

6th Grade Math

Unit 1: Proportional reasoning with ratios and rates

6.4B, 6.4C, 6.4D, 6.4E, 6.4H, 6.5A

Ratios and rates help us to decode everything from recipes and financial investments to sports statistics and technological advancements, and empower students to make sense of the fascinating interconnectedness of our everyday experiences.

- Write **part-to-part** and **part-to-whole** ratios
- Write ratios in **multiple forms**
- Model** ratios using visual strategies
- Find **equivalent** ratios
- Apply** ratios in real-world contexts



TEKS standards	Common misconceptions
<p>6.4B: Apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates</p>	<p>“The order doesn’t matter” The order of terms in a ratio matters a lot! Some students might think that the ratio 3:4 is the same as 4:3, but they’re actually different. In a recipe, 3 cups of flour to 4 cups of water will give a much different outcome than 4 cups of flour and 3 cups of water, for example.</p> <p>How to help: Talk often about how the order does matter. Give examples of ratios that are real for them and ask if they are the same if you switch the numbers. For example, you could ask students to write a ratio for the number of siblings Brian has to the number of siblings Henry has. If the ratio is 4:2, then you can discuss how if you wrote 2:4 then that would mean that Brian has two siblings, and that’s not accurate. It can also be a helpful visual reminder to students when you color code the ratio in words and in numbers.</p> <p style="text-align: center;"> Brian : Henry 4 : 2 </p> <p>“You can find equivalent ratios using addition” Students sometimes will try to use addition when finding equivalent ratios instead of multiplication. For</p>
<p>6.4C: Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute</p>	
<p>6.4D: Give examples of rates as the comparison by division of two quantities</p>	

<p>having different attributes, including rates as quotients</p>	<p>example, when finding equivalent ratios, they may add 5 to both sides instead of multiplying both sides by 5.</p>
<p>6.4E: Represent ratios and percents with concrete models, fractions, and decimals</p>	<p>How to help: Demonstrate for students that when you add the same number to both sides of a ratio, the ratios are no longer equivalent. Students making this mistake can draw out pictures of both ratios to see that they are not equivalent with addition.</p>
<p>6.4H: Convert units within a measurement system, including the use of proportions and unit rates</p>	<p>“Part-to-part and part-to-whole ratios are the same” Students often confuse these two types of ratios. For example, if there are 3 apples and 2 oranges, the part-to-part ratio of apples to oranges is 3:2, but the part-to-whole ratio of apples to all fruit is 3:5.</p>
<p>6.5A: Represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions</p>	<p>How to help: It can be fun and helpful to make ratios with class data. Students can see the difference between the ratio of the number of students with blue shirts:the number of students with red shirts and the number of students with blue shirts:the total number of students in class.</p>


















Unit resources

- Use this [Double number line workspace](#) for lessons 2, 3, and 4. This [Double number line graphic organizer](#) can be put in a sheet protector and used with dry erase markers.
- Use this [Table workspace](#) especially for lesson 3. This [Tables graphic organizer](#) can be put in a sheet protector and used with dry erase markers (double side with the double number line graphic organizer!).
- Students can practice using multiple strategies with ratios with this [template](#). Fill in one of the boxes and have students complete all of the others. It will be particularly helpful in Lesson 2 and is a good warm up activity.
- For the videos in this unit, use the [Learning summary video notetaking guide](#).
- For the articles in this unit, use the [Article notetaking guide](#).
- For the exercises in this unit, use the [Blank workspace template](#).
- To record key terms and information, use the [Vocabulary and notation notetaker](#).



Lesson overview

Lesson	Objective	Teaching tips
<p>Lesson 1: Intro to ratios</p> <p>TEKS standard: 6.4E, 6.5A</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Video</p>  <p>3</p> </div> <div style="text-align: center;"> <p>Article</p>  <p>1</p> </div> <div style="text-align: center;"> <p>Exercise</p>  <p>2</p> </div> </div>	<p>Students will be able to write part to part ratios and part to whole ratios.</p> <p>Students will be able to write ratios in multiple forms: $x: y$, x to y, x per y, x for every y.</p>	<ul style="list-style-type: none"> • Since this is the first lesson in the unit, review expectations when working on Khan Academy and how to use the unit resources with students. • This may be the first time students are seeing ratios. Spend time going over ratios with students and have them practice writing ratios for tangible items in the classroom (red markers:blue markers, paper clips:binder clips, eyes:toes, black backpacks:all backpacks, etc.). Students can bring in their own collections of items and write their own ratios—or quiz their friends! • It's important that students can write and interpret ratios in multiple forms ($x: y$, x to y, etc.), so provide practice writing ratios in all of the formats. • Key vocabulary: ratio, equivalent ratio, rate, and unit rate
<p>Lesson 2: Visualize equivalent ratios</p> <p>TEKS standard: 6.4B, 6.4E, 6.5A</p>	<p>Students will be able to write equivalent ratios using strip diagrams, double number lines, and tables.</p>	<ul style="list-style-type: none"> • Students learn three new visual strategies for working with ratios: strip diagrams, double number lines, and ratio tables. See “Best practices” for more on these strategies. Students will be expected to interpret these diagrams and know how to make their own. Use the worksheets provided above to scaffold students’ creation of diagrams.

<p>Video Article Exercise</p> <p> 5  0  5</p>		<ul style="list-style-type: none"> Encourage students to read each problem carefully to determine what the ratio is asking for, particularly for whether it is a part-to-part ratio or a part-to-whole ratio. In the videos, Sal talks about tape diagrams—they are the same as strip diagrams. 												
<p>Lesson 3: Equivalent ratios</p> <p>TEKS standard: 6.4B, 6.5A</p> <p>Video Article Exercise</p> <p> 5  1  5</p>	<p>Students will be able to find equivalent ratios and determine whether two given ratios are equivalent.</p>	<ul style="list-style-type: none"> This lesson moves students towards more efficient methods of finding equivalent ratios using tables and simple multiplication/division. For students who are struggling, it is fine if they continue to use one of the previous visual methods, but the goal is to move away from them once the conceptual understanding is developed. Help students to <i>organize their work</i> when they find equivalent ratios using more efficient methods. See “Best practices” for more ideas. 												
<p>Lesson 4: Ratio application</p> <p>TEKS standard: 6.4H, 6.5A</p> <p>Video Article Exercise</p> <p> 3  0  3</p>	<p>Students will be able to identify equivalent ratios on a coordinate grid.</p> <p>Students will be able to apply their knowledge of ratios in real-world contexts like unit conversion.</p>	<ul style="list-style-type: none"> Warm up activity: Students will need to plot points on a coordinate grid for this lesson, so be sure to review this. Give students a blank coordinate grid with a list of points to graph, or a graph with points and ask them to name the points as ordered pairs—or both! Students will be asked to perform unit conversion in some of the exercises. They may not know what the unit amounts are, so it will be helpful to demonstrate what they are by bringing in examples (gallon, quart, ounce, gram, mile, meter, etc.). 												
<p>Lesson 5: Intro to rates</p> <p>TEKS standard: 6.4B, 6.4C, 6.4D</p> <p>Video Article Exercise</p> <p> 5  1  3</p>	<p>Students will be able to apply their knowledge of ratios in real-world contexts like using rates.</p> <p>Students will be able to find unit rates.</p>	<ul style="list-style-type: none"> This lesson introduces rates as ratios (i.e. words per minute). Talk about the problems with students in their contexts. Act out situations and/or relate them to students’ experiences. For rate problems, it is usually easiest to find the unit rate first and then scale up. For example, if Charles can type 675 words in 9 minutes, how many words can he type in 13 minutes? There are no common factors of 9 and 13, so we can find how many words he can type in 1 minute and then scale that up to 13. <table border="1" data-bbox="1201 1711 1445 1900"> <thead> <tr> <th>Minutes</th> <th>Words</th> </tr> </thead> <tbody> <tr> <td>9</td> <td>675</td> </tr> <tr> <td>↓ ÷9</td> <td>↓ ÷9</td> </tr> <tr> <td>1</td> <td>75</td> </tr> <tr> <td>↓ ×13</td> <td>↓ ×13</td> </tr> <tr> <td>13</td> <td>975</td> </tr> </tbody> </table>	Minutes	Words	9	675	↓ ÷9	↓ ÷9	1	75	↓ ×13	↓ ×13	13	975
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9	675													
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Best practices

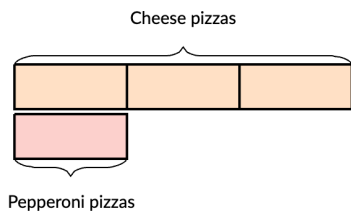


Let's look in more detail at each of the ratio strategies that are presented in this unit and used in future units.

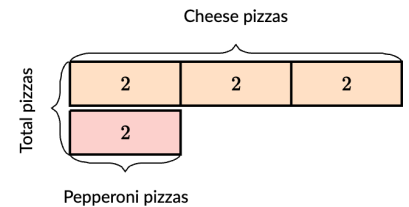
Strip diagrams

Students are likely to be familiar with strip diagrams as they are useful in many different contexts. Strip diagrams may look slightly different in different situations, but they are all essentially the same. In this unit, we will see strip diagrams used to represent relationships between two quantities (ratios).

For example, at a party, they are ordering pizzas. We can use a strip diagram to represent this situation:



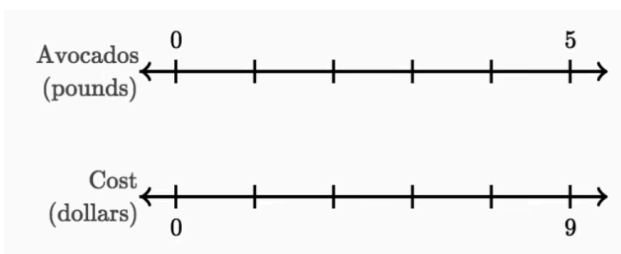
The strip diagram at left shows us that for every 3 cheese pizzas ordered, they are also ordering 1 pepperoni pizza (3:1). It is important to note that each rectangle represents the same quantity. If you made each rectangle equal to 2 pizzas, for example, then you could easily find an equivalent ratio (6:2).



Double number lines

Double number lines help us compare quantities visually. We use two number lines, one vertically below the other. As with all number lines, it is important that numbers are evenly spaced along the lines.

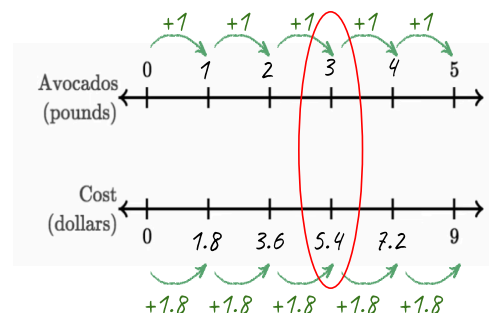
Here is a double number line showing that 5 pounds of avocados cost \$9. You can see that because the 5 on the Avocados number line (top) is vertically aligned with the 9 on the Cost number line (bottom). If we are asked to find out how much money 3 pounds of avocados would cost, we would want to fill in the tick marks on both number lines and look for the cost that vertically aligns with 3 pounds of avocados.



The number line on the top is broken up into five pieces and the largest value is 5, so each tick mark represents an increase of 1.

The bottom number line will not be so simple. It's also broken up into 5 parts but the largest value is 9. This means that each tick mark represents an increase of $9/5 = 1.8$. Below is the fully labeled diagram.

We can see in the double number line, at right, that 3 pounds of avocados would cost \$5.40.



Tables

Students will see tables in multiple formats in this unit. The most important thing to remember about tables is that all of the rows and columns must be proportional. Here is a ratio table showing that for every package of cookies in my classroom snack drawer, there are 3 packs of crackers. The table shows all of the ways that the entries are proportional.

We can write each of these rows as the following equivalent ratios:

1:3 2:6 5:15

	Cookies		Crackers
	1	$\times 3$	3
$\times 2$	2	$\times 3$	6
$\times 5$	5	$\times 3$	15

(Note: The diagram shows green arrows indicating multiplication factors: $\times 2$ from row 1 to row 2, $\times 5$ from row 1 to row 3, and $\times 3$ from column 1 to column 2.)

Efficient strategies - Use a consistent format

There are many ways to set up problems when finding equivalent ratios. Strip diagrams, double number lines, and tables are all examples of those, but can be time consuming to create. As students develop their conceptual understanding of ratios, move away from these visual representations and adopt more efficient strategies.

When using more efficient strategies, choose one format to use with students for consistency so their work is organized and clear. Let's look at one possible format for the following problem: If a snail can travel 15 centimeters in 6 hours, how long would it take to travel 45 cm?

The ratio can initially be set up like this:

cm	15	→	45
Hours	6	→	?

Then, we can show our thinking like this:

cm	15	$\xrightarrow{\times 3}$	45
Hours	6	$\xrightarrow{\times 3}$	18

This method can be used vertically, too!

cm	Hours
15	6
$\downarrow \times 3$	$\downarrow \times 3$
45	? = 18

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- [Weekly Khan Academy quick planning guide](#): Use this template to plan your week using Khan Academy.
- [Using Khan Academy in the classroom](#): Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- [Differentiation strategies for the classroom](#): Discover strategies to support the learning of all students.