Unit 5: Systems of linear equations and inequalities

Systems of equations let us solve problems with multiple unknowns. They're used in many fields like engineering and business, and give students a powerful tool for their future studies and careers.

- Find the solution to a system of linear equations by graphing, substitution, and elimination
- Write and solve systems of linear equations from problems in context
- Find the solution to a system of linear inequalities by graphing

<table>
<thead>
<tr>
<th>TEKS standards</th>
<th>Common misconceptions</th>
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<tbody>
<tr>
<td>A.2I: Write systems of two linear equations given a table of values, a graph, and a verbal description</td>
<td>Thinking solutions and intersections are totally different</td>
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<tr>
<td>A.3A: Determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including ( y = mx + b ), ( Ax + By = C ), and ( y - y_1 = m(x - x_1) )</td>
<td>Zero and infinitely many solutions confusion</td>
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<tr>
<td>A.3C: Graph linear functions on the coordinate plane and identify key features, including ( x )-intercept, ( y )-intercept, zeros, and slope, in mathematical</td>
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intersections will there be? None, so there are also no solutions. These are the only three possibilities for a system of linear equations: 0, 1, or infinitely many solutions. If students have trouble grasping this, challenge them to show where two parallel lines intersect or encourage them to explore two straight lines that intersect twice (which is not possible).

**Mixing up the methods** | Students sometimes confuse the elimination method with the substitution method. They might try to use parts of one method while actually using another, which can lead to errors.

**How to help:** It’s key to emphasize that in substitution, we solve one equation for one variable and then substitute that expression into the other equation. In elimination, we add or subtract the equations to eliminate one variable. While either of the methods can be used for any problem, each method has its own strengths and is best suited for certain types of problems. Give students lots of practice with each method individually and then allow them to choose which method to use, discussing which method might be most efficient in different situations. See “Best practices” for more.

**Overwhelm with inequalities** | When finding the solution for a system of inequalities, there is so much to think about! Students need to plot the line correctly, determine whether the line is dotted or solid, and shade the appropriate side of the line... that’s a lot! And they need to do this all for two inequalities at the same time!

**How to help:** Give students concrete steps to follow since there is an ordered procedure (you can’t shade if you haven’t graphed the line!). They should feel comfortable at this point graphing a line, but they might forget to make it dotted when the inequality is only “greater than” or “less than,” and not equal to. Determining which side of the line to shade can be tricky at first. Also, understanding that the solution is the entire region that is double-shaded is key!

**Shading the wrong side** | Students may frequently shade the wrong side of the graph, even after double checking their work. This may occur as a result of erroneously memorizing a certain inequality direction with up or down: but what if the variables change sides? This is why simple memorization is not appropriate here.

**How to help:** Encourage students to use the “origin test” for each inequality: Plug the point (0, 0) into the inequality, which is 0 for $x$ and 0 for $y$. If the inequality holds true, shade the origin side of the line. If it does not hold true, shade the other side of the line.
Unit resources

- For exercises that include graphing, specifically Lessons 1, 2, and 10, students can use this Graph paper workspace template. For an entire page of grid, use this graph paper.
- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

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<tr>
<th>Lesson 1: Introduction to systems of equations</th>
<th>Objective</th>
<th>Teaching tips</th>
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<tr>
<td>TEKS standard: A.3F, A.5C</td>
<td>Students will be able to determine whether a point is a solution to a system of linear equations algebraically. Students will be able to find the solution to a system of linear equations graphically.</td>
<td>- Warm up activity: Ask students whether given points are solutions to an equation. Discuss what makes a point a solution or not. For example: Given the equation, determine whether the points are solutions: ( y = 3x - 4 ) ((1, -1), (3, 4), (-2, -10)) (yes) (no) (yes)</td>
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<tr>
<td>Video</td>
<td>Article</td>
<td>Exercise</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>2</td>
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<tr>
<th>Lesson 2: Number of solutions to systems of equations</th>
<th>Objective</th>
<th>Teaching tips</th>
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<tr>
<td>TEKS standard: A.3F</td>
<td>Students will be able to determine the number of solutions to a system of equations graphically and algebraically.</td>
<td>- The focus here is to determine whether a system has 0, 1, or infinitely many solutions from the equation in slope-intercept form and/or the graph. (They will not see 0 or infinitely many solutions for substitution or elimination until Lesson 9). - Summarize with students the three possibilities for the number of solutions: ○ 1 solution: The slopes of the lines are different. The lines intersect at one</td>
</tr>
<tr>
<td>Video</td>
<td>Article</td>
<td>Exercise</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
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### Lesson 3: Write a system of equations from a table or graph

**TEKS standard:** A.2I, A.3A, A.3C

- Students will be able to write a system of linear equations given two tables.
- Students will be able to write a system of linear equations given a graph.

**Warm up activity:** Give students problems where they are given two points on a line and then write the equation of the line that passes through the two points.

**Students may need a refresher on writing equations from tables and graphs, which they saw in Unit 3. Encourage them to circle the $y$-intercept and label the slope in the table and graph. Students work with both vertical and horizontal tables.**

### Lesson 4: Systems of equations from context

**TEKS standard:** A.2I, A.3G

- Students will be able to write a system of linear equations given a problem in context.
- Students will be able to interpret the “better deal” from the graph of a system of linear equations.

**The biggest challenge in the next two lessons is translating word problems into a system of linear equations. Students need to read the questions carefully and focus on interpreting the problem.**

**Relatedly, they create graphs and determine where the “best deal” is from the graph. They will continue to build their skills of interpreting stories and relating the stories to given graphs to determine what different parts of a graph mean in context.**

### Lesson 5: Interpreting points relative to a system of equations

**TEKS standard:** A.3G

- Students will be able to interpret points on a coordinate grid in relation to a system of linear equations in context.

**In the exercise, encourage students to take their time interpreting the story, equations, and graph. While the equations are given, students need to figure out the meaning of each one so they can appropriately interpret the point asked for. This will prepare them for Lesson 9, where they write equations for situations.**

### Lesson 6: Solving systems of equations

- Students will be able to solve a system of linear equations

**Warm up activity:** To prepare for substitution, give students a linear equation and a value for either $x$ or $y$. 

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- **Infinite solutions:** The slopes and $y$-intercepts for the lines are exactly the same. The lines are the same and intersect at all points.
- **0 solutions:** The slopes of the lines are the same and the $y$-intercepts are different. The lines never intersect (they are parallel).
with substitution

TEKS standard: A.5C

using substitution.

or \( y \) and have them substitute to find the value of the other variable. When you introduce substitution with systems, relate it back to how they substituted a value for a variable in these problems.

Given \( y = 5x + 2 \), find the value of \( x \) when \( y = 6 \).

- The key in substitution is to solve one of the equations for one variable and substitute that expression into the other equation for that variable. Once you have one equation with one variable, you can simplify it!
- The first video may be confusing because it jumps into a problem that was started in a video that students will see in the next lesson. The second video and article provide examples of substitution and are great references for students.

Lesson 7: Solving systems of equations with elimination

TEKS standard: A.5C

Students will be able to solve systems of linear equations using elimination.

- Since the idea of elimination is new for students, this is a long lesson. In the first two exercises, students practice combining equations and deciding the best elimination strategy. The last two exercises have students use elimination to solve systems completely.
- Have students use the article as a reference for elimination if they want to see more examples or have additional practice.
- Walk through examples of elimination with students, especially where scaling is involved. Discuss how substitution and elimination are different and when they might choose to use one or the other for efficiency.

Lesson 8: Equivalent systems of equations

TEKS standard: A.5C

Students will be able to determine whether two systems of equations are equivalent.

- Students continue working with elimination, looking specifically at equivalent systems of equations. They won’t be solving systems here; rather, they’re looking for what operations were done to obtain new systems and determine if they are equivalent or not. The videos show examples of this; do examples together to model your thinking.
- Students may have difficulty understanding that even though two equations don’t look the same, they can still be equivalent. Here are some actions...
we can do to a given system of equations to obtain an equivalent system:
  - Swap the order of the equations
  - Replace one equation with the sum/difference of both equations
  - Replace one equation with an equation equivalent to it. This can be done in two ways:
    - Replacing one equation with a non-zero multiple of itself
    - Replacing one equation with itself where the same quantity is added to both sides

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<tr>
<th>Lesson 9: Systems of equations word problems</th>
<th>Students will be able to solve systems of linear equations word problems.</th>
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<tbody>
<tr>
<td>TEKS standard: A.5C</td>
<td>● This lesson is all about word problems! The most challenging part will likely be defining variables and writing equations. Do examples together and model this for students as they need to read the problems carefully. Encourage them to use different colored pencils to mark information that is related in the story.</td>
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<td>● Students see situations where there may be 0 or infinitely many solutions so review what this looks like with substitution and elimination.</td>
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<td>● Revisit the discussion about when substitution or elimination is more efficient when solving. Have students share their decision-making process.</td>
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<tr>
<th>Lesson 10: Graphing systems of two-variable inequalities</th>
<th>Students will be able to graph systems of linear inequalities.</th>
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<tbody>
<tr>
<td>TEKS standard: A.3H</td>
<td>● This lesson moves back to graphing, this time with inequalities. The solution to a system of inequalities is a region of a graph. Students will need to graph the lines correctly, determine whether the lines are dotted or solid, and then shade the appropriate side. The region where the shading overlaps is the solution region.</td>
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<tr>
<td></td>
<td>● Revisit graphing inequalities as necessary. They graphed single linear inequalities in Unit 3.</td>
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TRY THIS WITH YOUR STUDENTS
Best practices

Substitution vs. elimination: Which to choose when?
Understanding substitution and elimination as two separate methods for solving systems of linear equations is challenging and then having to choose which to use can feel like just too much! Any system can be solved with either substitution or elimination, but one method will usually be more efficient based on the form of the equations given. Note that graphing can also be used to solve any system of equations, but if the graph isn’t precise or if the lines intersect at non-integer points, it can be difficult to determine the exact solution.

Sometimes students may choose to use one method all the time because that is just what they prefer. Have students show their work on the board when they have solved the same system using different methods so the class can discuss which method is more efficient for that specific situation.

Substitution is generally more efficient when one equation is already solved for one variable or when the coefficients of the variables don’t easily lend themselves to elimination.

Elimination is generally more efficient when both equations are in standard form or when the coefficients of one of the variables in both equations are the same or easily made the same by multiplication.

These two systems are more suitable for substitution.
\[
\begin{align*}
15x + 31y &= -3 \\
x &= -y + 3
\end{align*}
\]
These two systems are more suitable for elimination.
\[
\begin{align*}
2x - 4y &= 8 \\
8x + 4y &= 2
\end{align*}
\]

Let’s solve the first example all the way through:
1. Substitute and simplify.
\[
\begin{align*}
15(-y + 3) + 31y &= -3 \\
-15y + 45 + 31y &= -3 \\
16y + 45 &= -3 \\
16y &= -48 \\
y &= -3
\end{align*}
\]
2. Substitute and simplify into the other equation.
\[
\begin{align*}
x &= (-3) + 3 \\
x &= 3 + 3 \\
x &= 6
\end{align*}
\]
3. Check the solution \((6, -3)\).
\[
\begin{align*}
15(6) + 31(-3) &= 90 - 93 \\
90 &= 90 \\
-3 &= -3
\end{align*}
\]
Since \((6, -3)\) satisfies both equations, it is the solution to the system.

Let’s solve the first example all the way through:
1. Add the equations and solve for a variable.
\[
\begin{align*}
2x - 4y &= 8 \\
8x + 4y &= 2
\end{align*}
\]
\[
\frac{10x + 0y}{10} = \frac{10}{10}
\]
\[
x = 1
\]
2. Substitute the value into one of the equations.
\[
\begin{align*}
2(1) - 4y &= 8 \\
2 - 4y &= 8 \\
-2 - 4y &= 0
\end{align*}
\]
\[
\frac{4y}{4} = \frac{6}{4}
\]
\[
y = -\frac{6}{4} = -\frac{3}{2}
\]
3. Check the solution in the other equation.
\[
8(1) + 4\left(-\frac{3}{2}\right) = 2
\]
\[
8 + \left(-\frac{12}{2}\right) = 2
\]
\[
8 - 6 = 2
\]
Since \((1, -3/2)\) satisfies both equations, it is the solution to this system.
Graphing systems of linear inequalities

When we graph systems of linear inequalities, each inequality must be graphed independently and the solution is found after both are graphed. Graphing inequalities includes graphing the line, determining whether the line is solid or dotted and calculating which side of the line should be shaded.

Let's look at the following system. \( \begin{cases} y > 2x - 3 \\ y \leq -x - 1 \end{cases} \)

We'll look at each inequality individually first.

The line has a \(y\)-intercept at \((0, -3)\) and a slope of 2. It is a dotted line because it is only greater than and not equal to.

The line has a \(y\)-intercept at \((0, 4)\) and a slope of -1. It is a solid line because it is less than or equal to. Since it can be equal, the line is solid.

To determine which side of the line to shade, substitute the point \((0, 0)\). If the statement is true, shade the side of the line that includes the point \((0, 0)\). If the statement is false, shade the side of the line that does not include the point \((0, 0)\). Note: You can use any point to check, it doesn't need to be \((0, 0)\). However, \((0, 0)\) will make the calculations the easiest.

\[
\begin{align*}
\text{For } y > 2x - 3: & \\
0 & > 2(0) - 3 \\
0 & > -3 & \checkmark
\end{align*}
\]

This is a true statement, so shade the side of the line that includes \((0, 0)\).

\[
\begin{align*}
\text{For } y \leq -x - 1: & \\
0 & \leq -(0) - 1 \\
0 & \leq -1 & \times
\end{align*}
\]

This is a false statement so shade the side of the line that does NOT include \((0, 0)\).

Lastly, when we combine the two graphs, we look for where the shaded regions overlap, and that is our solution set. When you make these graphs by hand, you only need to use one coordinate plane, but do them one by one. Note: the line above this double-shaded region is part of the solution set, while the line below is not, due to the dashed blue line.

This region is the solution because it is where both areas overlap.
GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- **Weekly Khan Academy quick planning guide**: Use this template to plan your week using Khan Academy.

- **Using Khan Academy in the classroom**: Learn teaching techniques and strategies to support your students and save time with Khan Academy.

- **Differentiation strategies for the classroom**: Discover strategies to support the learning of all students.