A Multi-Physics Eulerian Framework for Long-Term Contrail Evolution*

Contrails Analysis Workshop

Imperial College, London, Sept 16–18, 2025

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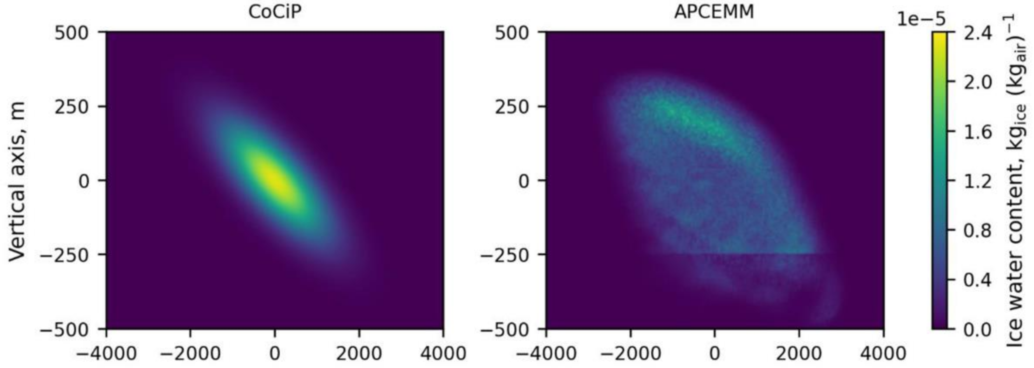
*Jafarimoghaddam, A., & Soler, M. (2025). A Multi-Physics Eulerian Framework for Long-Term Contrail Evolution. *arXiv preprint arXiv:2509.00965*.

*Jafarimoghaddam, A., & Soler, M. (2025). A Multi-Physics Eulerian Framework for Long-Term Contrail Evolution. Pre-print and interactive public peer review. *Atmo. Chem. Physics*. Egusphere-2025-4155

Motivation

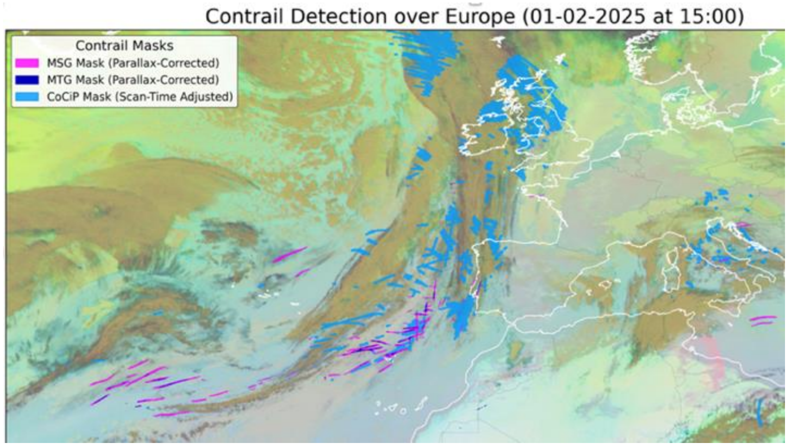
Current contrail climate impact estimation models being used in ATM studies exhibit significant discrepancies.

Predicted lifetimes and RF of CoCiP & APCEMM differ by factors of 2-5



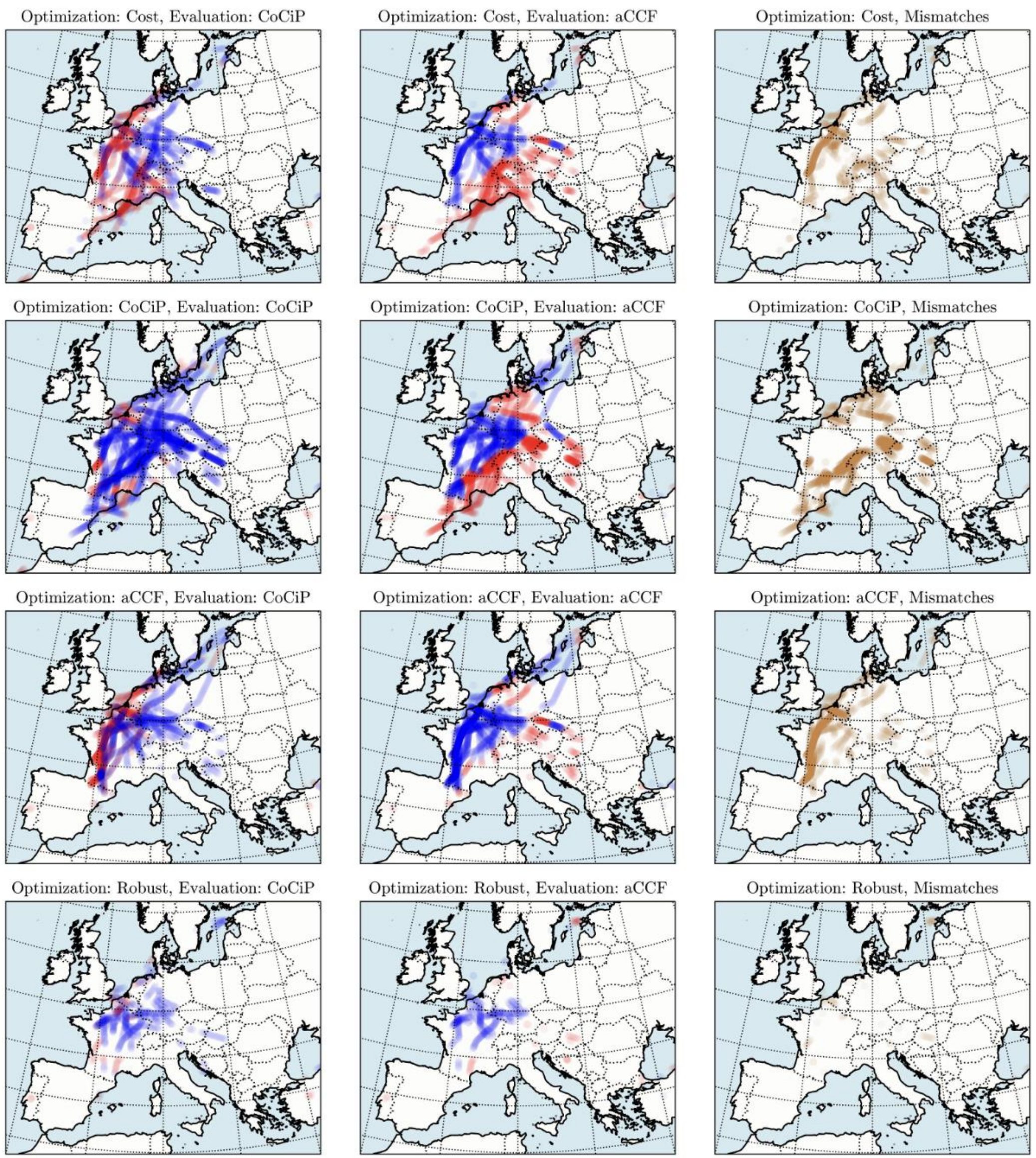
Akhtar Martínez el al. (2025). **Contrail models lacking post-fallstreak behavior could underpredict lifetime optical depth. EGU Sphere, 1-26.**

Data-Driven detections and Models differ.



Contrails detected by a deep learning model using data from Meteosat Second Generation (MSG) and Meteosat Third Generation (MTG), compared to those simulated by CoCiP. (Ortiz, Soler et. al, 2025 at EGU)

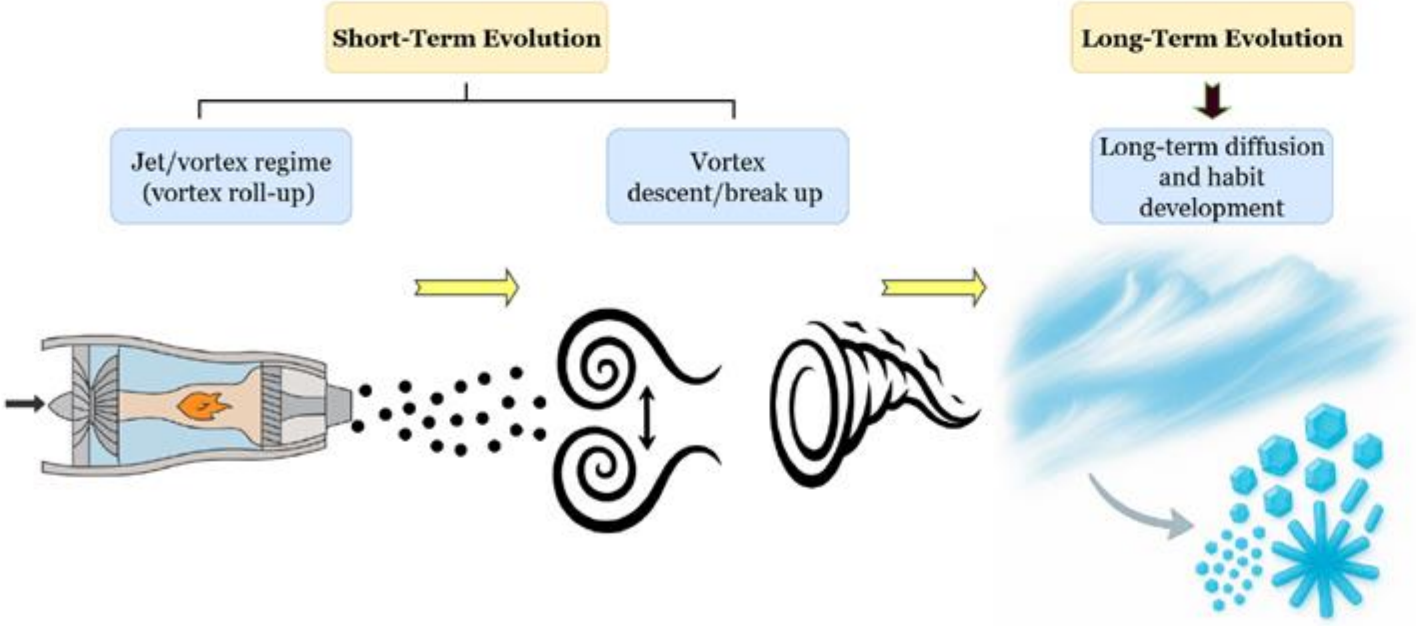
Contrail Avoidance strategies sensitive to model uncertainties



Simorgh, Abolfazl and Simorgh, Abolfazl and Soler, 2025, Manuel, Feasibility of Integrating Multiple Climate Impact Estimation Models to Enhance Confidence in Environmentally-Friendly Aircraft Trajectory Optimization (January 10, 2025). Available at SSRN: http://dx.doi.org/10.2139/ssrn.5409422

Contrail Modelling (Contributions)

Contrails undergo multiple regimes/stages right after their birth to eventually becoming cirrus clouds.



Phase 1: Short-Term

- Ice-Particle Transport (Macrophysics)
- Induced Engine Jet Flow (Macrophysics)
- Ice-Particle Growth (Microphysics)
- Nucleation (Microphysics)
- Aggregation (Microphysics)

Phase 2: Long-Term

- Ice-Particle Transport (Macrophysics)
- Ice-Particle Growth (Microphysics)
- Habit Development (Microphysics)
- Aggregation (Microphysics)
- Multi-Phase Flow Settling (Macrophysics)

Existing Contrail Models:

Are Lagrangian models
Do not incorporate habit development
Do not incorporate multi-phase flow settling

COCIP
(Schumann, 2012)

APCEMM
(Fritz et al., 2020)

CFD approaches (LES & RANS)
(Unterstrasser & Gierens, 2010 –I,II-)
(Lewellen, 2020)

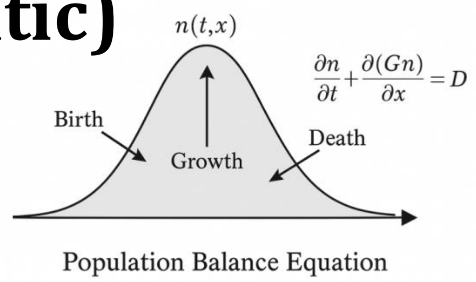
Our model is:

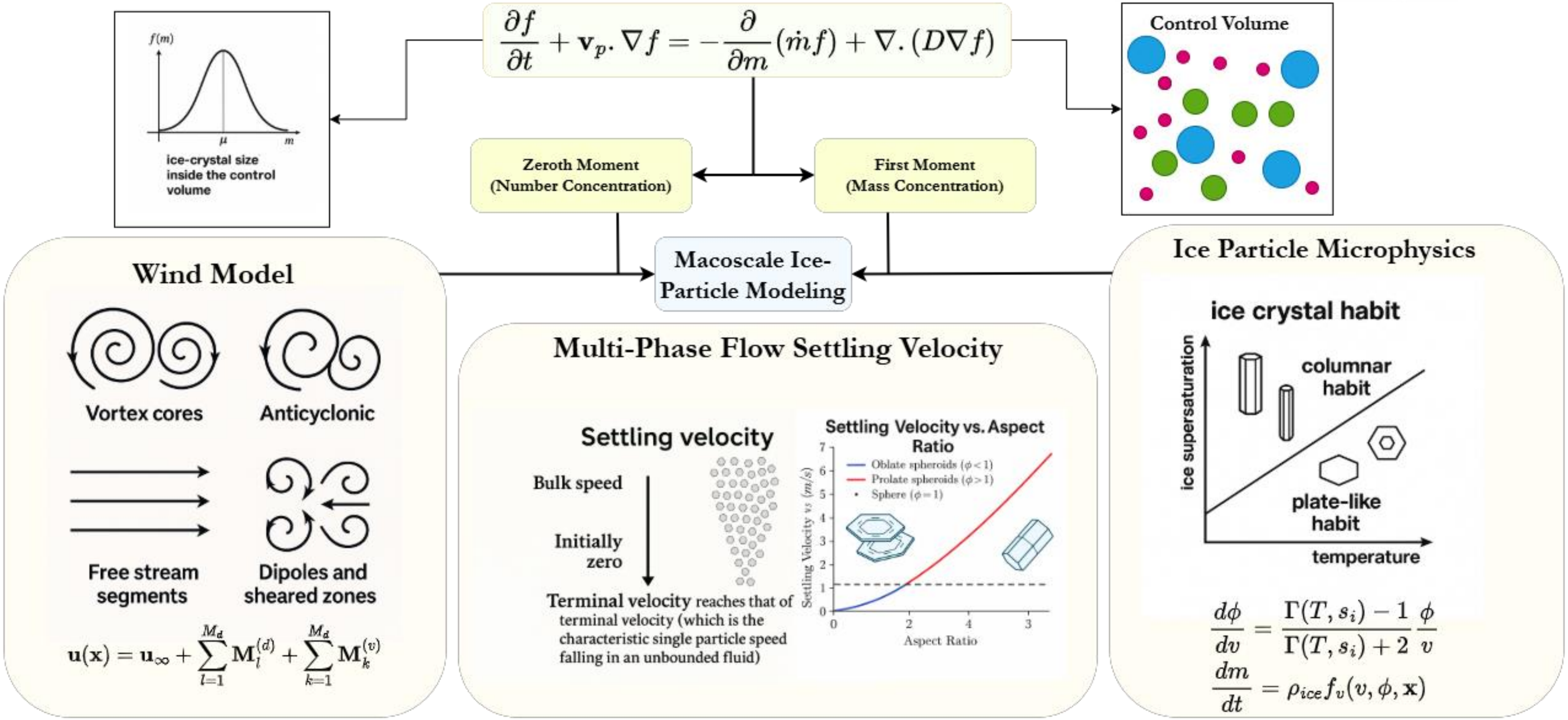
- Focus only on long-term
- Eurlerian
    - Quasy Analytic representation of the sol.
    - Includes habits and multi-phase flow settling.

Advection-Difussion Contrail Modelling (Schematic)

Population Balance Eq. (PBE)

$$\frac{\partial f}{\partial t} + \mathbf{v}_p \cdot \nabla f = -\frac{\partial}{\partial m}\Big(\dot{m}\, f\Big) + \nabla \cdot \Big(\tilde{\mathcal{D}}\, \nabla f\Big) + S_f,$$

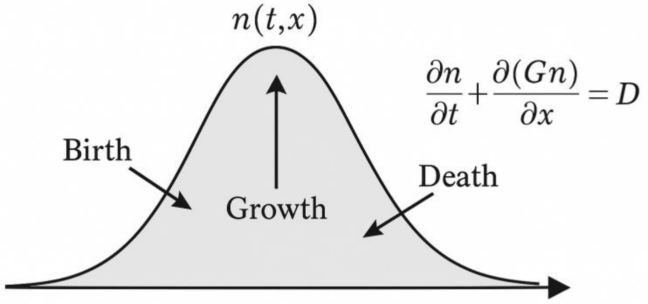




Problem Statement: Ice Particle Transport

Starting with PBE:

$$\frac{\partial f}{\partial t} + \mathbf{v}_p \cdot \nabla f = -\frac{\partial}{\partial m}\Big(\dot{m}\, f\Big) + \nabla \cdot \Big(\tilde{\mathcal{D}}\, \nabla f\Big) + S_f,$$



Population Balance Equation

Together with the particle velocity v_p *(long-term regime)*:

$$\mathbf{v}_p = \mathbf{v}_{slp} + \underbrace{(w_x, w_y, w_z)^\top}_{\substack{\text{background velocity} \\ \text{(wind components)}}} \approx (0,0,v_s)^\top + \underbrace{(w_x, w_y, w_z)^\top}_{\text{background velocity}} = (w_x, w_y, w_z + v_s)^\top.$$

We can derive the moment equations as:

$$\frac{\partial c_N}{\partial t} + \mathbf{v}_p \cdot \nabla c_N = \nabla \cdot \Big(\tilde{\mathcal{D}}\, \nabla c_N\Big) + S_{c_N}$$

$$\frac{\partial m}{\partial t} + \mathbf{v}_p \cdot \nabla m = \tilde{\mathcal{D}}\, \nabla^2 m + (\nabla \tilde{\mathcal{D}}) \cdot \nabla m + \frac{2\,\tilde{\mathcal{D}}}{c_N}\, \nabla m \cdot \nabla c_N + \rho_{dep} f_v + S_{C_M}$$

where, $m(\mathbf{x},t)$ is the representative particle mass in control volume, and c_N $(\mathbf{x},t)$ is number concentration.

Problem Statement: Ice Particle Growth and Habit Development

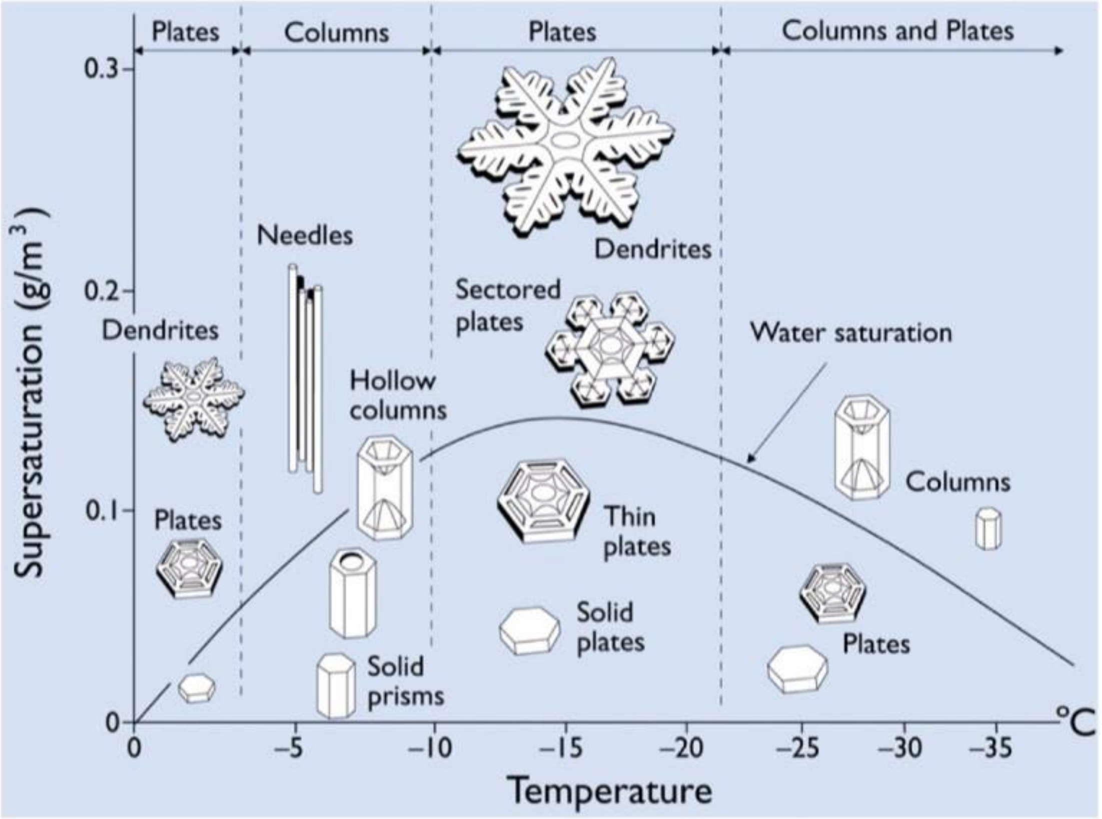
Mature contrail cirrus (on the order of 1–10 hours old in ice-supersaturated layers) transition into the same regime as natural cirrus, where habit classification based on temperature and supersaturation becomes appropriate.

To summarize:

Fresh contrail (< 5 min): ***Tiny polycrystalline droxtals***; no habit classification.

Early contrail cirrus (5 min–20 mins): ***Mixed phase***; spheres and rough aggregates.

Mature contrail cirrus (20 mins–10 h): ***Crystals ≥ 20–30 μm***; faceted habits consistent with natural cirrus and subject to the standard temperature-supersaturation-habit relationship.



Microphysics for ice-particle growth and shape evolution:

(Cheng and Lamb, 1994)

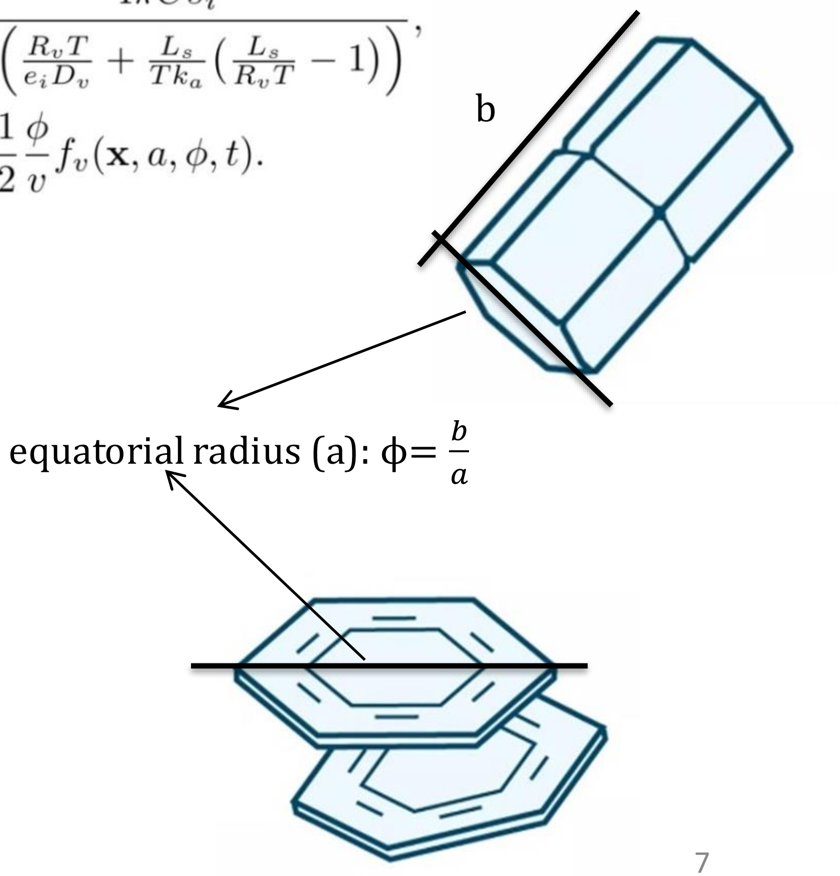
$$f_v(\mathbf{x}, a, \phi, t) = \frac{4\pi C s_i}{\rho_{dep}\left(\frac{R_v T}{e_i D_v} + \frac{L_s}{T k_a}\left(\frac{L_s}{R_v T} - 1\right)\right)},$$

$$f_\phi(\mathbf{x}, a, \phi, t) = \frac{\Gamma - 1}{\Gamma + 2}\frac{\phi}{v} f_v(\mathbf{x}, a, \phi, t).$$

$$\frac{dm}{dt} = \rho_{dep} f_v,$$

$$\frac{d\phi}{dt} = f_\phi,$$

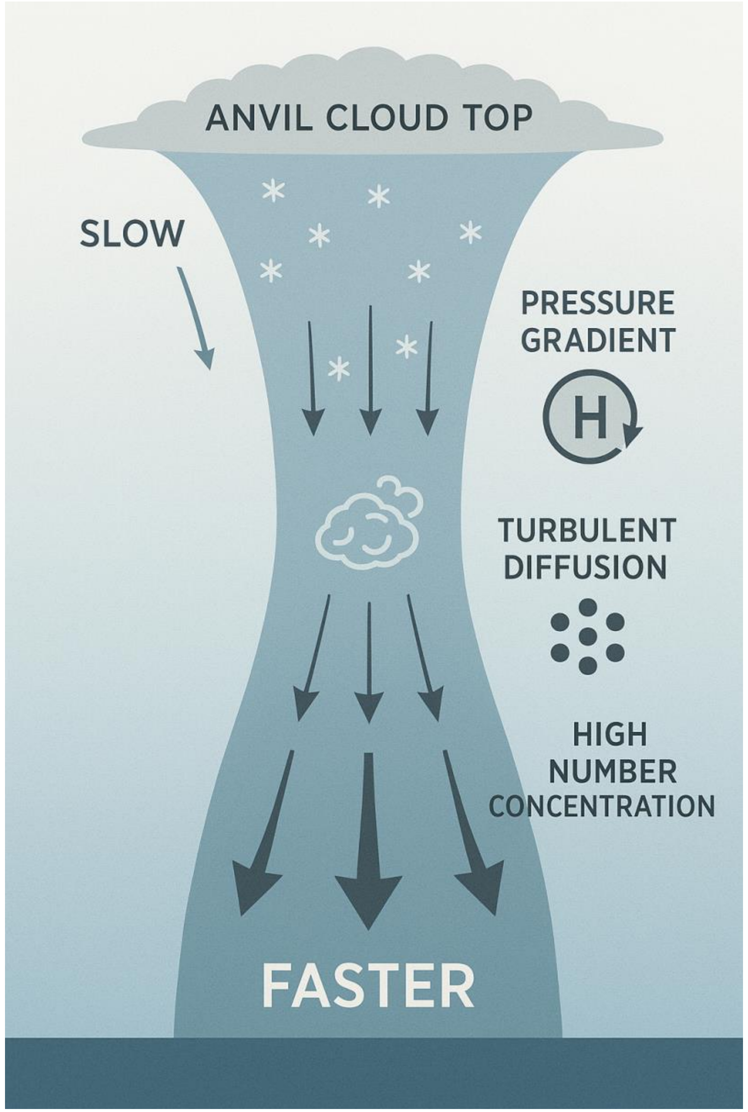
$$v = \frac{4}{3}a^3\phi.$$



Problem Statement: Multi-Phase Flow Settling

In Eulerian framework, settling velocity requires more careful implementation to account for self-diffusion effect:

- Bulk of particles in a turbulent mixing
- Loitering and sweeping
- V_s not equal to V_{ter} for all times (sol. to Stokes eq.)



Starting with the classical multi-phase flow equation in an Euler-Euler framework for the particle phase:

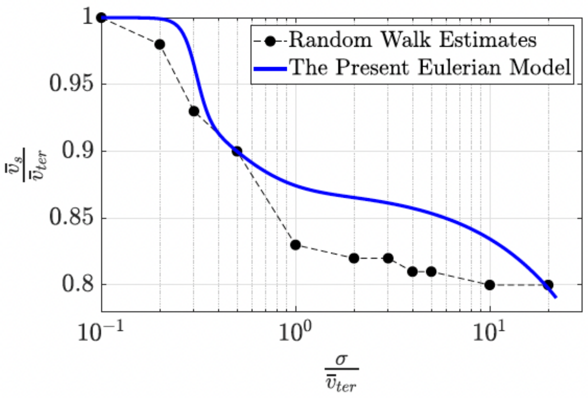
$$\frac{\partial(\epsilon_p \rho_p \mathbf{v}_{slp})}{\partial t} + \nabla \cdot \left(\epsilon_p \rho_p \, \mathbf{v}_{slp} \mathbf{v}_{slp}\right) = -\epsilon_p \, \nabla p + \nabla \cdot \tau_p + \epsilon_p \rho_p \, \mathbf{g} + F_d,$$

We can show that the equation can be reduce to the following reduced-order model (A Burgues-type PDE):

$$\frac{\partial v_s}{\partial t} + v_s(z,t)\,\frac{\partial v_s}{\partial z} = \nu_{t,ef}\,\frac{\partial^2 v_s}{\partial z^2},$$

$$v_s(z_{ref},0) = 0, \qquad \lim_{z\to-\infty} v_s = v_{ter}, \qquad \lim_{t\to\infty} v_s = v_{ter},$$

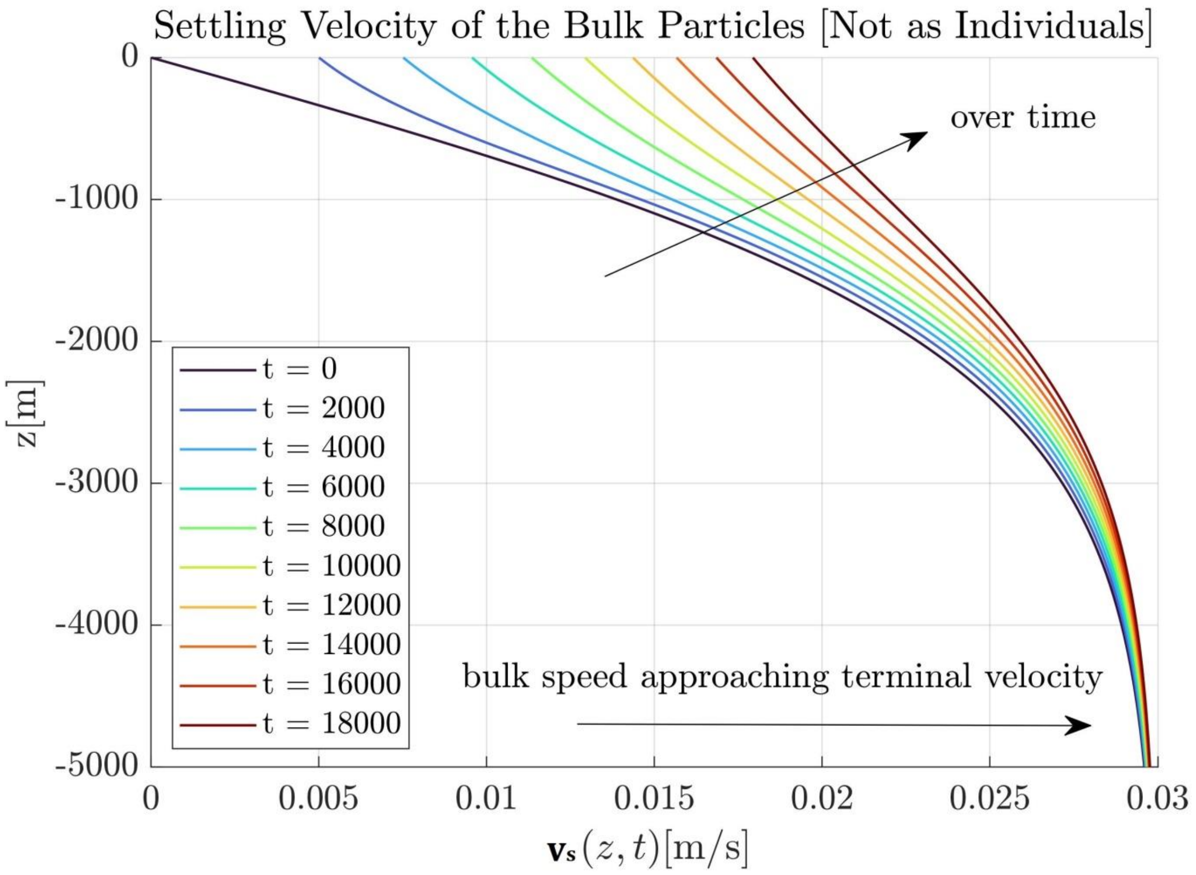
$$Re^2\, C_D(Re^*) = B. \qquad v_{ter} = \frac{Re\,\mu_{ef}}{\rho_f d_v}.$$



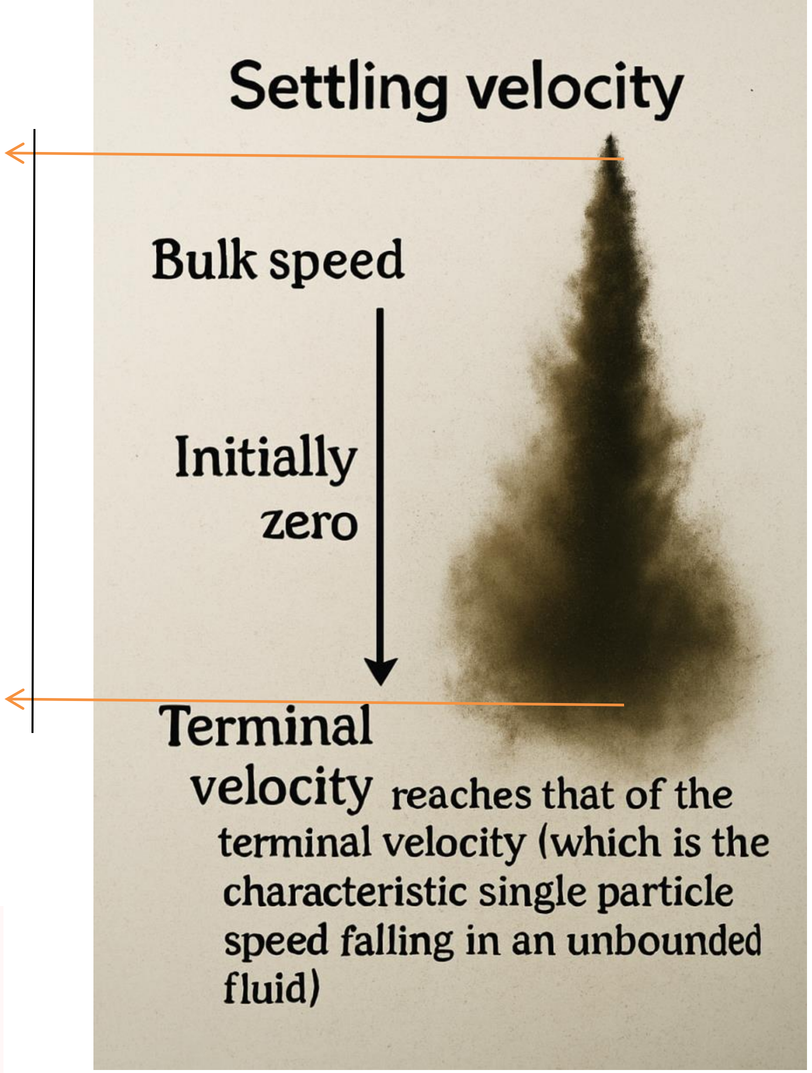
(Nielsen, 1995)

Problem Statement: Multi-Phase Flow Settling

In Eulerian framework, settling velocity requires more careful implementation to account for self-diffusion effect:



$$v_s(z,t) = v_{\text{ter}} \cdot \frac{\operatorname{erfc}\left(\dfrac{z - v_{\text{ter}} t}{2\sqrt{\nu t}}\right) - \exp\left(\dfrac{v_{\text{ter}}(z - v_{\text{ter}} t)}{\nu}\right) \cdot \operatorname{erfc}\left(\dfrac{z + v_{\text{ter}} t}{2\sqrt{\nu t}}\right)}{\operatorname{erfc}\left(\dfrac{z - v_{\text{ter}} t}{2\sqrt{\nu t}}\right) + \exp\left(\dfrac{v_{\text{ter}}(z - v_{\text{ter}} t)}{\nu}\right) \cdot \operatorname{erfc}\left(\dfrac{z + v_{\text{ter}} t}{2\sqrt{\nu t}}\right)}$$



The schematic shows *Free-Fall* of particles as a bulk

Problem Statement: System of Equations

It turns out the homogeneous boundary and initial conditions allow separation of variables through:

$$c_N(x,y,z,t) \;=\; F(x,y,t)\,g(z,t),$$

$$\mathcal{D}(c) \propto \frac{1}{1+\beta c}.$$

$$\widetilde{\mathcal{D}}_{ij}(c_N) \;\longrightarrow\; \begin{cases} \widetilde{\mathcal{D}}_{xx}\big(F(x,y,t)\big), \\ \widetilde{\mathcal{D}}_{yy}\big(F(x,y,t)\big), \\ \widetilde{\mathcal{D}}_{zz}\big(g(z,t)\big). \end{cases}$$

wind turbulence

$$w_x'(x,y,t) = \sigma_{w_x}\,\Re\Big\{\mathcal{F}^{-1}\Big[\sqrt{E(k_x,k_y)}\,\xi_{w_x}(k_x,k_y,t)\Big]\Big\},$$

$$w_y'(x,y,t) = \sigma_{w_y}\,\Re\Big\{\mathcal{F}^{-1}\Big[\sqrt{E(k_x,k_y)}\,\xi_{w_y}(k_x,k_y,t)\Big]\Big\}.$$

Therefore, following the assumption of local (piece-wise) constant temperature, we can show that the system breaks down into:

$$\frac{\partial F}{\partial t} + w_x(x,y,t)\,\frac{\partial F}{\partial x} + w_y(x,y,t)\,\frac{\partial F}{\partial y} = \frac{\partial}{\partial x}\Big(\widetilde{\mathcal{D}}_{xx}(F)\,\frac{\partial F}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(\widetilde{\mathcal{D}}_{yy}(F)\,\frac{\partial F}{\partial y}\Big),$$

$$\frac{\partial g}{\partial t} + v_s(z,t)\,\frac{\partial g}{\partial z} = \frac{\partial}{\partial z}\Big(\widetilde{\mathcal{D}}_{zz}(g)\,\frac{\partial g}{\partial z}\Big),$$

number concentration

$$\frac{\partial m}{\partial t} + v_s(z,t)\,\frac{\partial m}{\partial z} = \frac{\partial}{\partial z}\Big(\widetilde{\mathcal{D}}_{zz}(g)\,\frac{\partial m}{\partial z}\Big) \;+\; \frac{2\,\widetilde{\mathcal{D}}_{zz}(g)}{g}\,\frac{\partial m}{\partial z}\,\frac{\partial g}{\partial z} \;+\; \rho_{\text{dep}}\,f_v(z,t),$$

$$\frac{\partial \phi}{\partial t} + v_s(z,t)\,\frac{\partial \phi}{\partial z} = \frac{\Gamma\big(T(z,t)\big)-1}{\Gamma\big(T(z,t)\big)+2}\,\frac{\phi}{v}\,f_v(z,t),$$

microphysics evolution

$$\frac{\partial v_s}{\partial t} + v_s(z,t)\,\frac{\partial v_s}{\partial z} = \nu_{t,ef}\,\frac{\partial^2 v_s}{\partial z^2},$$

$$v_s(z_{ref},0) = 0, \qquad \lim_{z\to-\infty} v_s = v_{ter}, \qquad \lim_{t\to\infty} v_s = v_{ter},$$

settling velocity

$$\mathrm{d}X = -\frac{X-\mu}{\tau}\,\mathrm{d}t \;+\; \sigma_X\,\mathrm{d}W_t, \quad X \in \{\tilde{v}_s, \tilde{\mathcal{D}}_{xx}, \tilde{\mathcal{D}}_{yy}, \tilde{\mathcal{D}}_{zz}\},$$

$$v(z,t) = \frac{m(z,t)}{\rho_{\text{dep}}}.$$

stochasticity

Terminal Velocity

$$Re^2\,C_D(Re^*) = B.$$

$$v_{ter} = \frac{Re\,\mu_{ef}}{\rho_f d_v}.$$

Sol. Approach: Directional-ODE Discretization*

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + f(x,t).$$

traditional way

modern way

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} = D\cdot\frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{(\Delta x)^2} + f(x_i,t_n,u_i^n)$$

$$\frac{du_i}{dt} = D\frac{u_{i-1}(t)-2u_i(t)+u_{i+1}(t)}{(\Delta x)^2} + f(x_i,t_n,u_i^n), \quad t\in[t_n,t_{n+1}].$$

Constant Neighboring Nodes:

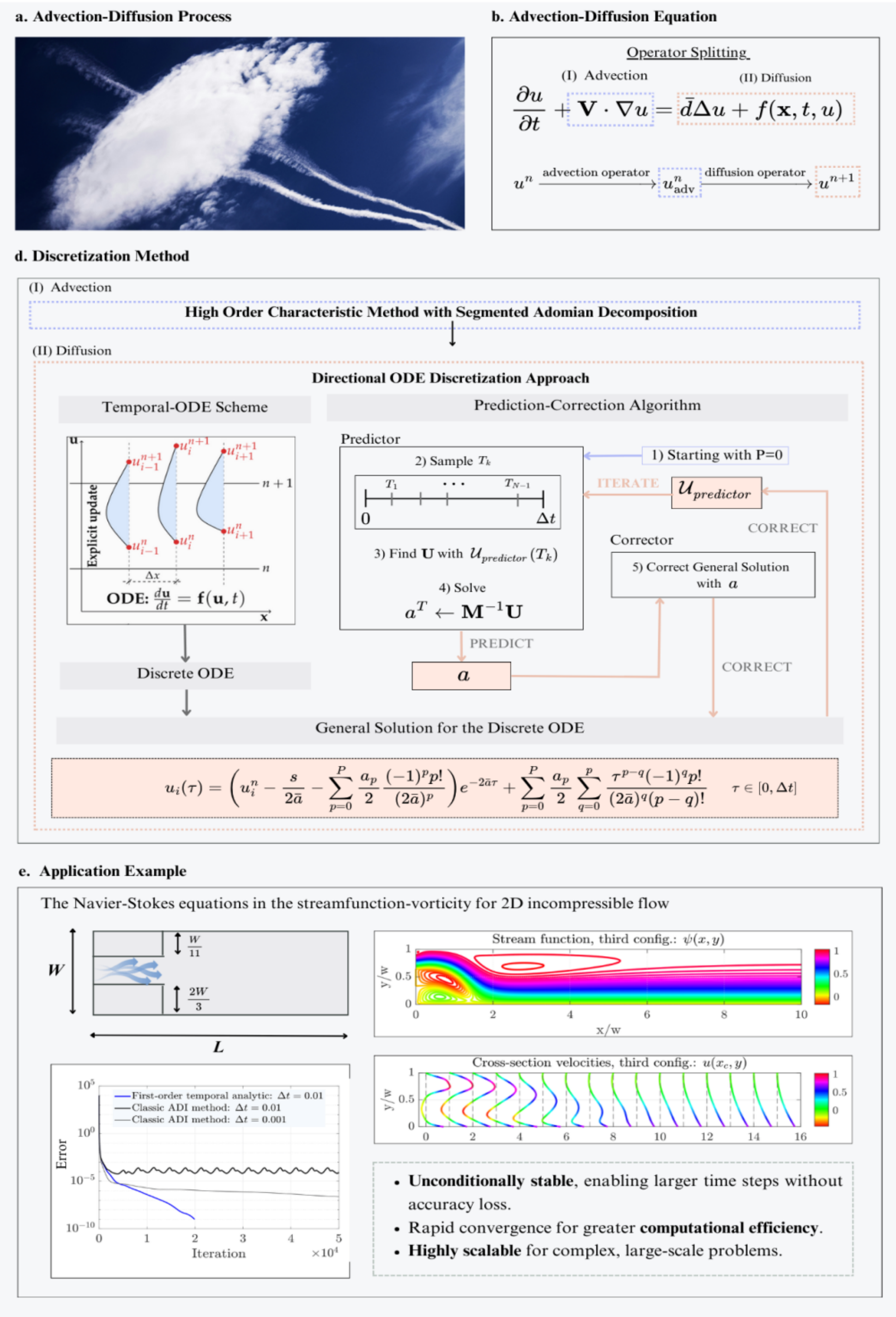
$$\frac{du_i}{d\tau} = -Au_i(\tau)+B,\quad u_i(0)=u_i^n,\quad \tau:=t-t_n,\quad \tau\in[0,\Delta t],\quad \Delta t:=t_{n+1}-t_n.$$

Solution with Constant Neighboring Nodes:

$$u(t) = -\frac{B'}{A'} + \left(\frac{B'}{A'}+u_i^n\right)e^{A'(t-t^n)}.$$

Global Update Formula with Non-Constant Neighboring Nodes:

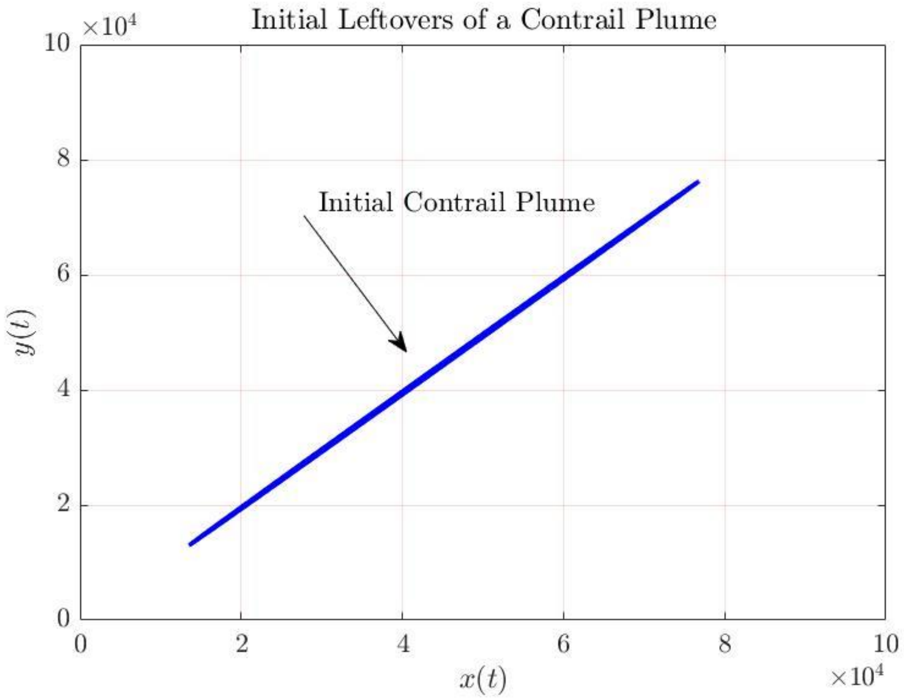
$$u_i(\tau) = \left(u_i^n - \frac{s}{2\bar{a}} - \sum_{p=0}^{P} \frac{a_p}{2}\frac{(-1)^p p!}{(2\bar{a})^p}\right)e^{-2\bar{a}\tau} + \sum_{p=0}^{P}\frac{a_p}{2}\sum_{q=0}^{p}\frac{\tau^{p-q}(-1)^q p!}{(2\bar{a})^q(p-q)!} + \frac{s}{2\bar{a}}.$$

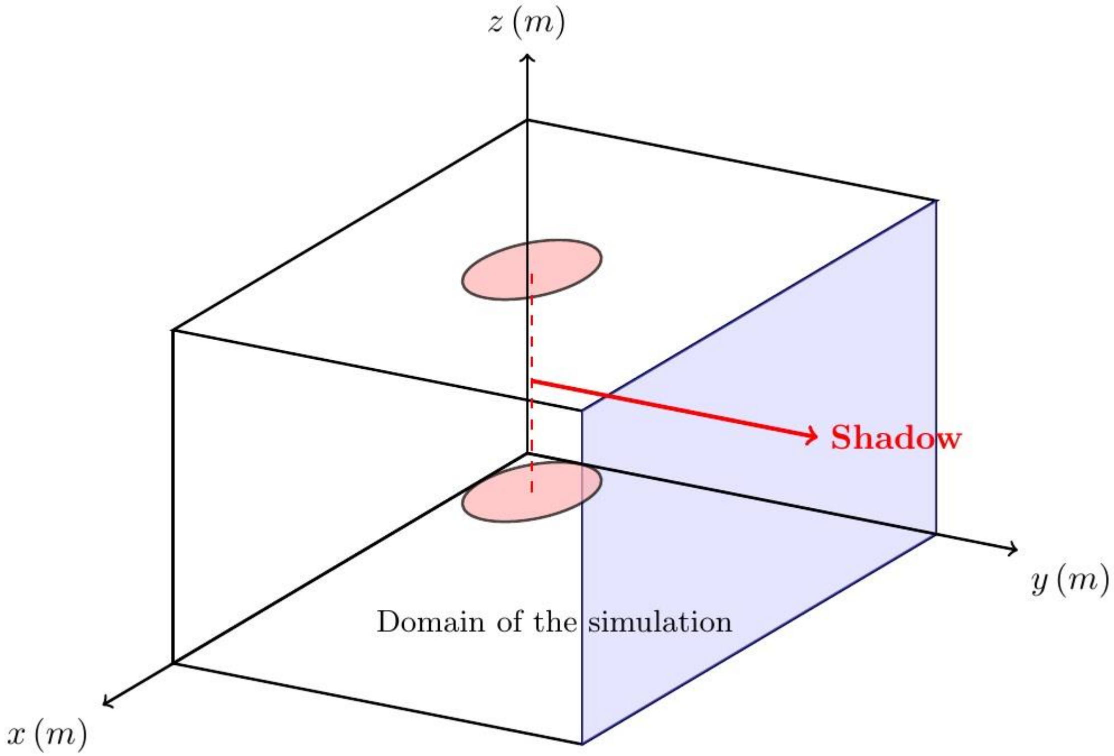


Amin Jafarimoghaddam, Manuel Soler, and Irene Ortiz. A directional-ode framework for discretization of advection-diffusion equations. arXiv preprint arXiv:2506.06543, Jun 2025. arXiv:2506.06543 [math.AP].

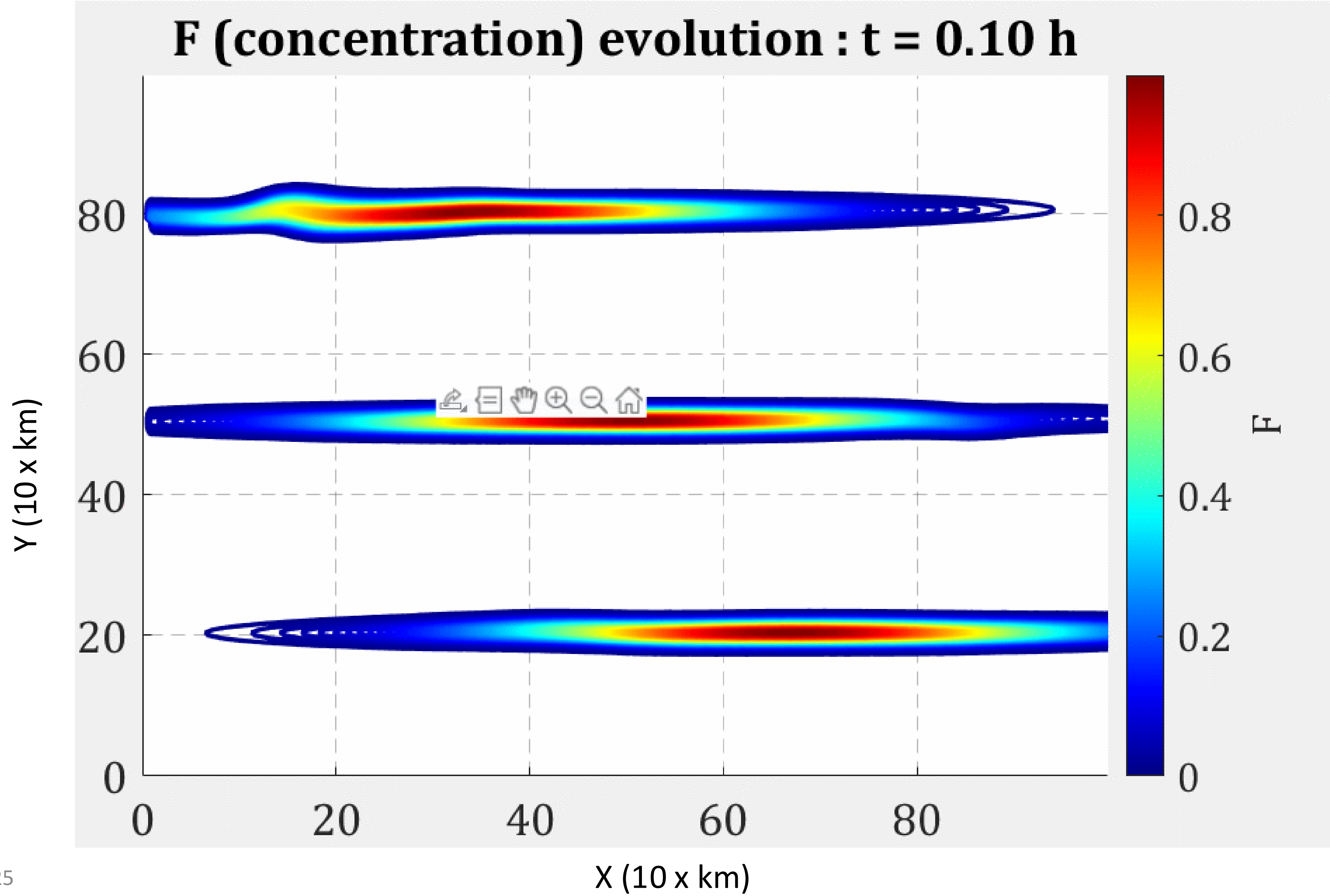
Simulation Results

Plume Initialization:
Reference altitude: 10 km,
Reference Temperature: -61 C *(Yang et al., 2010)*
Background ISSR: Varied Magnitude, ~1.3 km thick,
Background ISSR Profile: Gaussian-Like, Peaking up to 27% + Linear Negative Decay up to 8%
Initial Particles' Shape: Spherical
Initial Particles' Size: 1 micro-meter

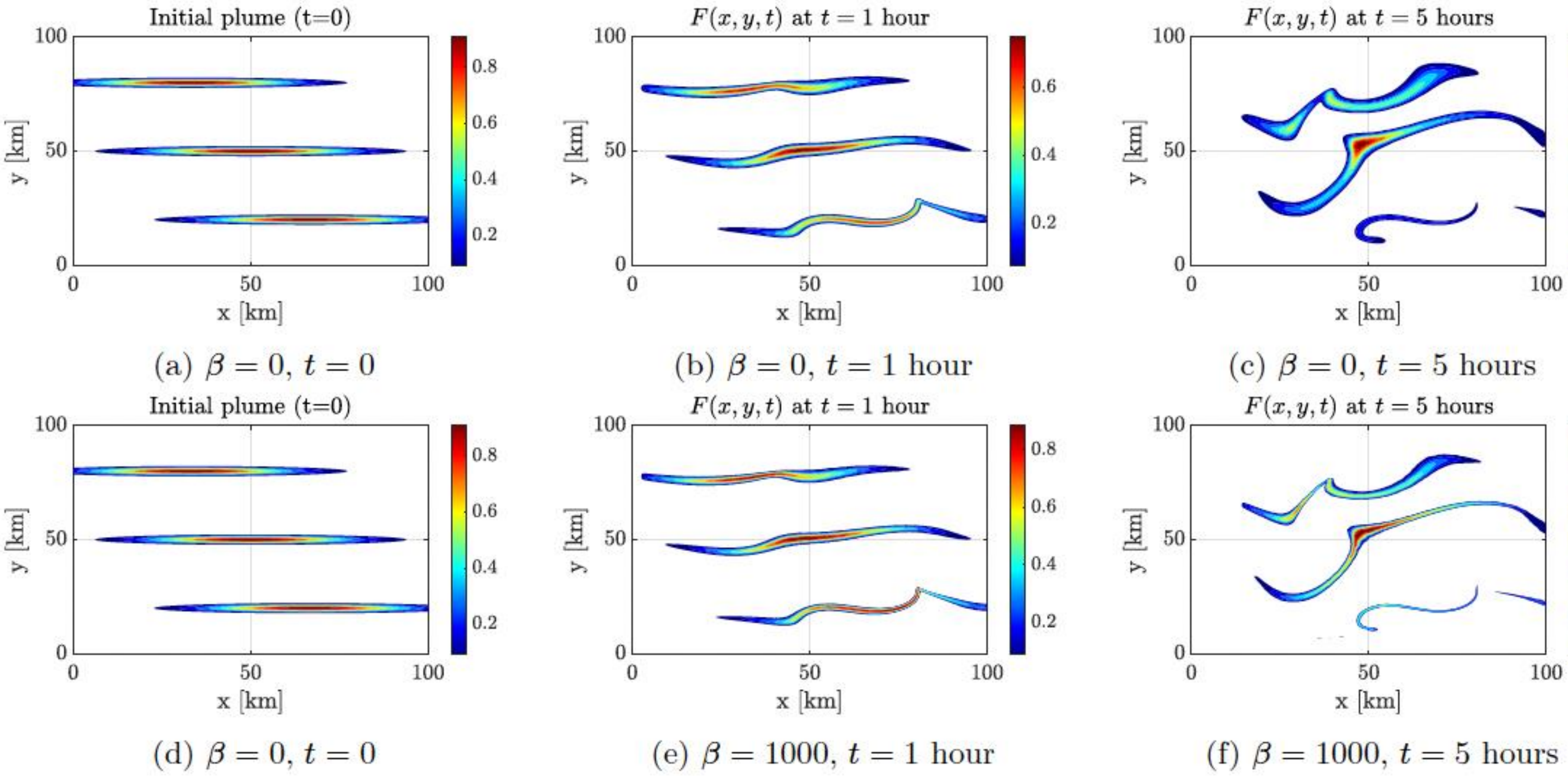




Horizontal Evolution of the plume



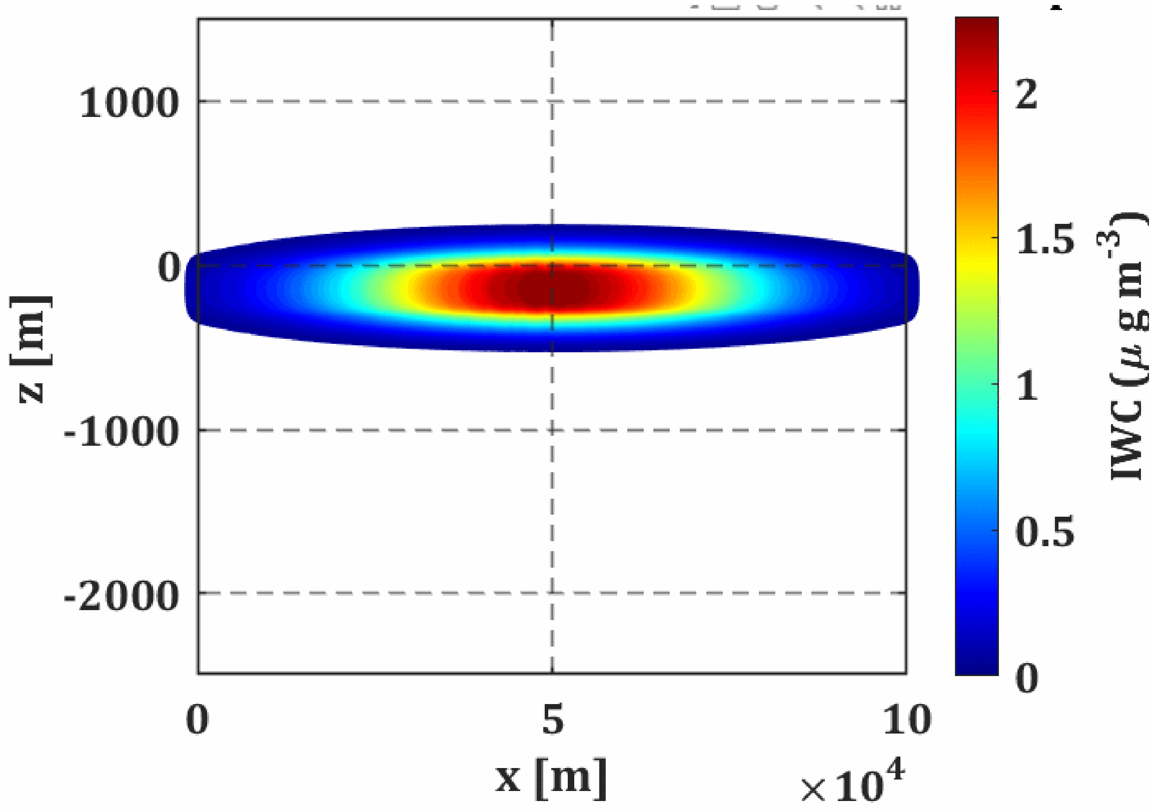
Horizontal Evolution of the plume (diffusion-blocking coefficient)



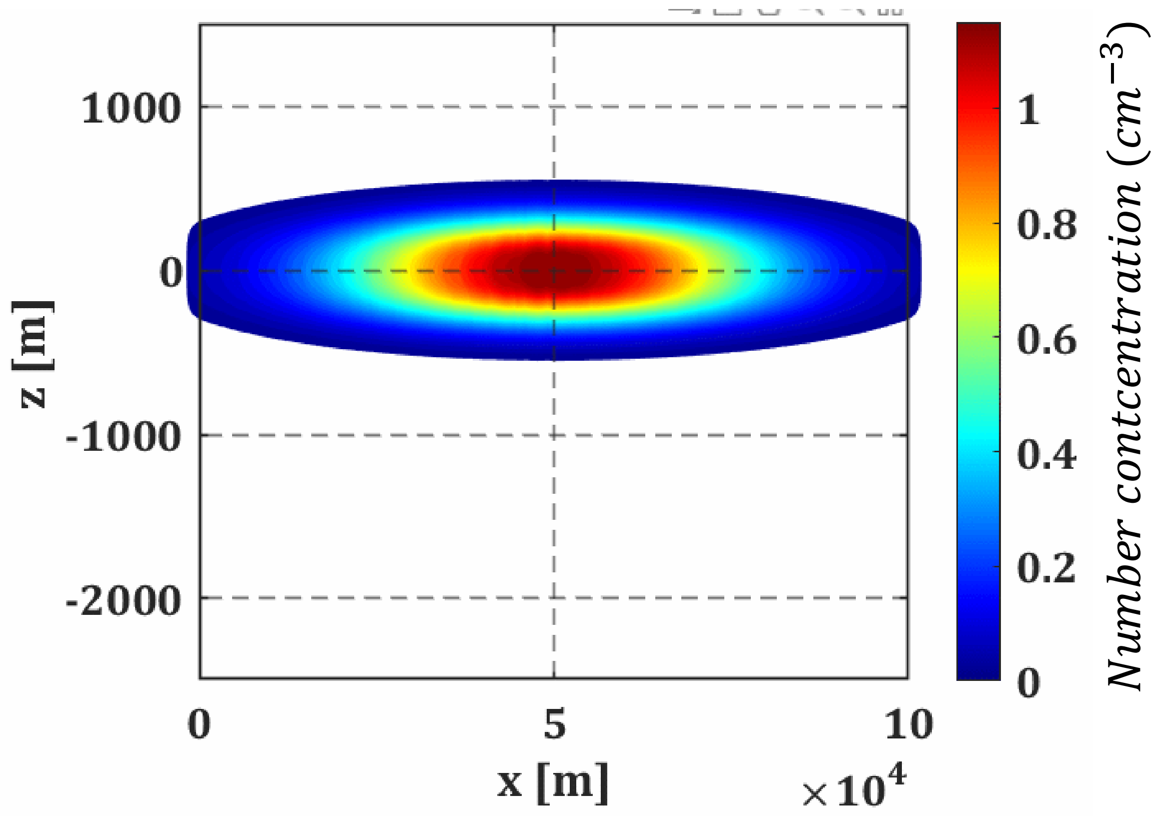
Horizontal evolution of F(x,y,t) –number concentration– for two difussion-blocking coefficients β

Vertical Evolution of the plume

T=-61 C, ISSR Peak=17%
Habit Dynamics Model



IWC: from about 3 mins to 6 hours with about 3 mins step

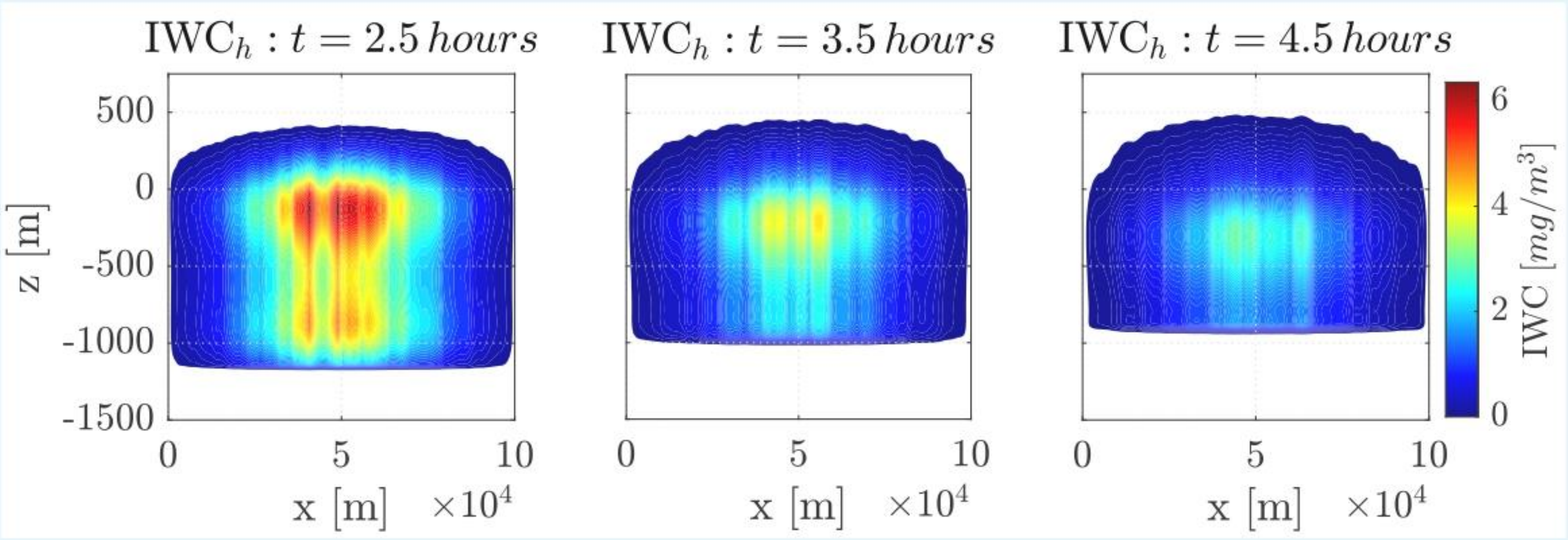


Number Cons.: from about 3 mins to 6 hours with about 3 mins step

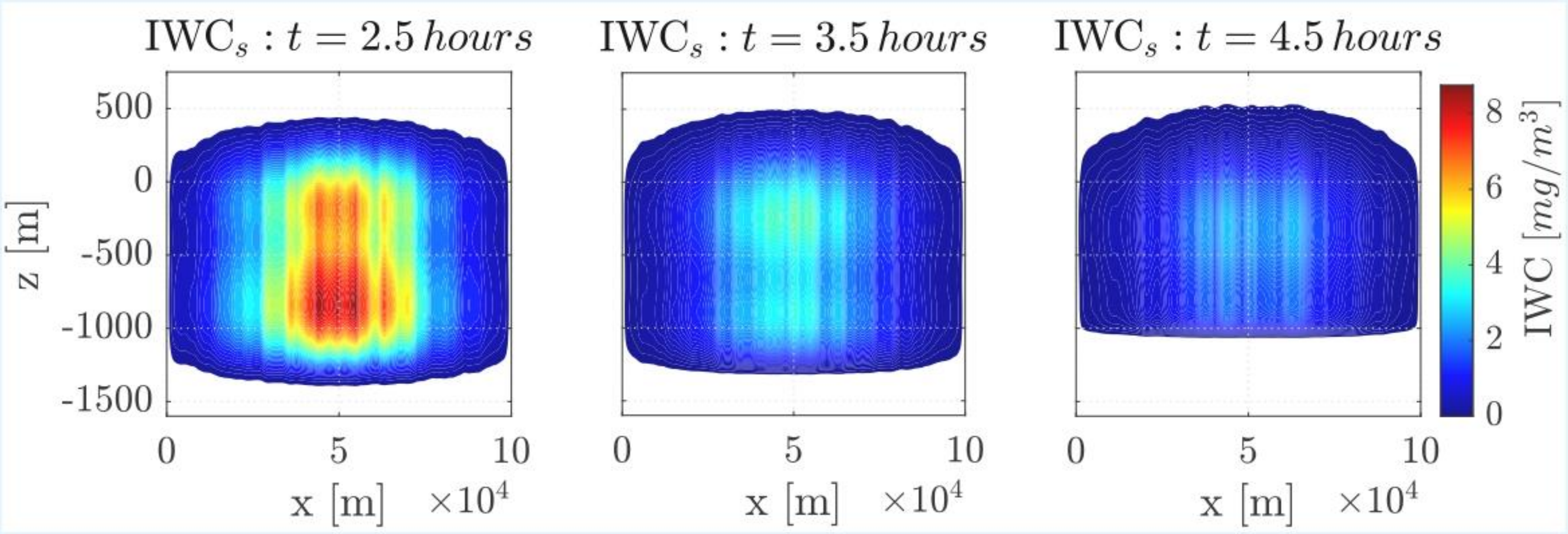
Vertical Evolution of the plume: Habit vs. spherical model

T=-61 C,
ISSR Peak=17%

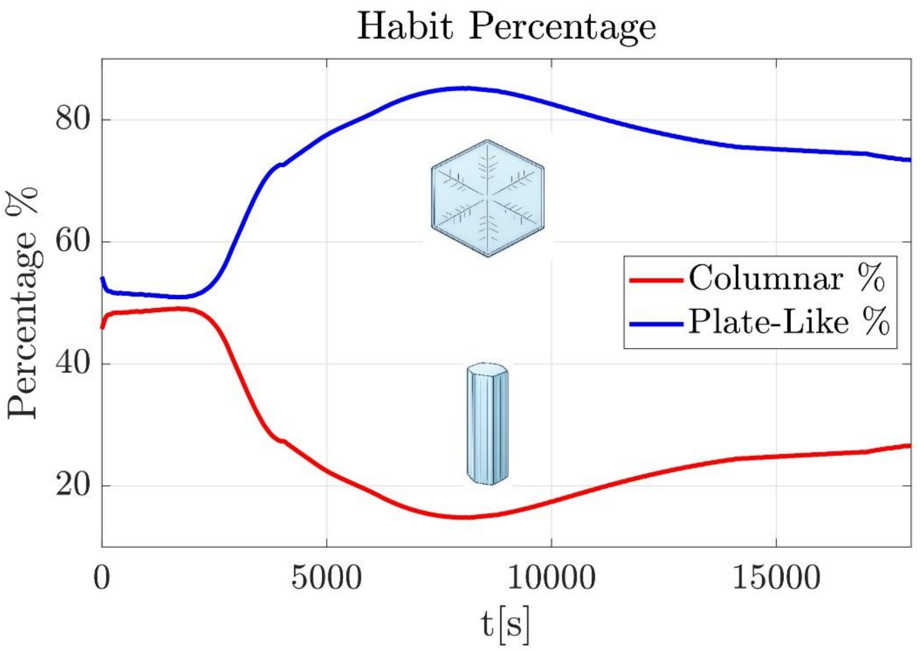
Habit Dynamics Model



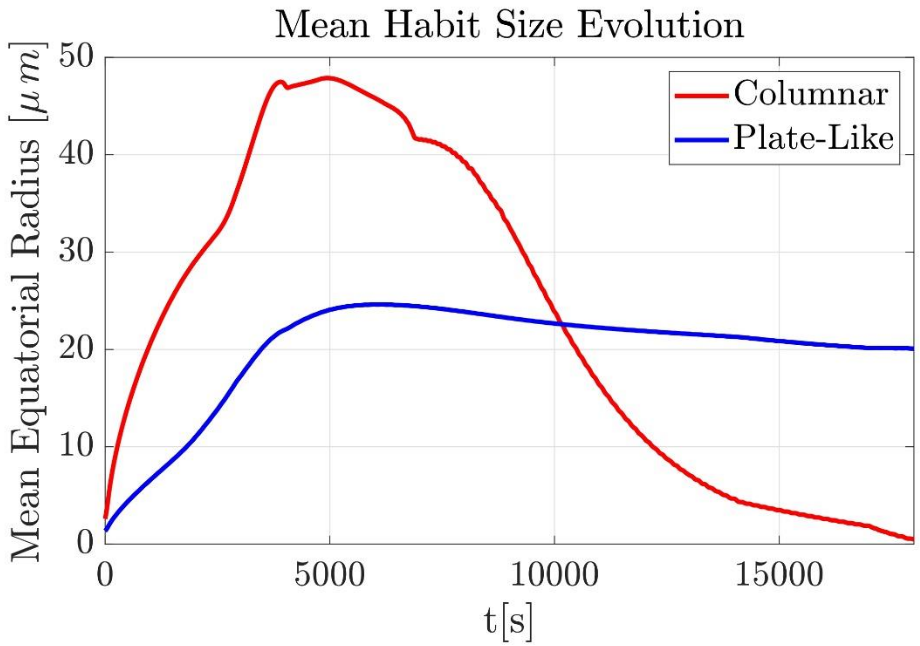
Spherical Model

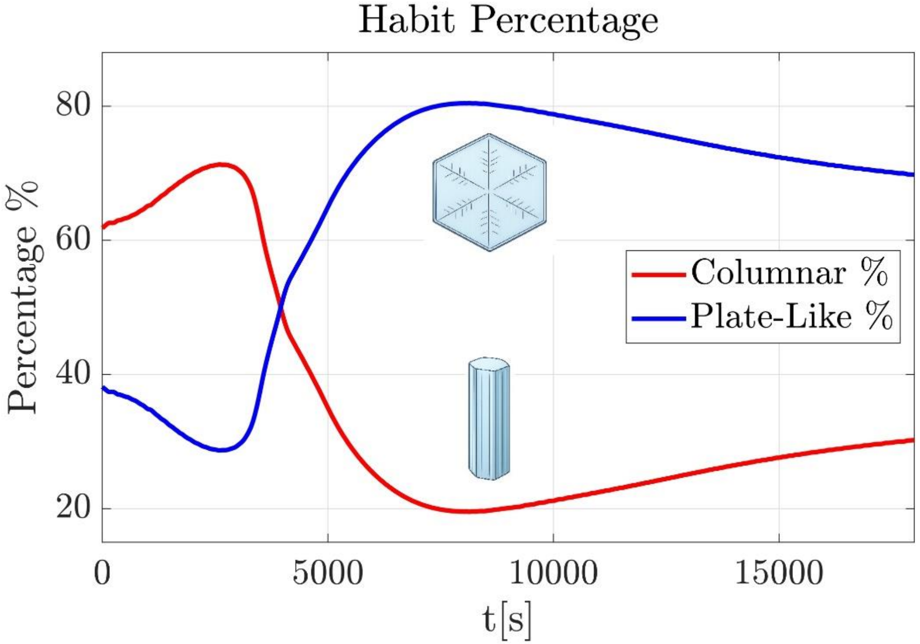


Habit Shape

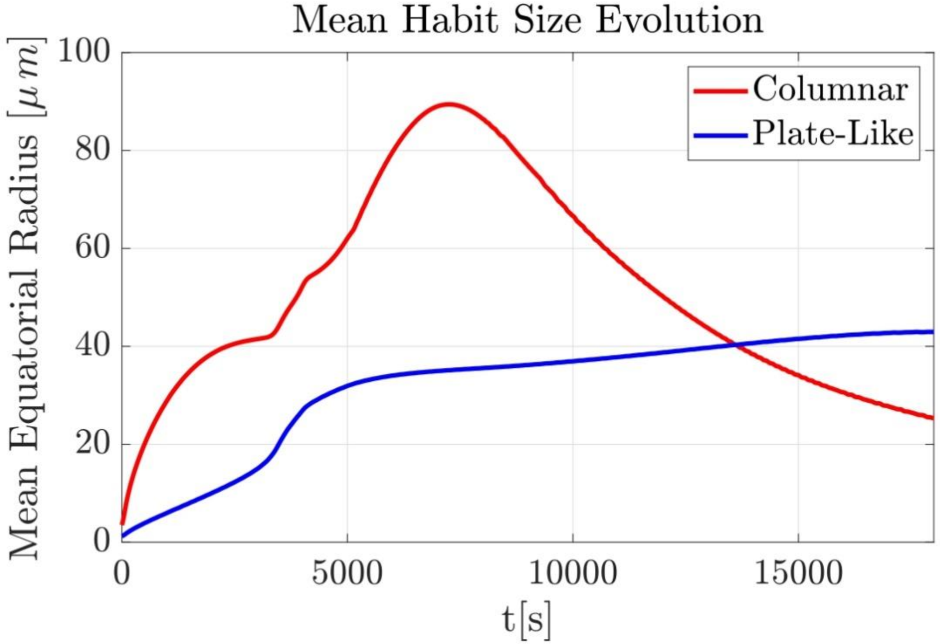


T=-61 C,
ISSR Peak=17%



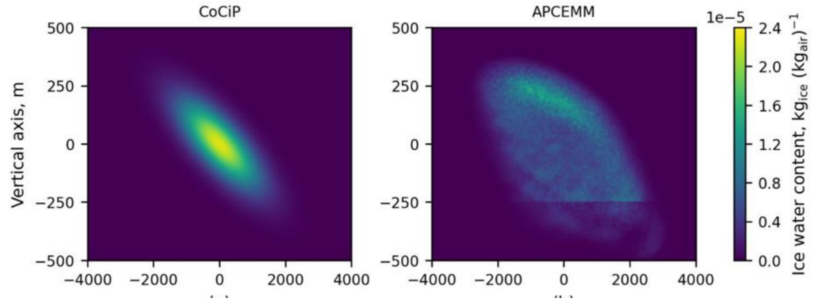


T=-61 C,
ISSR Peak=27%



(Some) Future Work

- Benchmarking w.r.t. state of the art contrail models (e.g.,CoCiP and APCEMM)
- Validation using Remote sensing devices (e.g., EarthCARE LIDAR) & Ground Visible and IR Cameras.
- PINNs for contrail detection.
- Integrating/parametrizing short-range phases.
- Application to Monitoring Reporting and Verification.



Akhtar Martínez el al. (2025).

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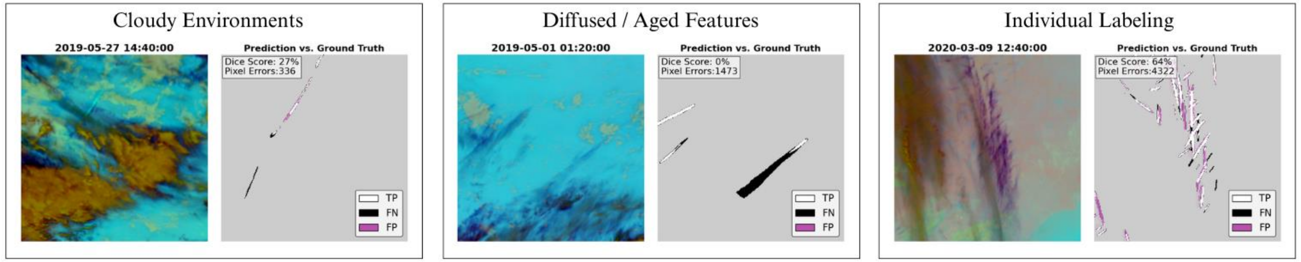


(Ortiz et al., 2025)

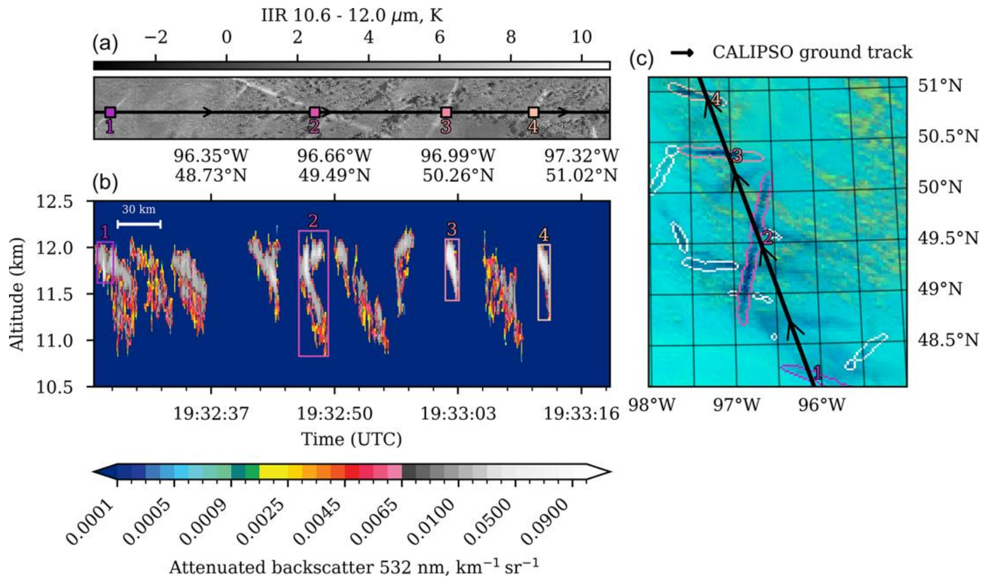
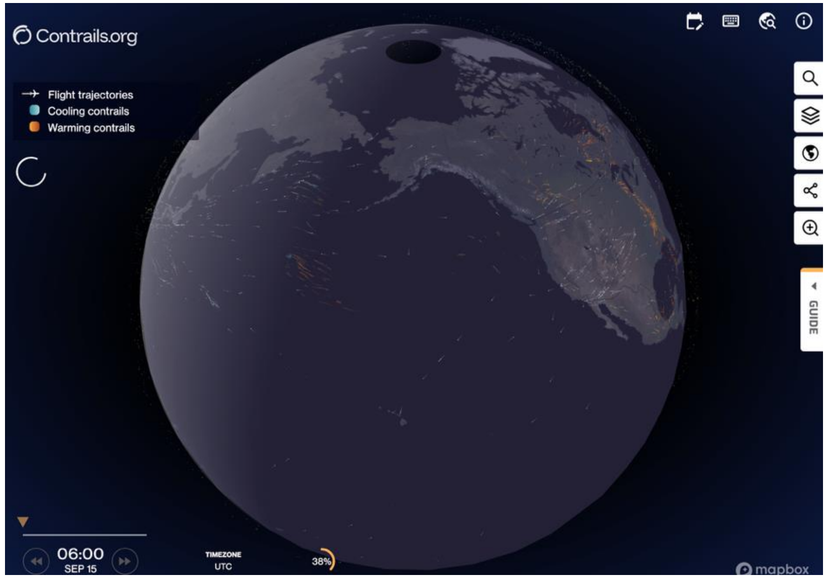


Figure 17: Example from Meijer et al. (2024) showing contrails detected in GOES-16 ABI imagery (c) located in CALIOP LIDAR data (b). Contrails are numbered to enable comparison between the LIDAR cross-sections and the "top view" from the GOES-16 satellite.



Contrails.org visualization

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Thanks!

References

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