Unit 6: Similarity
7.3A, 7.5C

Mastery of similar shapes and their properties is essential in fields ranging from engineering and architecture to computer graphics, art, and navigation.

- Identify corresponding parts in similar figures
- Write a proportion to show the relationships between sides of similar figures
- Determine whether two shapes are proportional
- Find the missing side of a triangle when given a pair of similar triangles by setting up a proportion

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5A: Generalize the critical attributes of similarity, including ratios within and between similar shapes</td>
<td>Difficulty identifying corresponding parts</td>
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<tr>
<td>7.5C: Solve mathematical and real-world problems involving similar shape and scale drawings</td>
<td>Misunderstanding scaling</td>
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<td></td>
<td>How to help: Do problems together where students determine that two figures are similar and then have them compare the lengths of each of the sides. They should see that every single side has the same ratio and the shape is still the same. If even only one side has a different ratio then the figures are NOT similar.</td>
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<td>“Scale factors must be whole numbers”</td>
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will get larger, but when a scale factor is a fraction less than 1, it will get smaller.

**How to help:** Provide problems to do together when a scale factor is a fraction less than 1. When you multiply each side length by a fraction, it will get smaller and the resulting shape will be smaller. Students will solve the problems in the same way they did with whole numbers, so they don’t need to do anything differently.

**Confusion with ‘size’ |** Students may be tempted to say that a scale factor of 2 will double the size, but it actually means each dimension will double in size, which affects the area differently.

**How to help:** Discuss with students what ‘size’ means when we solve problems with scale factors. There may be lots of different ideas! Try to come to a definition together and relate it to their work with scale factors so far. It may seem like a small distinction to make, but how we talk about scale factor and size is important. Do examples together where students scale up or down a shape and then compare the areas to look for patterns.

**“All triangles are similar” |** Some students might think that all triangles are similar. This is generally not the case because triangles come in all different proportions! The only case where this is true is for equilateral triangles, where the angles are all 60°. Even though right triangles always have one right angle, the other two angles can be different, so they are not always similar. Triangles are only similar if their corresponding angles are equal and their corresponding sides are proportional.

**How to help:** The best way to tackle this is with counterexamples. Show students two triangles that appear similar but aren’t and have them calculate if the sides are proportional. When they see multiple examples of triangles that are and are not similar, they will understand that we always need to actually do the calculations.

**“All images are to scale” |** Sometimes, students might assume that all drawings are to scale, which can lead to incorrect conclusions. Not every drawing is to scale.

**How to help:** Remind students that we determine similarity based on specific criteria, not just by eyeballing it. Just because they look similar doesn't mean they are. They still need to make sure that all of the sides are proportional to determine similarity.
Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objective</th>
<th>Teaching tips</th>
</tr>
</thead>
</table>
| Lesson 1: Scale copies  | Students will be able to identify corresponding parts in given scaled figures. Students will be able to determine whether one figure is a scaled copy of another figure. | ● Review how geometric diagrams are labeled. Vertices get a capital letter and we name sides as segments using those letters, for example, we could name side $AB$ in the diagram below.  

![Diagram](https://via.placeholder.com/150)  

- The first part of the lesson focuses on identifying corresponding parts of scaled copies. Corresponding parts have the same relative position. For example, if we have a diagram and its scaled copy, the longest sides of both figures will correspond, as will the shortest sides, as will the largest angles, and so forth.  

- When students determine whether figures are scaled copies of each other, they are looking to see if the corresponding sides all have the same scale.  

If we divide the length of each side of Figure A by 3, we get the length of each side of Figure B, so they are scaled copies. |
| Lesson 2: Scale factor  | Students will be able to determine if two shapes are scaled copies. Students will be able to find the scale factor of two scaled | ● Warm up activity: Have students solve proportions with missing values (from Unit 4). They will use the same skills when finding the missing side of a scaled figure.  

- When students find the scale factor from one |
<table>
<thead>
<tr>
<th>Lesson 3: Scale drawings</th>
<th>Students will be able to create scale drawings.</th>
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<tbody>
<tr>
<td>TEKS standard: 7.5C</td>
<td>Students will see scaled drawings used in real life situations in this unit. They will use information about an object and its scaled drawing to find the length of missing sides and also make their own drawings.</td>
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<td>This is a great lesson to have students make their own scale drawings. They can make a scale drawing of the classroom, another room in the school, or a room at their home.</td>
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<thead>
<tr>
<th>Lesson 4: Similar figures</th>
<th>Students will be able to identify corresponding parts of similar figures. Students will be able to write a proportion to show the relationships between sides of similar figures.</th>
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<tbody>
<tr>
<td>TEKS standard: 7.5A</td>
<td>Students will use their knowledge of corresponding parts to write proportions to show the relationships of corresponding sides of figures when the figures have been rotated or reflected. Encourage them to use colored pencils to mark corresponding sides on both figures.</td>
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<td>The exercise will use angle notation, so be sure to review that! See “Best practices” for more on notation.</td>
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<tr>
<th>Lesson 5: Similar triangles</th>
<th>Students will be able to determine whether a pair of triangles is similar. Students will be able to find the missing side of a triangle when given a pair of similar triangles by setting up a proportion.</th>
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<tbody>
<tr>
<td>TEKS standard: 7.5A, 7.5C</td>
<td>The videos use notation that may be new to students. Review each symbol and what it means. Make a poster for easy reference. See “Best practices” for more on notation.</td>
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<td>Students will be expected to write both within and between proportions when comparing two shapes.</td>
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Within proportions compare two sides of one triangle with the two corresponding sides of another triangle.

For example:

\[
\frac{AB}{AC} = \frac{XY}{XZ} \quad \text{or} \quad \frac{CB}{AC} = \frac{ZY}{XZ}
\]

Between proportions compare two corresponding sides (one from each triangle) to two other corresponding sides.

For example:

\[
\frac{AB}{XY} = \frac{AC}{XZ} \quad \text{or} \quad \frac{CB}{ZY} = \frac{AC}{XZ}
\]

- Introduce side-side-side (SSS) similarity, which states that the sides of similar triangles are always proportional (relate by the same ratio). It will be important for students to first identify which sides correspond to each other (for example, the longest side in one triangle corresponds to the longest side in the other triangle).

For example, to determine whether \( \triangle ABC \sim \triangle XYZ \), we need to determine whether each pair of corresponding sides relate by the same ratio. We will set up between ratios because they directly relate corresponding sides.

\[
\begin{align*}
\frac{AB}{XY} &= \frac{AC}{XZ} = \frac{BC}{YZ} \\
\frac{6}{3} &= \frac{10}{5} = \frac{8}{4} \\
2 &= 2 = 2
\end{align*}
\]

Yes, \( \triangle ABC \sim \triangle XYZ \) since the ratios are the same.
### Best practices

#### A note on notation

This unit uses some notation that may be unfamiliar to students. Be sure to review it with them so they are able to accurately interpret the questions and images.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Example/Explanation</th>
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<tbody>
<tr>
<td>(\overline{XY})</td>
<td>Line segment, in this case, line segment (XY)</td>
<td>To <strong>name</strong> the side of a figure, which is a line segment, we use the line segment notation and put a line over the letters and denote that side, (\overline{XY}). When we talk about the <strong>measure</strong> (or length) of the side of a figure, we don't put the line over the letters, (XY). It may be confusing for students to sometimes see the line and sometimes not, so be sure to explain the difference.</td>
</tr>
</tbody>
</table>
| \(\angle\) | Angle | The highlighted angle can be named with any of these:

\[
\angle ABC \\
\angle CBA \\
\angle B
\]

Note that the angle we are naming is always in the **middle** of the 3-letter conventions. Think about them as telling you where to start and end. For example, for \(\angle ABC\), start at vertex A, then go to vertex B, then vertex C. The angle that is highlighted when you take that path is \(\angle B\). |
| \(\triangle\) | Triangle | There are many ways to name this triangle. Pick a vertex to start and move either clockwise or counterclockwise and name the vertices in order. Some possible names: \(\triangle ABC\), \(\triangle BCA\), \(\triangle CBA\). It’s important to note that when naming two triangles that are similar, the corresponding vertices must be named in the same order!

For example, \(\triangle BAC \sim \triangle YXZ\) because \(\angle B \cong \angle Y\), \(\angle A \cong \angle X\), \(\angle C \cong \angle Z\), but \(\triangle BAC \sim \triangle ZXY\) because \(\angle B \not\cong \angle Z\), and \(\angle C \not\cong \angle Y\). |
| \(\sim\) | Similar - the figures are scaled copies | \(\triangle ABC \sim \triangle XYZ\)

\[
\frac{CB}{ZY} = \frac{CA}{ZX} = \frac{BA}{YX}
\]

Since the ratios are the same for each side, the two triangles are similar. |
Congruent - the figures are exactly the same

\[ \triangle ABC \cong \triangle XYZ \]

The lengths of all of the sides of the triangles are the same, as are the angle measures, so the two triangles are congruent.

CLASSROOM ACTIVITIES

Scale an object
Have students choose an object that is measurable (with a ruler or tape measure) or a room to create a scale drawing of. If they choose an object, it should be something that is not too complicated to draw. For example, they could draw a pencil or paperclip that is scaled bigger by a scale factor of their choice. Or, they could draw a notebook or chair that is scaled down by a scale factor of their choice. If they choose a room, they could make a scaled drawing of the floor plan, like a blueprint.

Scale a picture
Students can choose a simple picture or company/organization logo to scale up or down. A simple one-frame comic can work, too. Try not to make it too complicated as the emphasis should be on the scaling part of the activity, not necessarily the perfection/beauty of the result. For example, if a student has a logo on their shirt, they could measure the dimensions with a ruler and scale it up or down depending on how large it is on their clothing.

Examine real-world examples of scaling
Bring, show pictures of, and/or discuss scaling that students see in everyday life. For example, printing a picture on a large poster or billboard, creating an architectural blueprint, or building a model airplane (or other figure). Students can look for examples of scaled items and bring them in or take pictures of them to share with the class.

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.