Khan Academy

6th Grade Math

Unit 10: One-variable equations

6.6C, 6.7B, 6.9A, 6.9B, 6.9C, 6.10A

Learning to solve equations is like unlocking a secret language! Understand and solve problems you'll encounter in everyday life, from splitting a dinner bill to planning a road trip.

- ☐ Distinguish between **expressions** and **equations**
- ☐ **Write expressions** and **equations** that represent problem **situations**
- ☐ **Solve** one-step equations involving **addition** and **subtraction**
- ☐ **Solve** one-step equations involving **multiplication** and **division**

START HERE

TEKS standards	Common misconceptions
6.6C: Represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$	**"Equations and expressions are the same" |** The difference between an equation and expression is subtle, but an equation shows that two expressions are equal and will always have an equal sign (=), while an expression is a statement that values can change depending on the variable's value. **How to help:** Go over examples to show the differences between equations and expressions. An easy way to tell the difference is that equations have an equal sign (=) and expressions don't.
6.7B: Distinguish between expressions and equations verbally, numerically, and algebraically	**Misunderstanding variables |** The concept of a variable can be tricky for some students. They might think of it as a specific unknown number, rather than a symbol that can represent any number. Or, they may think that if $x = 3$, for example, in one problem, then $x = 3$ in every problem. **How to help:** A variable is a placeholder that can represent different numbers in different problems. Provide examples and demonstrate simplifying expressions using different values for the variable. It may be helpful, at the beginning, to replace the variable with an empty box, like problems that students saw in elementary school when finding a missing value. The box has the same meaning as a variable and this comparison can be helpful for students struggling with variables. $3 + \boxed{?} = 10 \quad \rightarrow \quad 3 + x = 10$
6.9A: Write one-variable, one-step equations and inequalities to represent constraints or conditions within problems	
6.9B: Represent solutions for one-variable, one-step equations and	

<table>
<tr>
<td>inequalities on number lines</td>
<td rowspan="3">
So many ways to multiply! | Students are introduced to a new symbol for multiplication: a dot ($\cdot$). They already know $\times$ and parentheses, like $4(2)$. With so many ways to imply multiplication, students can get confused or overwhelmed.
How to help: Explain to students that since they are now learning about variables, it can be confusing to use the $\times$ symbol for multiplication because it looks like an x. An expression like $6 \times x$ can appear confusing, especially when written by hand. It is much more clear to write $6 \cdot x$. From now on, students should primarily see and use a dot or parentheses to signify multiplication—move away from using a $\times$.
$$6 \times x = \underbrace{6 \cdot x = 6(x) = 6x}_{\text{Use these for multiplication}}$$
Forgetting to use inverse operations | When solving for a variable, students might not remember to use inverse operations to cancel a value. They may forget that addition and subtraction are inverses and multiplication and division are inverses. See “Best practices” for more.
How to help: Remind students that the goal is to get the variable on one side by itself, and to do that we need to eliminate whatever is on the same side as the variable. To eliminate a number, we need to do an inverse operation to “undo” it. Review inverse operations often and make sure students have lots of practice.
Forgetting to do the same thing to both sides of the equation | When solving equations, it’s crucial to maintain balance and equality. Some students might forget to perform the same operation on both sides of the equation. See “Best practices” for more.
How to help: Introduce a visual method for solving equations so students get the idea that both sides must remain balanced to be equal. Whatever we do to one side must also be done to the other side. Give plenty of practice to reinforce fluency.
Misinterpreting fractions | Fractions can be a tricky concept for students, especially with variables. They might have difficulty understanding that $\frac{3}{4}b$ means $\frac{3}{4}$ times b.
How to help: Encourage students to work slowly and not panic when they see fractions. The same rules apply as with integers! As usual, practice, practice, practice!
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<td>6.9C: Write corresponding real-world problems given one-variable, one-step equations or inequalities</td>
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<td>6.10A: Model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts</td>
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Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

<table>
<tr><th>Lesson</th><th>Objective</th><th>Teaching tips</th></tr>
<tr>
<td>Lesson 1: Variables, expressions, & equations

TEKS standard: 6.7B

Video: 1
Article: 0
Exercise: 1</td>
<td>Students will be able to identify expressions and equations.</td>
<td>
It may be helpful to use the Vocabulary and notation notetaker for this unit. Students will learn variable, expression, and equation in this lesson and will learn a new notation for multiplication in the next lesson (·). As they learn to translate words into expressions in the following lessons, record helpful language (sum/+, is/equals/totals, etc.).
The video includes examples with exponents and square roots that students may be unfamiliar with. Students should focus on understanding the big ideas of the video—1) variables are letters that represent numbers and 2) the differences between expressions and equations.
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<td>Lesson 2: Writing algebraic expressions

TEKS standard: 6.9A

Video: 1
Article: 1
Exercise: 1</td>
<td>Students will be able to write expressions with variables that represent problem situations.</td>
<td>
This may be the first time students are asked to write an expression for a situation that uses variables. If students have a hard time with this, encourage them to think about how they would write the expression if they knew the value of the variable (they can even temporarily substitute the variable with a value). Another option is to make a table to find a pattern:

Every gift bag needs 2 candies. How many candies are needed for n gift bags?
<table>
<tr><th>Bags</th><th>Candies</th></tr>
<tr><td>1</td><td>$2 \cdot 1 = 2$</td></tr>
<tr><td>2</td><td>$2 \cdot 2 = 4$</td></tr>
<tr><td>3</td><td>$2 \cdot 3 = 6$</td></tr>
<tr><td>n</td><td>$2 \cdot n = 2n$</td></tr>
</table>
The pattern is to multiply the number of bags by 2 to get the number of candies. So, if we have n bags, we will need to multiply that by 2 to get the number of candies.</td>
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Lesson 3: One-step addition & subtraction equations TEKS standard: 6.9B Video 2 Article 1 Exercise 2	Students will be able to solve one-step addition and subtraction equations.	• **Warm up activity:** Ask students to use the order of operations to simplify expressions. Be sure to include problems like $4(6) - 5$. • This lesson begins your students' journey into solving algebraic equations! The steps for solving one-step addition and subtraction equations are explained algebraically in the videos and article. For some students, visual examples may be helpful to demonstrate ***why*** you must do the same thing to both sides. See "Best practices" for examples. • There are two important concepts here to discuss and build. The first is the idea of inverse operations—operations that undo each other. The second is the idea that the same thing must be done on both sides of the equal sign in order for the equation to stay balanced. See "Best practices." • Encourage students to always check their answers by plugging their solution in for the variable and simplifying. This way they can know if their answer is correct before they submit it!
Lesson 4: One-step multiplication & division equations TEKS standard: 6.9B Video 4 Article 1 Exercise 2	Students will be able to solve one-step multiplication and division equations.	• Students will apply what they've learned about solving one-step addition and subtraction equations to multiplication and division. • The videos provide multiple ways of thinking about the problems as well as visual representations. • Encourage students to continue to check their answers before submitting them. They get extra practice simplifying and can build their mastery!
Lesson 5: One-step equation word problems TEKS standard: 6.6C, 6.9B, 6.9C, 6.10A Video 1 Article 0 Exercise 5	Students will be able to write and solve one-step equations that represent word problems. Students will be able to graph the solutions to one-step equations on a number line.	• Model with students how to annotate text in questions by underlining or circling important information and words. Students will need to read carefully and take their time interpreting the contexts. • There is a lot of vocabulary that goes along with translating a word problem into algebra. Review words as they are seen in problems and have students add them to a word wall or their vocabulary tracker. See "Best practices."

Best practices

CHECK THIS OUT

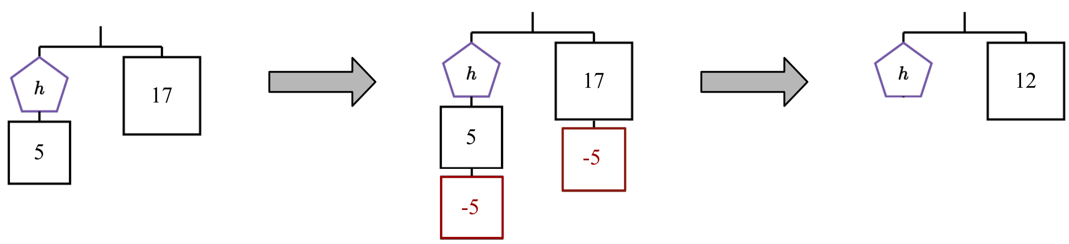
Visual models

There are many models that can be used to visualize what we are doing when we solve one-step equations. It's important that students aren't just memorizing steps—they should understand ***why*** what they are doing makes sense. Present multiple visual representations and allow students to choose the approach that makes the most sense to them. We will show examples of both hanger diagrams and tape diagrams, but feel free to research others.

Hanger diagrams

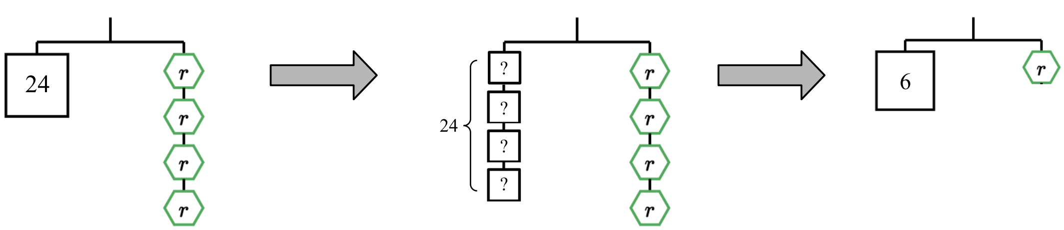
A hanger diagram is like a balance, we want to keep it even (balanced) on both sides. Each shape shows its value. We can add, subtract, multiply, or divide, but we must do the ***same thing to both sides***.

The first hanger diagram example shows us that $h + 5 = 17$. We want to find the value of the variable h, so we need to get it on one side by itself. What can we do to both sides to keep it balanced and also get h alone? We can add -5 to both sides!



$h + 5 = 17$ $\quad$ $h + 5 - 5 = 17 - 5$ $\quad$ $h = 12$

The hanger diagram below shows an example with multiplication, $24 = 4r$. We'll want to divide 24 into 4 equal pieces since there are 4 r's, and then figure out the value of each of those pieces.

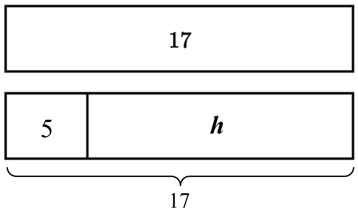


$24 = 4r$ $\quad$ $\frac{24}{4} = \frac{4r}{4}$ $\quad$ $6 = r$

Tape diagrams

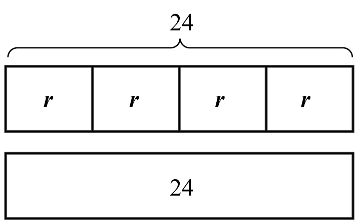
Tape diagrams are another way to visualize one-step equations. Tape diagrams don't emphasize equality "on both sides," like the hanger diagrams do. Students may prefer tape diagrams over hanger diagrams or vice versa, which is great as long as they are able to relate to one of them. We'll look at the same problems as above so you can see what they will look like as tape diagrams and hanger diagrams.

This tape diagram shows us that $h + 5 = 17$. Both tapes have the same length, so their values are equal. We need to figure out what $17 - 5$ is to figure out the value of h.

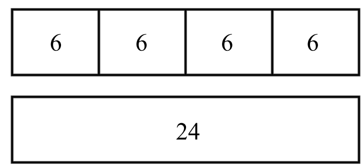


$$5 + h = 17$$
$$h = 17 - 5$$
$$h = 12$$

The tape diagram below shows that $4r = 24$. Both tapes have the same length, so their values are equal. We need to figure out what number 24 divided into 4 equal parts is equal to, or $24 \div 4 = r$.



$$4r = 24$$
$$r = 24 \div 4$$
$$r = 6$$



Inverse operations algebraically

When solving equations, students will have to recognize which operations they need in order to isolate the variable. This is the first step. An inverse operation is an operation that ***undoes*** another operation. If you take a step forward, the inverse operation would be taking a step back. You end up in the same place, right?

In math, addition and subtraction are inverse operations. If you add 3 to a number and then subtract 3, you're back to your original number. The same goes for multiplication and division. If you multiply a number by 3 and then divide by 3, you're back where you started.

When we use addition and subtraction as inverse operations to solve equations, we can think about them as "undoing" the value, or equalling to 0. When we add or subtract 0, the value doesn't change. Here are two examples with different ways to show the work.

$$x + 6 = 8$$
$$x + \underbrace{6 - 6}_{0} = 8 - 6$$
$$x = 2$$

To get x alone, we must "move" the 6. The inverse operation of +6 is -6. We will subtract 6 from both sides of the equal sign.

$$\begin{aligned} x - \cancel{5} &= 4 \\ \underbrace{\cancel{+5}}_{=0} &\quad +5 \\ x &= 9 \end{aligned}$$

To get x alone, we must "move" the - 5. The inverse operation of - 5 is +5. We will add 5 to both sides of the equal sign.

When we use multiplication and division as inverse operations to solve equations, we can think about them as "undoing" again, but this time to get to equal to 1. When we multiply or divide by 1, the value doesn't change. Here are two examples:

$$48 = 6r$$

$$\frac{48}{6} = \underbrace{\frac{\cancel{6}}{\cancel{6}}}_{=1}r$$

$$8 = r$$

To get ***r*** alone, we must "move" the 6. The inverse operation of multiplying by 6 is dividing by 6. We will divide both sides of the equation by 6.

$$\frac{a}{4} = 5$$

$$\cancel{4} \cdot \frac{a}{\underbrace{\cancel{4}}_{=1}} = 5 \cdot 4$$

$$a = 20$$

To get ***a*** alone, we must "move" the 4. The inverse operation of dividing by 4 is multiplying by 4. We will multiply both sides of the equation by 4.

CLASSROOM ACTIVITY

Guess my number

Students can write their own one-step equations by choosing a number and then doing an operation on that number (add, subtract, multiply, or divide) with another number. They can write their equation out and give it to a classmate to find their number. Take this one step further by having students write word problems that represent their equations!

> Magic number = 8. Let's say I add 25 to my magic number: $8 + 25 = 33$
> So, the equation $m + 25 = 33$ is passed along to a classmate to solve.

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.