6th Grade Math

Unit 13: Geometry and measurement
6.4H, 6.8A, 6.8B, 6.8C, 6.8D

The concepts in this unit help students understand space and design, solve practical problems, and serve as stepping stones to more advanced math.

- Convert units using unit rate and proportions
- Determine valid side lengths for triangles
- Calculate the area of triangles, parallelograms, and trapezoids
- Calculate the missing angle of a triangle when two angle measures are given
- Calculate the volume of rectangular prisms with fractional side lengths

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.4H:</strong> Convert units within a measurement system, including the use of proportions and unit rates</td>
<td>Finding the area of a parallelogram and a rectangle is the same</td>
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<tr>
<td><strong>6.8A:</strong> Extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle</td>
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<tr>
<td><strong>6.8B:</strong> Model area formulas for parallelograms, trapezoids, and triangles by</td>
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Finding the area of a parallelogram and a rectangle is the same

\[
A = 5 \cdot 2 = 10
\]

\[
A \neq 5 \cdot 2.5 \\
A = 5 \cdot 2 = 10
\]

How to help: Encourage students to think about making every parallelogram into a rectangle (as shown in the video and article in Lesson 3). When they have the conceptual understanding behind why the equations are the same, they can always imagine (or draw) the parallelogram as a rectangle and see which length is the height.
### Confusing “base” and “height” in triangles

The base of a triangle can be any of its sides, and the height is always perpendicular to the base (just like in parallelograms). If it’s a right triangle then the height may also be one of the sides, but if not, then the height will be drawn separately.

**How to help:** Clarify the terms “base” and “height” with students and make sure they understand what perpendicular means. Use the visual model in the article in Lesson 4 so they can see the relationship between the base and the height more clearly. It is crucial that the base and the height used when finding the area are perpendicular to each other.

### Confusion between area and volume

Students might confuse the concept of area with the concept of volume. Area is the space inside a 2D shape while volume is the amount of space a 3D shape takes up.

**How to help:** Start with a conversation about dimensions. This will help students to think about length as one dimensional, area as two dimensional, and volume as three dimensional. The discussion will also help students to better understand the units that each dimension is measured in (unit length, unit square, unit cube). Area is about covering up a space while volume is about filling up a space.

### Challenge with fractional edge lengths

Fractions can be a tricky concept for students, and when they’re used as edge lengths, it can add an extra layer of complexity. They might think that a fractional edge length means the prism is somehow incomplete or not a “full” prism.

**How to help:** Explain that fractional edge lengths just mean the edges are not whole units long, and may even be less than one unit long. Have students measure the side lengths of various rectangular prisms and write the dimensions as fractions, so they can see examples in real life. A standard die, which is a special rectangular prism (cube), has side lengths that are less than one inch.

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<table>
<thead>
<tr>
<th>decomposing and rearranging parts of these shapes</th>
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<tbody>
<tr>
<td><strong>6.8C:</strong> Write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers</td>
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**6.8D:** Determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers
Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objective</th>
<th>Teaching tips</th>
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<tbody>
<tr>
<td>Lesson 1: Converting units of measure</td>
<td>Students will be able to use unit rate and proportion to convert units within the US customary system and the metric system.</td>
<td><strong>Warm up activity</strong>: This lesson works with units of measure in both the US customary and metric systems. Bring as many examples (or use pictures!) as you can of common items with unit measures (gallon jug, liter bottle, ruler, meter stick, etc.). Have students label the ones they know and categorize them as US customary or metric, and as measures of mass, length, or volume. &lt;br&gt;Model conversion problems together, showing your work for each step and explaining as you go. Always go back to make sure your answer makes sense so students get in the habit of this, too.</td>
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<tr>
<td>Lesson 2: Constructing triangles</td>
<td>Students will be able to use the triangle inequality theorem to determine valid side lengths for triangles.</td>
<td>Give students straws or popsicle sticks of varying lengths to determine which ones can make triangles together. Have students summarize their discoveries and make a conjecture about what must be true about the relationship between the three sides of a triangle. Discuss the triangle inequality theorem (any side of a triangle is always shorter than the sum of the other two sides) together and give examples of three lengths that do not make a triangle. Show physical examples and/or draw them on the board.</td>
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<tr>
<td>Lesson 3: Angles in a triangle</td>
<td>Students will be able to find the missing angle of a triangle when given the measures of the two other angles.</td>
<td><strong>Activity</strong>: Have students draw any triangle (at least an inch on each side) on a piece of paper. With a colored pencil, have them shade each vertex and then label each vertex 1, 2, and 3. Cut out the triangle. On a separate piece of paper, draw a straight line with a ruler. Now, rip off the vertices of</td>
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the triangle. What happens when students place all three vertices against the straight line? They should all line up and sum to 180°. This is a quick activity to show that the three angles of a triangle sum to 180° so students can see that it’s true.

**Lesson 4: Area of a parallelogram**

TEKS standard: 6.8B, 6.8C, 6.8D

Students will be able to calculate the area of a parallelogram.

- **Warm up activity:** Have students find the area of various rectangles. Also give the area and the length of one side and have them find the length of the other side.

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<thead>
<tr>
<th>Video</th>
<th>Article</th>
<th>Exercise</th>
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<tr>
<td>1</td>
<td>0</td>
<td>1</td>
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- Print out various parallelograms (not rectangles or squares) so each student gets one. Challenge them to make one cut and move the piece around so it makes a rectangle. Have students share their solutions with the class.

- Once students see and understand that do some examples together to demonstrate which lengths are the base and height of a parallelogram. Discuss what perpendicular means.

**Lesson 5: Area of a triangle**

TEKS standard: 6.8B, 6.8C, 6.8D

Students will be able to calculate the area of triangles.

- **The article provides an excellent visual demonstration for why the formula for the area of a triangle makes sense.** Give students time to explore the demonstration and work through the practice problems. Spend time discussing the formula to find the area of a triangle and have multiple students explain why it makes sense.

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<tr>
<th>Video</th>
<th>Article</th>
<th>Exercise</th>
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<td>3</td>
<td>1</td>
<td>4</td>
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- Before the second exercise, review what a 90° angle is and what perpendicular means. Explain how this helps to correctly identify the height of a triangle. The most common mistake may be students using an incorrect length for the height.

<table>
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<tr>
<th>Area of parallelogram</th>
<th>Area of rectangle</th>
<th>Length</th>
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<tbody>
<tr>
<td>3 cm</td>
<td>6 cm</td>
<td>7 cm</td>
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<table>
<thead>
<tr>
<th>Area = ?</th>
<th>Length = ?</th>
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<tbody>
<tr>
<td>35 cm²</td>
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**Lesson 6: Area of a trapezoid**

TEKS standard: 6.8D

Students will be able to calculate the area of trapezoids.

- **This lesson transitions back to quadrilaterals; now we’ll investigate trapezoids.** A trapezoid is a quadrilateral with one pair of parallel sides. Sal gives a visual explanation for the formula of the
area of a trapezoid. Have students draw or cut out trapezoids and have them experiment with finding the area on their own before watching the video.

- There are many ways we can think of and write the area of a trapezoid, as Sal discusses in the video. Here are two common ways to write the formula. Note that the second one shows taking the average length of the two bases and then multiplying that by the height. Students may need extra support with the $b_1$ and $b_2$ notation.

$$\text{Area of a trapezoid} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$$

$$= \frac{(b_1 + b_2)}{2} \cdot h$$

<table>
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<tr>
<th>Lesson 7: Volume of a rectangular prism</th>
</tr>
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<tbody>
<tr>
<td>TEKS standard: 6.8D</td>
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Students will be able to calculate the volume of a rectangular prism.

- **Warm up activity:** Give students a variety of problems where they multiply two and three fractions and integers together.

$$\frac{1}{3} \cdot \frac{4}{5} = \frac{3}{7} \cdot \frac{2}{5} \cdot 4 =$$

- Have examples of cubes (dice, blocks, etc.) and rectangular prisms (different boxes, etc.) on hand as concrete examples. Students did find the volume of rectangular prisms in the 5th grade course, so this shouldn't be brand new.

- If you have building blocks or counting cubes, they may be helpful for students who would benefit from building the actual figures to find their volume, at least at the start.

- Discuss the rectangular prism volume formula together, especially why it is the area of the base times the height. See “Best practices” for more.
Best practices

Volume formula for rectangular prisms, explained!
The formula for the volume of a rectangular prism is easy to memorize but it’s more important to understand where it comes from. Knowing how the formula is derived will help to understand how to find the volume of other 3D figures. The idea of finding the area of the base and multiplying that by the height is used to find the volume of any right prism or cylinder and is related to finding the volume of pyramids. Let’s dig deeper.

Find the volume of a rectangular prism with dimensions of 3 cm, 4 cm, and 5 cm.

We can label the sides however we wish. If we were to rotate this prism so the base is 4 cm by 5 cm and the height is 3 cm, the volume would still be the same.

First, we will find the area of the base. In this diagram, the base is 3 cm by 5 cm. So, we can fit $3 \times 5 = 15$ cubes at the base.

Second, we will multiply by the height of the prism. Since the height is 4 cm, we will have 4 layers, where each layer contains 15 cubes. We will multiply the number of cubes in the base layer by the number of layers (the height).

There will be $4 \times 15 = 60$ cubes in the entire figure. So, the volume of a cube with dimensions 3 cm by 4 cm by 5 cm is 60 cm$^3$. We can also think about this with repeated addition where each layer has 15 cubes, so the volume is $15 + 15 + 15 + 15 = 60$.

Here is an example to see that how we label the length, width, and height of a rectangular prism does not matter when finding the volume:

The dimensions of the three rectangular prisms below are 2 by 4 by 6. The diagrams are each rotated so the bases have different dimensions, but the volumes are always the same.
Working in 3 dimensions
It may be challenging for students to work with three-dimensional figures in a two-dimensional space like a computer screen or a piece of paper. It's helpful for students to have access to cubes so they can build the figures and work with them physically. As they practice building shapes from a diagram, they will improve their ability to visualize and understand diagrams on paper.

CLASSROOM ACTIVITIES

Shapes all around them
Have students look for examples of triangles, parallelograms, and trapezoids (and rectangular prisms if you want to add them) around them when they are not in class. They can take pictures or describe where they saw the shapes and why they think that shape may be important where they see it being used. Students can share their findings with the class to encourage everyone to investigate the world around them.

Volumes of boxes
There are rectangular prisms everywhere around us! Commonly, they are in the form of cardboard boxes. Have students bring in a box (and/or provide boxes) for students to use in this assignment. Boxes can be any size, from candy boxes to jewelry boxes to granola bar boxes (or other food packaging), to shoe boxes, to mailing boxes.

Have students work individually or in pairs to find the volume of their box. They will need rulers to measure (specify if they should use inches or centimeters or something else). Have students label the length of each side on the box and then show their work to find the volume. They can make a poster for this if you’d like.

There may be interesting discussion possibilities if two or more boxes have the same volume but different dimensions, or if some boxes have one or two dimensions that are the same but other dimensions are different. Allow students some time to compare their boxes and volumes and share their findings.

Volume of the classroom
If your classroom is a rectangular prism then you can find the volume of it! Also find the area and perimeter of the floor and/or walls! This may take some creative thinking to find the height of the ceiling, but it is a good challenge for your students.

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.