### Algebra 1

#### Unit 2: Introduction to functions


Functions are rules that link inputs to outputs. They're key in math and beyond, helping us understand patterns and relationships.

- Determine whether a relation is a function from equations, lists, diagrams, tables, graphs, and words
- Evaluate functions from equations and graphs
- Find the domain and range of a function

### TEKS standards

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.2A:</strong> Determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities</td>
<td><strong>Determining whether a relation is a function</strong></td>
</tr>
<tr>
<td>This standard is only partially covered here. Knowledge of quadratic functions is not necessary in this unit.</td>
<td>There are many misconceptions that students might have about functions: “all relations are functions,” “all functions must be linear,” and “all functions must be equations,” to name a few. Determining whether a relation is a function may seem simple, but at first they can seem quite complicated and confusing. How to help: Consistently review with students that a relation is a function if and only if every domain (x) value is mapped to exactly one range (y) value. All functions are relations but not all relations are functions, similar to how all squares are rectangles but not all rectangles are squares. Be sure to discuss real-world examples, like the soda machine example discussed in the “Lesson overview.” Because the idea of a function is so abstract, it may take a long time for students to fully grasp it.</td>
</tr>
<tr>
<td><strong>A.6A:</strong> Determine the domain and range of quadratic functions and represent the domain and range using inequalities</td>
<td>“A graph is a function if it passes the vertical line test anywhere”</td>
</tr>
<tr>
<td>This standard is only partially covered here. Knowledge of quadratic functions is not necessary in this unit.</td>
<td>The vertical line test is a great tool, but it must be used correctly. If any vertical line can touch the graph at more than one point, then it’s not a function. How to help: Have students use a pencil for the vertical line test, holding it vertically and passing it along the (x)-axis. If the pencil “.touches” the graph in more than one place, anywhere, it is not a function. Even if the graph appears to be a function everywhere except at one point, it is still not a function. It must pass the vertical line test at every location.</td>
</tr>
<tr>
<td>A.9A: Determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities</td>
<td></td>
</tr>
</tbody>
</table>

This standard is only partially covered here. Knowledge of exponential functions is not necessary in this unit.

| A.12A: Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function |

| A.12B: Evaluate functions, expressed in function notation, given one or more elements in their domains |

**Misunderstanding function notation** | Students often confuse the notation $f(x)$ with multiplication. They might think that $f(x)$ means $f$ times $x$, rather than understanding it as the function $f$ applied to the input $x$ (said “$f$ of $x$”). This confusion is of course very understandable, so be sure to acknowledge it and emphasize that context is key.

**How to help:** Reiterate often that function notation is simply a shorthand, similar to using $y$. The beauty of function notation is that it tells us what value was substituted for $x$ after it’s been simplified. For example, if we simplify a function and get $y = 3$, we don’t know what was plugged in to get that value. If we write $f(5) = 3$, then we know that when $x = 5$, and 5 is substituted into the function, we will get out 3. Function notation is generally isolated by itself on one side of the equation. It is also useful in naming different functions. If you are working with two functions, they could be named $f(x)$ and $g(x)$ to note which is which.

**Mixing up domain/range or input/output** | Domain and input both describe $x$-values and range and output both describe $y$-values. It’s important to note that domain and input are not synonymous, even though they both describe $x$-values. An input is a specific number that is substituted into a function for the $x$-value. The domain is the set of all inputs that a function can accept. Similarly for output and range, output is the specific resulting $y$-value when an input is plugged in. The range is the set of all possible outputs.

**How to help:** Associate the words domain, input, and $x$-value together, along with range, output, and $y$-value. Create a poster with the axes and words for easy reference. As students use these words regularly, they will become more automatic.

**Confusion with domain and range for continuous vs discrete graphs** | A continuous graph is one where the line is unbroken, you can graph it without lifting your pencil. A discrete graph is a graph that is made of distinct, separate points (or lines) that you would need to lift your pencil in order to draw. When we find the domain and range of continuous graphs, we can use inequality notation to show all the possible values, and when we find the domain and range of discrete graphs containing points for example, we can write a list of all possible values.

**How to help:** Students may have difficulty with the domain and range in general, especially for continuous graphs. It can be helpful to use colored pencils to mark the possible $x$- and $y$-values. Discuss why different notation is used for different types of graphs. Give lots of practice! See “Best practices” for more on domain and range.
## Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

## Lesson overview

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objective</th>
<th>Teaching tips</th>
</tr>
</thead>
</table>
| **Lesson 1: Recognizing functions** | Students will be able to determine whether a relation is a function from equations, lists, diagrams, tables, graphs, and words. | - Determining whether a relation is a function can be confusing for students as it is quite abstract. Provide a concrete example for students to reference as they work through the lesson and unit. For example, a soda machine (or snack machine) is a function because when you press a button, you know what you’re getting out. If you press the grape soda button, you get out a grape soda, not an orange soda. A soda machine would NOT be a function if you could get different types of soda when pushing the same button—the grape soda button could give you either a grape soda or an orange soda. See “Best practices” for more.  
- There is a lot of new notation in the first exercise, including set notation. This notation will not be seen again in another exercise in this unit, so don’t spend too much time here if it’s too confusing for students.  
- This lesson shows many different examples of how functions can be presented, in pictures, lists, tables, graphs, and words. Show the connections between each presentation and how one function can be represented in multiple ways. When graphing, be sure to discuss the vertical line test!  
- When students get to the word problems, encourage them to work slowly and read carefully! They will need to do some logical reasoning! |
| **Lesson 2: Evaluating functions** | Students will be able to evaluate functions from equations and graphs. | - Evaluating a function is like solving a puzzle! For equations, you substitute the input, simplify, and get the output. For graphs, you look at the input on |
| TEKS standard: A.12B | the x-axis and then find the corresponding y-value for the output.  
● For students who are struggling with understanding functions, this lesson should help them develop a more concrete understanding. |
|---|---|
| Lesson 3: Introduction to the domain and range of a function | Students will be able to identify the domain and range of a function from a graph.  
● Warm up activity: Give some problems where students practice graphing one-variable inequalities on a number line. You can use this number line template.  
Graph  
$x \geq -3$  
-4 -2 0 2 4 6 8 10  
● Domain and range may seem as abstract as determining whether a relation is a function. However, it is concrete to show on a graph. Have students use colored pencils to mark the domain and range on the graph before trying to describe it numerically.  
● The videos introduce different notations for representing the domain and range from functions. Students should be familiar with these notations, but they don't appear in the exercise. They will need to use inequalities, which are not shown in either of the videos. See "Best practices" for more on domain and range with inequalities.  
● The domains and ranges of continuous graphs will be written with inequality notation while the same for discrete graphs will be written as a list. See "Best practices," below, for more. |
| TEKS standard: A.2A | Students will be able to identify the domain and range of a graph.  
● Students will find the domain and range of linear, quadratic, and exponential functions from graphs.  
● These types of problems will be investigated further in the later units devoted to quadratics and exponential functions respectively, so feel free to view or present this as an introduction of what's to come. |
| Lesson 4: Domain and range with inequalities | Students will find the domain and range of linear, quadratic, and exponential functions from graphs.  
● Students will find the domain and range of linear, quadratic, and exponential functions in this lesson. Note that no prior knowledge of quadratics or exponential functions is expected or needed here to solve. Encourage students to use colored pencils to mark the domain and range, as they did in Lesson 3, above.  
● These types of problems will be investigated further in the later units devoted to quadratics and exponential functions respectively, so feel free to view or present this as an introduction of what's to come. |
Best practices

Functions, explained

A relation is a relationship between a set of values. All functions are relations but not all relations are functions—a function is a special kind of relation. A relation is a function if and only if every domain (x) value is mapped to exactly one range (y) value. This definition is quite abstract and may sound more complicated than it is, so let’s dig into it further. And when in doubt, we can always use the vertical line test by making a quick graph!

A “function machine” is typically used to demonstrate the difference between a function and a general relation.

\[
\begin{align*}
\text{This is a function because when you put a number into} & \quad \text{the rule, it always gives one answer out.} \\
\begin{array}{c}
\text{This is NOT a function because when you put a number} \\
\text{into the rule, you could get multiple answers out.}
\end{array}
\end{align*}
\]

The soda machine example, mentioned above in the lesson overview, is a great example to share with students. Another example is height because every person only has one height. If the input is a student’s name, the output would be their height and there is only one possible outcome. Some students may ask, what if two students have the same height? And this is a very important nuance. It is still a function if two inputs have the same output, so it is still a function if two students have the same height. What would make this example not a function is if one student had two heights!

An example of a relation that is not a function would be the sister function: the input is the student’s name, and the output is their sister’s name. It is okay for multiple students to have “none,” because multiple inputs can have the same output, but if a single student has two sisters this is not a function! Because then one input would map onto multiple outputs, the two sisters’ names. Have students come up with their own examples of functions and non-function relations!

Here is an algebraic example to show the difference between a relation and a function:

\[
\begin{align*}
\text{This is a function because every x-value will have exactly one y-value. When } x = 2, f(x) = 9 \text{ for example.} \\
\text{This is NOT a function because when } x = 2, g(x) = 8 \text{ and } -8.
\end{align*}
\]
Let's look at the same data set in a picture, table, and graph to see how they compare.

**Pictures**

These are functions because every input has exactly one output.

These are not functions because when we input 4, we can get either 3 or 4 as the output.

**Tables**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**Graphs**

**Domain and Range**

The domain is all possible x-values of a function and the range is all possible y-values of a function. The domain and range tell us about the possible inputs (x) and outputs (y) of the function. Graphs help us make this concept visual. When we work with a continuous graph (an unbroken line), we can use inequality notation to make sure that we include all values between the endpoints. When we work with a discrete graph (points), we can simply list all of the values.

**Continuous graph**

- Domain: \(-6 \leq x \leq 6\)
- Range: \(-3 \leq y \leq 7\)

**Discrete graph**

- Domain: \(x = -6, 0, 2, 4, 7\)
- Range: \(y = -5, 1, 3, 4\)
GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- **Weekly Khan Academy quick planning guide**: Use this template to plan your week using Khan Academy.

- **Using Khan Academy in the classroom**: Learn teaching techniques and strategies to support your students and save time with Khan Academy.

- **Differentiation strategies for the classroom**: Discover strategies to support the learning of all students.