6th Grade Math

Unit 7: Multiplying and dividing integers

6.3D

Understanding how to multiply and divide negative numbers is key to solving real-world problems in finance, science, and everyday life.

- Multiply positive and negative integers
- Divide positive and negative integers

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
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<tbody>
<tr>
<td><strong>6.3D:</strong> Add, subtract, multiply, and divide integers fluently</td>
<td>Misunderstanding the meaning of negative numbers</td>
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<tr>
<td></td>
<td>“A negative times a negative is negative”</td>
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<tr>
<td></td>
<td>Ignoring the negative sign</td>
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How to help: Encourage students to work slowly and carefully so they don’t accidentally ignore a negative sign. Instill the importance of using all of the negative signs in the computations and the solution.

Operation confusion | Students may confuse the rules for adding and subtracting negative numbers with those for multiplying and dividing.

How to help: This is one reason why it’s so important for students to understand what they are doing and not simply memorize rules. When students memorize rules, they tend to apply them inappropriately when they are rushing or not thinking through the problem. Encourage students to take the problems slowly and think about what the problem is asking and what operation(s) need to be used.

Real-world context | When negative numbers are used in real-world contexts, such as temperatures below zero or debts, students might have difficulty understanding how multiplying or dividing these numbers works.

How to help: Remind students that they should first focus on determining the steps for solving the word problem and writing an expression. After they solve the mathematics, they should check their answer (and the sign of their answer) to make sure it makes sense in the situation.

Misunderstanding Zero | Zero is an important number that has special properties when it comes to positive and negative numbers as well as multiplying and dividing. Students may misapply or forget these when they see a zero.

How to help: Talk to students about the importance of zero. Zero is the only number that is neither positive nor negative. When any number is multiplied by zero, the solution is always zero (a number repeated zero times is zero, as is zero repeated any number of times). When zero is divided by any number, the solution is always zero (zero divided into any number of pieces is still zero). When a number is divided by zero, the solution will always be undefined (it doesn’t make sense to divide a number into zero pieces).
# Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

## Lesson overview

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<th>Lesson</th>
<th>Objective</th>
<th>Teaching tips</th>
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| **Lesson 1: Multiplying with negatives**<br>TEKS standard: 6.3D | Students will be able to multiply positive and negative integers. | - **Warm up activity:** As a review of using integer chips from Unit 6, give problems where students add and subtract positive and negative numbers. They will continue to use integer chips in this unit.  
- The two videos explain why a negative number multiplied by a negative number is positive using two different numeric methods. If students are confused with these methods, demonstrate examples with integer chips. See "Best practices."
- The articles reiterate the methods used in the videos and offer visual representations with number lines and integer chips.  
- Encourage students to use one of the methods presented in the videos or articles and look for patterns to predict the sign of the answer. Don't show them any shortcuts yet! |
| ![Video](2).png | ![Article](2).png | ![Exercise](1).png |
| **Lesson 2: Dividing negative numbers**<br>TEKS standard: 6.3D | Students will be able to divide positive and negative integers. | - The shortcuts for multiplying and dividing positive and negative numbers will be presented in the video and article. Some students will pick up on these quickly and use them right away, others may need more support. Encourage those students to continue to use integer chips for solving the problems to build their conceptual understanding.  
| ![Video](1).png | ![Article](1).png | ![Exercise](1).png |

<table>
<thead>
<tr>
<th>Expression</th>
<th>Product</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive × positive</td>
<td>positive</td>
<td>$2 \times 3 = 6$</td>
</tr>
<tr>
<td>negative × negative</td>
<td>positive</td>
<td>$-2 \times (-3) = 6$</td>
</tr>
<tr>
<td>negative × positive</td>
<td>negative</td>
<td>$-2 \times 3 = -6$</td>
</tr>
<tr>
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<td>$2 \times (-3) = -6$</td>
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The next lesson focuses entirely on using the shortcuts so be sure students are comfortable using conceptual models before moving on.

Lesson 3: Multiplying and dividing negative numbers

TEKS standard: 6.3D

Students will be able to determine the sign of multiplication and division problems with positive and negative numbers.

- The video explains the shortcuts again and provides examples. Make a poster of the shortcuts so students can easily reference them. There are other visual ways to help students remember the shortcuts that you can share, like this:

<table>
<thead>
<tr>
<th>Sign of the first number</th>
<th>Sign of the second number</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
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- The second exercise may be the most challenging in the lesson. The task is simple—students only need to determine the sign of the answer. But the format given includes variables, number lines, and constraints that may be unfamiliar to students. Encourage them to choose numbers that could be possible for the variables and simplify to determine the sign for the solution.

Lesson 4: Multiplying and dividing negative numbers word problems

TEKS standard: 6.3D

Students will be able to solve word problems that use multiplication and division of positive and negative numbers.

- Students will get more practice multiplying and dividing positive and negative numbers with word problems. They will need to read each problem carefully to determine if they should use multiplication or division and remember to check if the solution should be positive or negative before submitting it.
Best practices

Visual examples for negative $\times$ negative = positive

The videos in the first lesson provide mathematical explanations for why two negative numbers multiplied together results in a positive number. Let’s look at a visual model with integer chips for another way to think about it.

We’ll begin by looking at a model for multiplying a positive number and a negative number. For general multiplication, we can think about the first number in the multiplication problem as the number of groups we have and the second number as the number of items in each group. Since order doesn’t matter when we multiply, either number could be considered the “first” or “second.” However, when multiplying a positive and negative number, it is easiest to think about it when the positive number is the number of groups and the negative number is the amount in each group.

For example, $6 \times (-8)$ is like saying “how many is 6 equal groups of -8?” A diagram with integer chips would look like this.

$$6 \times (-8) = -48$$

Now, let’s look at multiplying two negatives. This time, both numbers are negative, so we need to think about it a little differently. The first number is the number of groups we are going to take away and the second number is still the number of items in each group.

For example, $-3 \times (-7)$ is like saying “how much do we have after we take away 3 equal groups of -7?” How do we remove groups of objects when we don’t have any? Great question!

We can start with zero-pairs. The first diagram represents 0 because there is one negative integer chip for each positive integer chip.

$$-3 \times (-7) = 21$$
Dividing negative numbers
Division is one of the more challenging basic operations and incorporating negative numbers can make students feel even more confused. Provide ways to help contextualize division with negative numbers so they can think about and sketch diagrams to represent any given problem. Unlike multiplication, we cannot switch the numbers around—order matters in division. Let’s look at some examples.

Negative ÷ positive:  \[ -20 ÷ 5 = -4 \]
Suppose we split 20 negative integer chips into 5 equal groups. How much is in each group?

Each group has 4 negative integer chips, which represents -4.

Positive ÷ negative:  \[ 12 ÷ (-6) = -2 \]
For this problem, using the same wording doesn’t make sense: how can we split positive 12 into -6 groups? What does a negative group mean? Or, how can we split positive 12 into groups that sum to -6? This is getting tricky! We’ll need to think about it another way:

How many groups of -6 do we take away from 12 to equal 0?

We can add 2 groups of -6 to 12 to equal 0.
So we can take away -2 groups of -6.

Negative ÷ negative:  \[ -15 ÷ (-3) = 5 \]
How many groups of -3 does it take to equal -15?

Or, another way similar to above, suppose we split 15 negative integer chips into groups that total -3. How many groups would we need?

It takes 5 equal groups of -3 to equal -15.

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:
- [Weekly Khan Academy quick planning guide](#): Use this template to plan your week using Khan Academy.
- [Using Khan Academy in the classroom](#): Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- [Differentiation strategies for the classroom](#): Discover strategies to support the learning of all students.