# 7th Grade Math

## Unit 2: One-variable equations and inequalities

7.4D, 7.10A, 7.10B, 7.10C, 7.11A, 7.11B, 7.13D, 7.13E, 7.13F

Equations and inequalities can help us with everyday problem-solving, like managing time and money, and making decisions.

- Solve two-step equations and inequalities using algebra tiles and algebraically with fractions and decimals
- Calculate percent discounts, markups, and commission
- Calculate simple and compound interest

### TEKS standards vs. Common misconceptions

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4D: Solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems</td>
<td>Misunderstanding when to use the operation shown vs the inverse operation</td>
</tr>
</tbody>
</table>
| 7.10A: Write one-variable, two-step equations and inequalities to represent constraints or conditions within problems | Forgetting to do the same thing to both sides of the equation/inequality | When solving equations and inequalities, it’s crucial to maintain balance and equality. It’s easy to forget to perform the same operation on both sides of the equation. **How to help:** Remind students of the hanger diagrams they used in 6th grade, where both sides must remain balanced to be equal. Whatever we do to one side must also be done to the other side. Give plenty of practice to reinforce fluency and have students show their work.
### Misinterpreting fractions

Fractions can be a tricky concept for students, especially with variables. They might have difficulty understanding that \( \frac{3}{4} \cdot b \) means \( \frac{3}{4} \) times \( b \). Students may similarly struggle with decimals and/or negative numbers.

**How to help:** Encourage students to work slowly and not panic when they see fractions (or decimals or negative numbers). The same rules apply as with positive integers! The only thing to remember is to flip the sign when multiplying or dividing by a negative number in an inequality. As usual, practice, practice, practice!

### Confusing the inequality symbols

There may be confusion between all of the inequality symbols. Students may not only confuse which way they open, but also whether they can be equal or not.

**How to help:** Spend time going over each symbol with students, especially spending time on the difference between less/greater than and less/greater than or equal to. Provide examples for each where they may see them in daily life. For example, if Matty wants to get at least an 90% on his next quiz \( q \), we could write that as \( q \geq 90 \) because even if he earned a 90% \( (q = 90) \) he would still be happy. However, if Matty wants to get higher than 90%, \( q > 90 \), then he would not be happy with a 90% because it is not in his goal range.

Another challenge is rewriting inequalities with the values on the opposite side: if \( 7 > x \) then \( x < 7 \). Students can convince themselves using concrete numbers, like if \( 7 > 5 \) then \( 5 < 7 \). Practice is crucial!

Make a poster to display the symbols and their meanings for students to reference. Include helpful reminders, like the alligator or PacMan—in both cases, it “eats” the bigger number.

### Forgetting to flip

This is a biggie! When we multiply or divide both sides of an inequality by a negative number, we have to flip the inequality symbol.

**How to help:** Show some examples for why we need to flip the symbol that use numbers as opposed to variables. Reiterate that to keep the statement true, the inequality must flip. Give lots of practice!

### Confusion between percent decrease (or increase) and decrease (or increase) by a certain amount

Some students might confuse “20% off” (a percent decrease) with “off by $20” (a decrease by a certain amount).

**How to help:** With percent decrease, the amount of decrease changes depending on the initial amount. Students must read problems carefully to determine if they’re working with percent decrease/increase or decrease/increase of a certain amount, as that changes how to think about and solve the problem. Do examples together for each type.
Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objective</th>
<th>Teaching tips</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson 1: Solutions to equations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TEKS standard: 7.11B</td>
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<tr>
<td>Students will be able to use substitution to determine whether a given number is a solution to an equation.</td>
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<tr>
<td><strong>Warm up activity:</strong> Give students problems where they simplify expressions using the order of operations. Include problems with fractions, variables, and parentheses. For example: ( \frac{4x - 8}{2} = \frac{4(2 + 3) - 8}{2} )</td>
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<tr>
<td><strong>Warm up activity:</strong> Review how to determine if a value is a solution of an equation by substituting and simplifying. Students may need a reminder of what it means for a value to be a solution of an equation and the order of operations (PEMDAS). Do examples together.</td>
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<p>| <strong>Lesson 2: Two-step equation intro</strong> |
| TEKS standard: 7.10B, 7.11A |
| Students will be able to solve two-step equations with a visual model (algebra tiles) and algebraically. Students will be able to graph solutions to two-step equations on a number line. |
| <strong>Warm up activity:</strong> Have students graph positive and negative numbers on a number line, including fractions and decimals. |
| <strong>Warm up activity:</strong> Students work with virtual algebra tiles in this lesson and in Lesson 6. If you have algebra tiles in your classroom, make them available for students to use as they work, or print them yourself! Algebra tiles provide a concrete model for students to build their conceptual understanding—don’t skip this foundational step! See &quot;Best practices&quot; for more on working with algebra tiles. |
| <strong>Warm up activity:</strong> In 6th grade math, students used hanger diagrams to model solving one-step equations. Hanger diagrams and algebra tiles similarly use shapes to represent variables and constants, but hanger diagrams had a visual reminder to keep both sides balanced where algebra tiles do not. Show how hanger diagrams and algebra tiles are related so students see connections between the two models. |</p>
<table>
<thead>
<tr>
<th>Lesson 3: Two-step equations with decimals and fractions</th>
<th>Students will be able to solve two-step equations with fractions and decimals.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEKS standard: 7.11A</td>
<td><strong>Warm up activity:</strong> Have students simplify expressions with fractions and decimals that include addition, subtraction, multiplication, and division. For example: $5 - 1.34$, $6 \div 0.25$, $\frac{3}{5} \cdot 10$, $\frac{2}{3} + \frac{1}{6}$</td>
</tr>
<tr>
<td>Video</td>
<td>Article</td>
</tr>
</tbody>
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This lesson may be challenging for two particular reasons. First, students solve equations algebraically, without a visual model. If you have students who aren’t ready for this step, they may continue to model with algebra tiles on their own, but many of the problems here have fractions and decimals, which are difficult to represent with algebra tiles. Second, the equations include fractions and decimals, which may be challenging for some students. Remind them that they solve equations with the same steps for fractions and decimals as they do for whole numbers!

**Do examples together where you show each step for solving. Spend time reviewing operations with fractions and decimals as necessary.**

<table>
<thead>
<tr>
<th>Lesson 4: Two-step equation word problems</th>
<th>Students will be able to write equations to represent situations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEKS standard: 7.10A, 7.11A</td>
<td><strong>Encourage students to read the problems slowly and carefully, making sure they understand the situation and what the question is asking of them. Model careful reading of problems and annotating using the drawing tool in the exercise. It may be helpful for students to draw pictures to represent the problem situations.</strong></td>
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<tr>
<td>Video</td>
<td>Article</td>
</tr>
</tbody>
</table>

The drawing tool is in the lower left corner of the screen during exercises.

<table>
<thead>
<tr>
<th>Lesson 5: Solutions to inequalities</th>
<th>Students will be able to determine whether a number is a solution to an inequality.</th>
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<tbody>
<tr>
<td>TEKS standard: 7.11B</td>
<td><strong>Warm up activity:</strong> To refresh inequalities, give students pairs of numbers and ask them to fill in the correct symbol (&lt;, &gt;, =).</td>
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<tr>
<td>Video</td>
<td>Article</td>
</tr>
</tbody>
</table>

$5 \ ? 9 \rightarrow 5 \ < \ 9$
Students worked with inequalities back in 6th grade, and they may need a reminder here.

- Review compound inequalities with students. They appear in the exercises as two separate inequalities, like “15 > x and x > 8.” Go over a few together so students understand what they mean.

### Lesson 6: Two-step inequalities

**TEKS standard:** 7.10A, 7.10B, 7.11A

- Students will be able to solve two-step inequalities using algebra tiles and algebraically.
- Students will be able to graph solutions to two-step inequalities.

**Warm up activity:** Have students graph inequalities on a number line. For example:

- We return to algebra tiles, now for solving two-step inequalities. The first two exercises use the visual models and the last exercise focuses on solving algebraically.
- As you review the steps for solving two-step inequalities, remind students that they must flip the sign when they multiply or divide by a negative number. See the 6th grade math Unit 11 Unit guide for an explanation.

### Lesson 7: Two-step inequalities and equations word problems

**TEKS standard:** 7.10A, 7.10C, 7.11A

- Students will be able to match given equations or inequalities with situations they represent and answer questions about them.

**We return to word problems, now with both inequalities and equations. Students need to understand each context and also determine if the situation is best represented as an equation or inequality.**

- Encourage students to continue to use annotation methods with the drawing tool and to sketch problem situations as is helpful.

### Lesson 8: Discounts

**TEKS standard:** 7.4D, 7.13F

- Students will be able to calculate percent discounts, markups, and commission.

**Warm up activity:** Have students calculate percentages. For example: What is 40% of 80?

- Review vocabulary that is used in this lesson, like tax, tip, discount, sale, markup, and commission. Students’ experiences with these words are varied, so explain each one and give examples.

- Do examples together so students can see how to solve different types of problems. They will need to read the problems carefully to figure out what each is asking for.
<table>
<thead>
<tr>
<th>Lesson 9: Scaling a budget</th>
<th>Students will be able to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEKS standard: 7.13D</td>
<td>● Have a discussion with students about what a household budget is and what items might go into a monthly budget.</td>
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<td>● The video uses an external website, the Economic Policy Institute's <strong>Family Budget Calculator</strong>. Students can explore the website, investigate their location, and use the budget calculator.</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Lesson 10: Simple and compound interest</th>
<th>Students will be able to calculate simple and compound interest.</th>
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<tbody>
<tr>
<td>TEKS standard: 7.13E</td>
<td>● <strong>Warm up activity:</strong> Have students solve or write in expanded form simple exponent problems to remind them of exponents and what they mean. For example, have them write $4^5$ as $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ or give them expanded form and have them write it with an exponent.</td>
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<tr>
<td></td>
<td>● Review exponent vocabulary and do examples together. It’s important for students to see the expanded form step before multiplying.</td>
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<tr>
<td></td>
<td>● Discuss the difference between simple and compound interest and review the formulas for each. Students may not be familiar with interest, so give examples of where they might encounter it. The article provides an explanation of compound interest with examples.</td>
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<td></td>
<td>Simple interest (amount of interest): $I = Prt$</td>
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</tbody>
</table>
|                                        | $I$ is the amount of interest  
$P$ is the principal, which is the original amount deposited  
$r$ is the interest rate expressed as a decimal  
$t$ is the time in years |
|                                        | Compound interest (total principal with interest): $A = P(1 + r)^t$ |
|                                        | $A$ is the accumulated amount of money  
$P$ is the principal, which is the the original amount deposited  
$r$ is the interest rate expressed as a decimal  
$t$ is the time time (or number of periods) |
Best practices

Modeling with algebra tiles
Algebra tiles provide a visual model for equations and inequalities with variables. Many students will benefit from having this model when they solve two-step equations and inequalities. If you have algebra tiles, make them available for students to use as they solve the problems. If not, students can draw them or you can print them on cardstock and cut them out. We work with two types of algebra tiles in this course, variables and units. Generally, yellow represents positive quantities and red represents negative quantities.

A single unit is represented by a yellow square. Its side lengths are both 1 unit and its area is 1 unit$^2$.

Negative numbers are represented by red tiles.

A variable is represented by a rectangle that is 1 unit wide and $x$ units long, so the area is $x$ units$^2$.

Let’s do an example to see how algebra tiles can be helpful when solving a two-step equation.

Solve $3x - 4 = 11$. Here is a model of this equation with algebra tiles.

Note that students may try to estimate the height of an $x$-tile with the unit tiles and think that is the value of $x$, but remind them that the tiles represent an unknown quantity, and that quantity will be different in different problems. This is a model.

We can get the $x$-term alone by “undoing” the -4. Let’s add 4 to both sides of the equation to keep it equal. Each 1 and -1 pair forms a zero pair. We’re left with a shorter equation.

Since we have 3 $x$-tiles, we can make 3 groups from the tiles on the other side of the equation.

Each row shows that 1 $x$-tile represents 5 unit tiles.

Let’s check our solution!

$3\cdot(5) - 4 \stackrel{?}{=} 11$

$15 - 4 \stackrel{?}{=} 11$

$11 = 11$
Solving two-step equations algebraically
When solving two-step equations, students will have to recognize which operations they need in order to isolate the variable. An inverse operation is an operation that **undoes** another operation. If you take a step forward, the inverse operation would be taking a step back. You end up in the same place, right?

To solve an equation, we work backwards until we get the variable alone. For example, if the equation **adds** a value to the variable, we **subtract** that variable from both sides. When we’re done, we know the value of the variable.

Let’s solve $2x + 7 = -9$ algebraically.

**Planning how to solve**
The variable $x$ is only on one side of the equation. Let’s look at that side. In $2x + 7$, here’s what happens to $x$ according to the order of operations:

- Multiply by 2
- Add 7

We’ll work **backwards** to solve the equation:

- Subtract 7
- Divide by 2

**Solving the equation**
Each time we change the value of one side of the equation, we change the value of the other side in the same way. This keeps the sides equal.

\[
\begin{align*}
2x + 7 &= -9 \\
-7 &= -7
\end{align*}
\]

Subtract 7 from both sides

\[
\begin{align*}
x + \frac{16}{2} &= -8 \\
\frac{16}{2} &= \frac{16}{2}
\end{align*}
\]

Divide both sides by 2

Lastly, let’s check our solution!

\[
\begin{align*}
2(-8) + 7 &= -9 \\
-16 + 7 &= -9 \\
-9 &= -9
\end{align*}
\]

**GENERAL CLASSROOM IMPLEMENTATION RESOURCES:**

- **Weekly Khan Academy quick planning guide:** Use this template to plan your week using Khan Academy.
- **Using Khan Academy in the classroom:** Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- **Differentiation strategies for the classroom:** Discover strategies to support the learning of all students.