Khan Academy

6th Grade Math

Unit 9: Equivalent expressions

6.7A, 6.7C, 6.7D

This unit provides students with essential skills and knowledge that form the basis for success in algebra. They promote critical thinking, problem-solving, and mathematical fluency—valuable assets inside and outside the classroom.

- ☐ Find the **greatest common factor** of two or more whole numbers
- ☐ Use the **distributive property** to simplify expressions
- ☐ Use the distributive property to **factor** out the greatest common factor
- ☐ Rewrite **subtraction as adding the opposite** and use the **commutative** and **associative** properties to write equivalent expressions

TEKS standards	Common misconceptions
6.7A: Generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization	**Confusing the greatest common factor and the least common multiple |** The greatest common factor (gcf) is the *largest factor* that all the numbers share. The least common multiple (lcm) is the *smallest multiple* that all of the numbers go into evenly. Even though this unit works with the gcf, students might get confused since they've learned about the lcm previously. **How to help:** Review both gcf and lcm so students are clear about the difference. Provide plenty of examples and include questions in the warm ups. For example, the *gcf*(5, 20) = 5 because 5 goes evenly into 20 and itself. The *lcm*(5, 20) = 20 because both 5 and 20 go evenly into 20. The gcf will be smaller than or equal to the given numbers while the lcm will be greater than or equal to the given numbers. **Incorrect application of the distributive property |** Students might apply the distributive property incorrectly by only distributing to one term inside the parentheses. **How to help:** Remind students to distribute to ***every*** term inside the parentheses. This is critical for keeping the value of the expression the same. Have students use arrows or arcs to keep track of the distributions to make sure they've completed them all. $5(3x + y - 2)$ $3(7x + 1)$
6.7C: Determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations	
6.7D: Generate equivalent expressions using the properties of operations: inverse, identity, commutative,	

associative, and distributive properties	**Overgeneralizing the commutative property |** The commutative property only applies to addition and multiplication so it is important that students only use it for those operations. The commutative property of addition tells us that we can add the terms in any order without changing the value. **How to help:** Show students examples of how the commutative property does and doesn't work with addition and subtraction. For example, $5 + 3 = 8$ and $3 + 5 = 8$ for addition. However, the same doesn't work for subtraction: $5 - 3 = 2$ and $3 - 5 = -2$. **Overgeneralizing the associative property |** Like the commutative property, the associative property applies only to addition and multiplication, not subtraction or division. The associative property of addition says that we can group terms in different ways without changing the value. **How to help:** Present the associative property and show examples of how it works with addition but not with subtraction. For example, $(9 + 4) + 2 = 15$ and $9 + (4 + 2) = 15$ for addition. However, the same doesn't work for subtraction: $(9 - 4) - 2 = 3$ and $9 - (4 - 2) = 7$. **Ignoring negative symbols |** Some students may think that negative symbols are just not that important - but they are! Negative symbols change the value of a number. **How to help:** Consistently remind students that negative numbers are different from their positive counterparts and they have different values. We must always pay close attention to negative symbols! Negative numbers can be tricky for students, but the more practice they get, the better! Using a number line or integer chips may be helpful tools for students to use as they did in Unit 3.

Unit resources

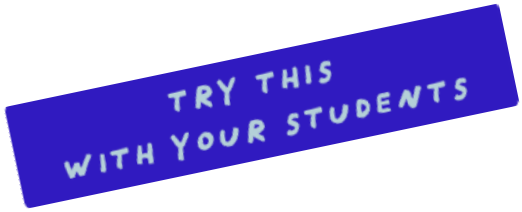
- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

Lesson	Objective	Teaching tips
Lesson 1: Greatest common factor TEKS standard: 6.7A Video 3 · Article 1 · Exercise 2	Students will be able to find the greatest common factor (divisor) of two or three whole numbers.	• **Warm up activity:** Have students find the factors of a variety of numbers. They may list them out or use factor trees. This is a central skill in this lesson. • The terms "greatest common factor" and "greatest common divisor" can be used interchangeably as they mean the same thing. • There are two strategies students can use to find the greatest common factor (gcf) of two or more numbers: lists and factor trees. At this point, students can use whichever method they prefer. Find the *gcf*(8,20) List: $8: 1, 2, 4, 8$ $20: 1, 2, 4, 5, 10, 20$ The gcf is 4 because 4 is the largest factor of both 8 and 20. Factor tree: 8 → 4, 2 → 2, 2 $8 = 2 \cdot 2 \cdot 2$ 20 → 10, 2 → 5, 2 $20 = 2 \cdot 2 \cdot 5$ Both numbers, when written as their prime factorizations, have two 2s. Thus, since $2 \cdot 2 = 4$, 4 is the GCF of 8 and 20. • Review the idea that two numbers are *relatively prime* if they have no common factors besides 1. For example, 5 and 17 are relatively prime because they have no common factors besides 1.

		• The second exercise asks students to do the same thing they did in the previous exercise, find the greatest common factor of two numbers, but the answer should be written in factored form. Using the example from above, apply the distributive property to factor out the greatest common factor. $8 + 20$ We already know the gcf is 4 $4(2 + 5)$ The numbers in parentheses do not have any common factors other than 1, so we know this is fully simplified.
Lesson 2: Prime factorization TEKS standard: 6.7A Video 2 Article 0 Exercise 1	Students will be able to find the prime factorization of a given whole number.	• Creating factor trees will be the easiest way to solve these problems. • It will be helpful for students to know divisibility rules through 10 as they find factors for their trees. Review these with students to help them find factors more easily. A multiplication table is a helpful support for students who aren't yet fluent with multiplication facts.
Lesson 3: Distributive property TEKS standard: 6.7D Video 3 Article 0 Exercise 3	Students will be able to use the distributive property to simplify expressions. Students will be able to use the distributive property to factor out the greatest common factor.	• This may be the first time students are seeing the distributive property, but it certainly won't be their last. This lesson introduces students to the basics of the distributive property and gives them practice distributing and also factoring. • Review with students that a number in front of parentheses indicates multiplication. $6(2) = 6 \cdot 2 = 6 \times 2 = 12$ • See "Best practices" for more on the distributive property and factoring.
Lesson 4: Properties of addition and subtraction TEKS standard: 6.7D Video 2 Article 0 Exercise 3	Students will be able to rewrite expressions with subtraction as adding a negative number. Students will be able to use the associative and commutative properties to simplify expressions.	• **Warm up activity:** Give students practice adding and subtracting negative numbers. They will be working with negative numbers heavily in this lesson, so make sure to provide a refresher. • Students will build their fluency and flexibility with simplifying expressions in this lesson. They will use the commutative and associative properties to group numbers to make zero pairs and perform operations more easily. Help students to see

		patterns for efficiency. $-3-8+6+3+18$ — Rewrite all cases of subtraction as addition of the opposite $-3+(-8)+6+3+18$ $-3+3+(-8)+18+6$ — Commute related values together $(-3+3)+((-8)+18)+6$ — Associate the related values so we can evaluate them first $0+10+6$ 16
Lesson 5: Equivalent expression TEKS standard: 6.7C Video 1 · Article 0 · Exercise 1	Students will be able to write models as expressions.	• In this lesson, students will be given a set of images that represent values or variables. They will use the images to write expressions using the values they represent. This is their first introduction to substitution, which will be a foundation of algebra moving forward. Write an expression equivalent to the model: ▮ = 1 ▲ = n ▯ = -1 △ = -n ▮▮△▯▮▲△ $1+1+(-n)+(-1)+1+n+(-n)$ — Substitute numbers for symbols $1+1+1+(-1)+n+(-n)+(-n)$ — Reorganize with like terms together $2+(-n)=2-n$ — Simplify



Best practices

The distributive property
Using the distributive property is a foundational algebra skill. Let's look at it in more depth as we see how it relates to factoring.

The distributive property is used when we want to multiply a sum or difference by a number. In situations where we don't have variables, we get the same solution by using *either* the order of operations or the distributive property and can simplify using either method.

Order of operations:

$$3(2+4)$$
$$3(6)$$
$$18$$

Distributive property:

$$3(2+4)$$
$$3\cdot 2+3\cdot 4$$
$$6+12$$
$$18$$

The distributive property shows us how to "distribute" the number being multiplied to all of the terms inside parentheses. It's particularly useful when we have variables within parentheses that cannot be combined, as the order of operations would require us to do. Here's how we can use the distributive property to rewrite some expressions:

The general case is:

$$a(b+c)$$
$$ab+ac$$

$$5(3x-2)$$
$$5(3x)-5(2)$$
$$15x-10$$

$$2(6+y-3z)$$
$$2\cdot 6+2\cdot y-2\cdot 3z$$
$$12+2y-6z$$

Note that either parentheses or a dot (·) can be used to represent multiplication.

Factoring - Reversing the distributive property
Factoring is like the distributive property but in reverse. It can "undo" the distributive property. When we factor, we find the greatest common factor (GCF) of *all* of the terms, and then we divide that number out of *each* term.

$$10-15x$$
$$5\cdot 2-5\cdot 3x$$
$$5(2-3x)$$

← In this problem, 5 is the GCF of 10 and 15. We rewrite both terms as multiplication by 5 and then "factor out the 5" by writing the 5 as multiplication in front and put the terms in parentheses. If we apply the distributive property to the answer, the result will be the original expression. →

$$5(2-3x)$$
$$5\cdot 2-5\cdot 3x$$
$$10-15x$$

Here are a few more examples. Remember that we can check our work using the distributive property to make sure we get back the original expression.

$$18a-6$$
$$6(3a)-6(1)$$
$$6(3a-1)$$

The GCF of 18a, and -6 is 6. We will "factor out" a 6.

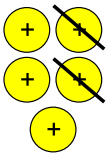
$$32x+40y$$
$$8(4x)+8(5y)$$
$$8(4x+5y)$$

The GCF of 32x and 40y is 8. We will "factor out" an 8.

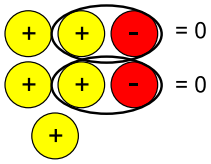
Subtraction: Adding the opposite
In Lesson 4, students will learn to rewrite subtraction as addition of the opposite of the number. This is useful so students can rewrite expressions with addition only, which can make them easier to simplify and allows the use of the commutative and associative properties. Fluency with changing subtraction into addition of the opposite is an important building block for more advanced math and numeracy in general.

Let's look at an example with integer chips. You can see that we get the same answer both ways.

$5 - 2 = 3$



$5 + (-2) = 3$



We will also explore writing **equivalent expressions** with addition and subtraction with parentheses. Some of the problems will also include the distributive, associative, and/or commutative properties. For example, all of the expressions in each set below are equivalent.

$$-14 = 3 - 5 - 12$$
$$-14 = 3 + (-5) + (-12)$$
$$-14 = (-12) + (-5) + 3$$
$$-14 = 3 + ((-5) + (-12))$$
$$-14 = 3 - (5 + 12)$$

$$2 = 8 - 10 + 4$$
$$2 = 8 + (-10) + 4$$
$$2 = 8 + 4 + (-10)$$
$$2 = 8 - 10 - (-4)$$
$$2 = 8 - (10 - 4)$$

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.