equal pieces, how big will each piece be? This is difficult to think about because what does $\frac{1}{3}$ equal pieces mean? We'll need to think of it another way.
- “How many times does $\frac{1}{3}$ go into 5?” is easier to visualize.

**Area model: $5 \div \frac{1}{3}$**

We start with 5 wholes, in this case circles, and divide each one into thirds.

To find out how many times $\frac{1}{3}$ goes into 5, we can count the number of pieces, because that's how many thirds are in 5 wholes.

There are 15 pieces, so $5 \div \frac{1}{3} = 15$.

**Number line: $5 \div \frac{1}{3}$**

First, label a number line from 0 to 5, counting by 1s. Then, divide each segment into thirds.

This shows us that we can fit 15 thirds into 5 wholes.

There are 15 pieces, so $5 \div \frac{1}{3} = 15$.

**Strip diagram: $5 \div \frac{1}{3}$**

First, divide a strip into 5 pieces that each represent 1. Then, divide each piece further into three pieces to represent thirds.

There are a total of 15 thirds that fit into 5 wholes.

There are 15 pieces, so $5 \div \frac{1}{3} = 15$.

**GENERAL CLASSROOM IMPLEMENTATION RESOURCES:**

- **Weekly Khan Academy quick planning guide:** Use this template to plan your week using Khan Academy.
- **Using Khan Academy in the classroom:** Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- **Differentiation strategies for the classroom:** Discover strategies to support the learning of all students.