Khan Academy

Algebra 1

Unit 1: Linear expressions, equations, and inequalities

A.5A, A.5B, A.10A, A.10C, A.10D, A.12E

The algebra essentials in this unit will empower your students with tools and techniques they'll use throughout the rest of their journey in mathematics!

- ☐ Simplify expressions and solve equations using the **distributive property**
- ☐ Simply expressions and solve equations by **combining like terms**
- ☐ Solve **multi-step equations and inequalities**
- ☐ Add, subtract, and divide **polynomials** of degree one
- ☐ **Solve formulas** and equations for specific variables

TEKS standards	Common misconceptions
A.5A: Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	**Confusion with subtraction and negatives when distributing |** Students may apply the distributive property to addition but neglect to pay attention to subtraction and negative numbers, which results in dropping negatives. **How to help:** Provide diverse examples that demonstrate the distributive property's application to both addition and subtraction. Students may need to review multiplication with negative numbers. Make sure students have plenty of practice! **Not distributing a coefficient of -1 |** It may be confusing at first for students to distribute when there doesn't appear to be a coefficient, especially with subtraction, like this $8 - (x + 5 - y)$. **How to help:** Get students in the habit of writing a coefficient of 1 when there isn't a given coefficient . Use the problem above to model the step-by-step application of the distributive property: $8 - 1(x + 5 - y)$ $8 - 1(x + 5 - y) = 8 + (-1)(x) + (-1)(5) + (-1)(-y)$ $= 8 + (-x) + (-5) + y$ $= 3 - x + y$
A.5B: Solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	

A.10A: Add and subtract polynomials of degree one and degree two

Degree two polynomials are covered in Unit 6.

A.10C: Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend

Degree two polynomials are covered in Unit 6

A.10D: Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property

Degree two polynomials are covered in Unit 6

A.12E: Solve mathematic and scientific formulas, and other literal equations, for a specified variable

When to combine like terms when solving equations with variables on both sides | There are two common mistakes that you might see here, both using what they have learned but in incorrect ways. First, when given an equation with variables on both sides, students may ignore the equal sign and combine like terms across the entire equation $3 + 4x = 5 - 2x \rightarrow 8 + 2x$ (this is **not** correct). Second, they may use "balanced" operations even when an equal sign is not present. See the first step in attempting to solve the equation below, where instead of combining like terms, they subtract as if there were an equal sign between the terms.

$$\begin{aligned} 2x + 6 + 5x &= 9 - 3x \\ -2x \quad\quad -2x & \\ 6 + 3x &= 9 - 3x \end{aligned}$$

This is **not** correct ✖

How to help: Be sure to model problems for students and give them frequent feedback. Have students draw a vertical line through the equal sign that extends an inch or two through where their work will be to differentiate between combining like terms (staying on one side of the line) and performing balanced algebraic operations (crossing the line). Practice is key!

$$\begin{aligned} 7x - 2 &= 9 + 6x \\ -6x & \quad\quad -6x \\ x - 2 &= 9 \\ +2 & \quad\quad +2 \\ x &= 11 \end{aligned}$$

"Too many variables!" | Students might struggle with the concept of isolating a specific variable in a formula. They may see many variables with few numbers and believe it's not possible to "solve". They will want nice, clean answers, but rearranging for a specific variable doesn't always make it look simpler.

How to help: Break down the process of isolating a variable by applying inverse operations, factoring, and simplifying the formula when possible. Students should view these problems more as ***rearranging*** the variables rather than solving. They already know how to do all of the algebra—it just looks a little different!

Solve for ***b*** in $A = \frac{1}{2}bh$, the formula to find the height of a triangle.

$$A = \frac{1}{2}bh$$

$$2 \cdot A = \frac{1}{2}bh \cdot 2$$ Multiply both sides by 2 (same as dividing by ½)

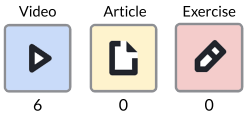
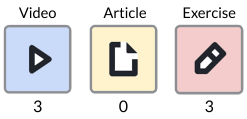
$$\frac{2A}{h} = \frac{bh}{h}$$ Divide both sides by *h*

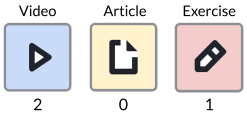
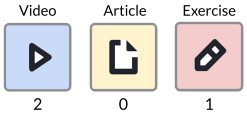
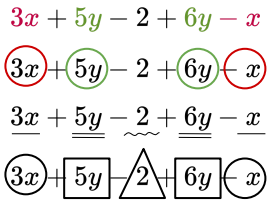
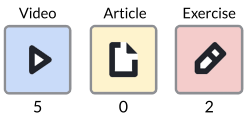
$$\frac{2A}{h} = b$$

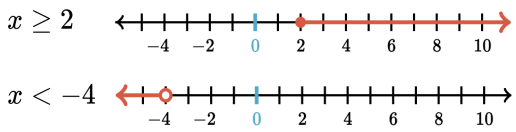
Unit resources

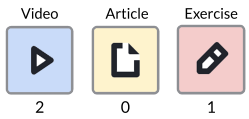
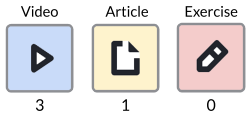
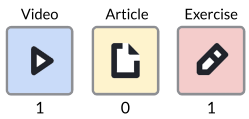
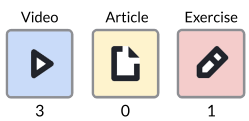
- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

Lesson	Objective	Teaching tips
Lesson 1: Overview and history of algebra 	Students will be able to explain the origins and history of algebra and why it is important.	• Students often ask questions related to the topics explained in these engaging videos! The history of algebra and the coordinate plane; the meaning of "abstract"; why algebra is beautiful, useful, and requires creativity; why we use letters in algebra. Enjoy!
Lesson 2: Distributive property with variables TEKS standard: A.10D 	Students will be able to use the distributive property to write equivalent expressions.	• **Warm up activity**: Students should be familiar with the distributive property from previous years. Give them some simple distribution problems like this: $5(2 + 3)$, with both addition and subtraction, to see how they solve them (do they simplify what is in parentheses first or use the distributive property?). Include some problems with a variable to see what they do, like this: $4(2x - 1)$. This will help you to know how much support students will need in this lesson. • Students may need review of distributing negative numbers, fractions, and decimals in both binomials and trinomials. Do examples together and model showing each step. • Students will be introduced to factoring (using the distributive property backwards—undistributing!). Review prime factorization and how to find the greatest common factor.
Lesson 3: The distributive property & equivalent expressions	Students will be able to use the distributive property to write equivalent expressions.	• Students will continue to practice with the distributive property and factoring in this lesson. • Go over with students the many notations that can

TEKS standard: A.10D 		indicate to use the distributive property. For example, all of these expressions would use the distributive property and simplify to the same answer: $5(3a-1+b)$ $(3a-1+b)5$ $5\times(3a-1+b)$ $5\cdot(3a-1+b)$ • This is a good place for teacher led groups since there are so many places students might make mistakes.
Lesson 4: Combining like terms TEKS standard: A.10D 	Students will be able to simplify expressions using the distributive property and combining like terms.	• Combining like terms may seem simple, but it can be very difficult for some students to grasp at first. Do examples using classroom items that can be combined and not combined (5 paperclips + 7 paperclips = 12 paper clips, but 3 erasers + 1 stapler = 3 erasers and 1 stapler (they can't be combined)). Also encourage students to use colored pencils or different underlining techniques to mark like terms. $3x+5y-2+6y-x$  • Encourage students to show each step of their work so mistakes can be more easily found.
Lesson 5: Linear equations with variables on both sides TEKS standard: A.5A 	Students will be able to solve linear equations that have variables on both sides.	• This lesson is a foundation for understanding balancing equations. Solving equations should not be new for students, but they will likely need a review. The first video uses a scale as a model that may be helpful to reference throughout the lesson. • When variables start on both sides, students may get stuck deciding which side to move them to. Discuss that there is freedom and choice here! Students can move them to either side, but we tend to prefer positive over negative, when possible. • This is a good time for teacher-led groups. Learning gaps from previous grades and lessons may surface, and students will need help with a variety of challenges. • Encourage students to show all of their work and check their answer to be sure it is correct.

Lesson 6: Linear equations with parentheses TEKS standard: A.5A Video 1 · Article 1 · Exercise 2	Students will be able to solve linear equations that include the distributive property.	• Students continue solving equations with more layers of difficulty. Students will see more negative numbers, fractions, and decimals. • Review problems featuring the distribution of negative numbers. Encourage students to show their work at every step to ensure they're paying attention to negatives, as well as distributing and combining like terms properly.
Lesson 7: Multi-step inequalities TEKS standard: A.5B Video 3 · Article 0 · Exercise 2	Students will be able to solve multi-step inequalities.	• **Warm up activity:** Have students graph single-variable inequalities on a number line and/or provide graphs and have them write the inequality. This will prepare them to work with inequalities in this lesson.  • Remind students that solving inequalities is just like solving equations: they can use all the same strategies. The only difference is that when they multiply or divide by a ***negative*** number, the inequality switches direction. • Students may need help using the calculator and entering their answer. Model this for students.
Lesson 8: Add and subtract polynomials of degree one TEKS standard: A.10A Video 2 · Article 0 · Exercise 1	Students will be able to add and subtract polynomials of degree one.	• Adding polynomials is just like combining like terms. For subtraction, you will need to distribute the negative first. • Students will see notation where they will need to differentiate between capital letter variables and lower case. Remind students to pay attention to that! See the example below where the capital letter denotes an **equation**: $A = 3a + 5b - 1$ $B = a - 3b$ $A - B = (3a + 5b - 1) - (a - 3b)$ $A - B = (3a + 5b - 1) - a + 3b$ $A - B = 2a + 8b - 1$

Lesson 9: Divide polynomials of degree one TEKS standard: A.10C 	Students will be able to divide polynomials of degree one.	• Dividing polynomials of degree one may not be intuitive for students. After giving time to explore and find patterns, students can use the algorithm. See "Best practices" for more information. • This is a good lesson for teacher led groups as students will face different challenges with problems like these. They require a fundamental understanding of division as well as factoring. This is the most challenging exercise in the unit.
Lesson 10: Division by zero 	Students will be able to explain why dividing by zero is undefined.	• The videos in this lesson provide explanations for why dividing by zero results in an undefined solution. • The article includes answers to many commonly asked questions in algebra that your students may be asking. Use this as a resource when answering students' questions, and have them read it themselves!
Lesson 11: Linear equations with unknown coefficients TEKS standard: A.12E 	Students will be able to solve equations for a given variable.	• Students will practice their algebra skills in this unit by using what they know about the distributive property, combining like terms, and solving equations to isolate a given variable. In these problems, they are not finding a numerical answer, but simply rearranging equations to get a given variable alone. Students may have a challenging time with this because they are used to finding a numerical answer and will want to do that. They will need to first expand the equation (do all distribution) before simplifying and solving. • Continue to model for students how to enter their answers correctly on Khan Academy. They will need to be able to use parentheses and division for their answers to be correct.
Lesson 12: Manipulating formulas TEKS standard: A.12E 	Students will be able to rewrite formulas for a given variable.	• Similar to Lesson 11, students will manipulate formulas to solve for a given variable. Again, they are not finding numerical answers, but are using their algebra skills to rearrange a formula and isolate a given variable.

Best practices

The distributive property
Using the distributive property is a foundational algebra skill. Let's look at it in more depth, along with factoring.

The distributive property is used when we want to multiply a sum or difference by a number. In situations where we don't have variables, we'll get the same solution by using either the order of operations or the distributive property and can simplify using either method.

Order of operations:

$$3(2+4)$$
$$3(6)$$
$$18$$

Distributive property:

$$3(2+4)$$
$$3\cdot 2+3\cdot 4$$
$$6+12$$
$$18$$

The distributive property shows us how to "distribute" the number being multiplied to all of the terms inside parentheses. It's particularly useful when we have variables within parentheses that cannot be combined, as the order of operations would require us to do. Here's how we can use the distributive property to rewrite some expressions:

The general case is:

$$a(b+c)$$
$$ab+ac$$

$$2(3a+8b-5)$$
$$2(3a)+2(8b)-2(5)$$
$$6a+16b-10$$

$$\frac{1}{3}(6e-18f-12)$$
$$\frac{1}{3}(6e)-\frac{1}{3}(18f)-\frac{1}{3}(12)$$
$$2e-6f-4$$

$$-5(x-3y+2z)$$
$$(-5)(x)-(-5)(3y)+(-5)(2z)$$
$$-5x+15y-10z$$

Factoring - Reversing the distributive property
Factoring is like the distributive property but in reverse. It can "undo" the distributive property. When we factor, we look for the greatest common factor (GCF) of *all* of the terms, and then we divide that number out of *each* term.

$$10-15x$$
$$5\cdot 2-5\cdot 3x$$
$$5(2-3x)$$

← In this problem, 5 is the GCF of 10 and 15. We rewrite both terms as multiplication by 5 and then "factor out the 5" by writing the 5 as multiplication in front and put the terms in parentheses. If we apply the distributive property to the answer, the result will be the original expression. →

$$5(2-3x)$$
$$5\cdot 2-5\cdot 3x$$
$$10-15x$$

Here are a few more examples. Remember that we can check our work using the distributive property to make sure we get back the original expression.

$$18a-12b+6$$
$$6\cdot 3a-6\cdot 2b+6\cdot 1$$
$$6(3a-2b+1)$$

The GCF of 18, -12, and 6 is 6. We will "factor out" a 6.

$$39+24n-21m$$
$$3\cdot 13+3\cdot 8n-3\cdot 7m$$
$$3(13+8n-7m)$$

The GCF of 39, 24, and -21 is 3. We will "factor out" a 3.

Manipulating formulas

The last two lessons use students' algebra skills to rearrange equations and formulas. Students have been building these skills throughout the unit, but some may find this particular application difficult. It can be confusing when there are only variables with few, if any, numbers, but the order of operations and distributive property remains the same. Factoring will be used more often, as seen in the next example:

On the left is a problem similar to what students were solving earlier in the unit, while on the right is a problem from these last two lessons involving more than one variable . Two of the numbers in the problem on the left have been replaced with different variables on the right. In both equations, we solve for *y*.

$$5(3+y) = 10y + 18$$

$$15 + 5y = 10y + 18$$
$$-15 \quad\quad -10y$$

$$5y - 10y = 18 - 15$$

$$\frac{-5y}{-5} = \frac{3}{-5}$$

$$y = -\frac{3}{5}$$

You can see that the first three steps are the same for both of these problems.

The difference comes with combining like terms. On the right, we can combine like terms while on the left we cannot. On the left, we need to factor out the *y*. Once the *y* is factored out, the steps are again the same.

$$a(n+y) = 10y + 18$$

$$an + ay = 10y + 18$$
$$-an \quad\quad -10y$$

$$ay - 10y = 18 - an$$

$$\frac{y(a-10)}{a-10} = \frac{18-an}{a-10}$$

$$y = \frac{18-an}{a-10}$$

Here is another common example with the equation for converting degrees Celsius to Fahrenheit. Let's solve for C.

$$F = \frac{9}{5}C + 32$$
$$-32 \quad\quad -32$$

Subtract 32 from both sides

$$\frac{F-32}{\frac{9}{5}} = \frac{\frac{9}{5}C}{\frac{9}{5}}$$

Divide both sides by 9/5. Remember that dividing by 9/5 is the same as multiplying by 5/9.

$$\frac{5(F-32)}{9} = C$$

Take each step slowly and carefully to help to minimize mistakes. Encourage students to show their work for each step so that finding errors is possible. The more practice, the better!

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.