

# Solution to Practice Exam 3

2022 Fall

## 1 Quick warm-up calculations

Let  $i, r, \pi, t$  denote the nominal interest rate, the real interest and the inflation rate and the tax rate respectively. Without tax:

$$r \approx i - \pi.$$

After-tax real interest rate:

$$r_t = i(1 - t) - \pi$$

1.  $r = 0 - 6\% = -6\%$
2.  $r = 6\% - 6\% = 0\%$
3.  $r^t = 0 - 6\% = -6\%$
4.  $r^t = 6\% \cdot \frac{2}{3} - 6\% = -2\%$

When the nominal rate is positive, taxation lowers the real interest rate that an investor receives. The inflation rate is not affected by taxation. But the nominal interest rate becomes lower after taxation if it is positive.

## 2 Rolling over a traditional IRA to a Roth IRA

Traditional accounts: pay no taxes on money you contribute, but pay taxes later when you withdraw money (at rate 30% in this question).

Roth accounts: pay taxes now on contributions (at a rate of 20%), but no taxes later on withdrawals.

a. After 10 years, the money in your savings account grows to

$$\$30,000 \times 1.05^{10} = \$48,866.84.$$

When you withdraw the money from your traditional account, you are taxed at 30%, so your after-tax savings is

$$\$48,866.84 \times 70\% = \$34,206.79$$

**b.** If you convert your account to Roth, your past contributions are taxed at 20%, you have to pay tax of

$$\$30,000 \times 20\% = \$6,000$$

and are left with

$$\$30,000 - \$6,000 = \$24,000$$

to save in your Roth account.

**c.** After 10 years, the money in your Roth account grows to

$$\$24,000 \times 1.05^{10} = \$39,093.47.$$

You won't be taxed when you withdraw the money, so this is the total amount you get to consume.

**d.** It is better to convert my traditional IRA to a Roth IRA. Because my current tax rate is lower than my future tax rate, I benefit from converting to a Roth account now as it allows me to pay tax based on my current tax rate.

**e.**

- There can be a change in the tax policies.
- Filing as single, filing jointly as married, married filing separately, or filing as a head of household have different tax brackets and rates.

### 3 Retirement saving with a deferred payout annuity

Note that through the entire question, when you draw money from your 401k account, you have to pay taxes at a rate of 28%.

**a.** To spend \$80k annually, your monthly spending is approximately \$6666.67. You will receive \$2000 of social security benefits every month. So you need to receive \$4666.67 from the annuity every month. So you need to invest

$$\frac{\$4666.67}{\$2400} \times \$100,000 = \$194,444.58.$$

This is the after-tax amount you need, so the pre-tax amount you have to withdraw from your 401k traditional account is

$$\$194,444.58 \div (1 - 28\%) = \$270,061.92.$$

b. The after-tax amount of money you need to withdraw from your 401k traditional account each year is

$$\$80,000 - \$2,000 \times 12 = \$56,000.$$

The corresponding pre-tax value is

$$\$56,000 \div (1 - 28\%) = \$77,777.78$$

The present value of this payment stream is

$$\$77,777.78 \times \frac{1 - (\frac{1}{1+3\%})^{15}}{3\%} = \$928,506.09.$$

c. The total amount of money you need to have in your account now is

$$\$928,506.09 + \$270,061.92 = \$1,198,568.01$$

## 4 Adjusting asset allocation to your age

a. In 40 years, the balance will grow to

$$\$15,000 \times 1.1^{40} = \$678,888.83$$

b. When you retire, a third of your balance is expected be in equities:

$$\frac{1}{3} \times \$678,888.83 = \$226,296.28.$$

c. I would like to keep as close as I can to a constant dollar allocation and I know when I retire, the dollar value in equities should be \$ 226,296.28. My current saving is \$15,000 and I cannot use leverage, so the best I can do is to put all the savings in equities. That is, I will keep holding 100% equities until my balance grows to \$ 226,296.28. After this point, my balance will still grow at 10% per year. So I will have to gradually decrease the proportional allocation in equities every year to maintain that the dollar value of equities at \$ 226,296.28. Each year I have to reduce my equity holdings by  $\frac{1}{11}$  until the proportion reaches  $\frac{1}{3}$ .

d. It will take approximately 28.47 years for my balance to reach \$ 226,296.28. So from year 29, I will stop holding 100% and start to reduce the weight.

$$\$15,000 \times 1.1^x = \$226,296.28 \implies x \approx 28.47$$

e. **Time diversification:** your expected dollar return on equities over your lifetime depends on your average lifetime dollar allocation to equities, and not the timing of that allocation. But the risk you take with

your equity investing is less if you spread the dollar investment out over time rather than concentrating it in a single period. “Half stocks all the time” is safer than “All stocks half the time”.

f. Target date funds implement the strategy of time diversification. They start out with major weights in equity and shift into bonds as your retirement date approaches. The glide path of weights in equity also diminishes financial risk as retirement approaches. This makes sense because young people have future earning power and it is relatively safe compared to the stock market. Therefore young people already have a large implicit investment in safe assets and should take on more risk in the financial portfolio for compensation. As people move closer to retirement, their human capital diminishes and their financial savings increase. Hence they should gradually increase their investment in safe assets while reduce holdings of equities.

## 5 Adverse selection and insurance demand

a. The actuarially fair premium for a dollar of insurance is

$$P_{AF} = \frac{Pr(Loss)}{(1 + R)^N} = Pr(Loss) = 0.015.$$

b. The expected probability of loss from the insurance company’s view is

$$\frac{1}{3}(0.01 + 0.015 + 0.02) = 1.5\%.$$

The premium is therefore \$0.015 for a dollar of insurance.

c. For the first group, their actuarially fair premium is \$0.01. Their  $m$  is 0.5. For the second group, their  $m$  is 0 as they get their actuarially fair premium. For the third group, their actuarially fair premium is \$0.02. Their  $m$  is -0.25.

d. For the first group,  $\frac{m}{y} = 0.1$ . They will not buy any insurance. For the second group,  $\frac{m}{y} = 0$ . They will choose full coverage. For the third group, their  $m$  is negative. According to this formula, the third group will also buy full insurance.

e. The pool of customers are the second and third group. So now the expected probability of loss is

$$\frac{0.015 + 0.02}{2} = \$0.0175,$$

which is also the actuarially fair premium for a dollar of insurance.

f. With the new premium, the second group will no longer insure fully as the insurance company is charging them a positive markup now. They only insure partially meanwhile the third group still insure fully. As they change their behaviors, the insurance company cannot break even at this new premium \$0.0175. Therefore the insurance company has to increase the premium again, which is going to drive down

the coverage of the second group further. The iteration will continue until the premium charged is too high for the second group to insure.

Thus in equilibrium, the pool of customers is the third group only. The insurance company will set the premium based on their probability and the actuarially fair premium will be \$0.02 for one dollar of insurance. The third group will choose to insure fully or partially based on the markup charged by the insurance company.

## 6 Financial advice

a. The goal of the study is to understand whether advisers actively de-bias their clients or instead exaggerate existing biases, especially biases that help the adviser's own interests by increasing fees and turnover. The auditors were randomly assigned to four different portfolios. The auditors tracked the information requested from them, the advice given and other features of the interaction.

The four different portfolios are "Chasing Returns", "Company Stock", "Index Funds" and "Cash". For "Chasing Returns", the auditor holds a portfolio which is heavily invested one sector ETF that performed well in the previous year and the auditor expresses the interest to invest in those industries that had performed well in the recent years. In "Company Stock", an auditor's portfolio is heavily invested in stocks of his employer. In "Index Funds", an auditor holds well-diversified, low-fee portfolio that is consisted of index funds.

b. From the table, we can see that the advisers were significantly more supportive for the returns-chasing portfolios than for either the company stock or the index portfolio. The likelihood of the adviser against the current strategy is much higher for the index portfolios than for the returns-cashing portfolios or for the company stock. But we all know that for a beneficial adviser, he or she should encourage the index portfolios and discourage the returns-chasing portfolio and the company stock portfolio. If all investors were helpful, we would have observed that, 100% supportive responses for index portfolios and 0% supportive responses for the returns-chasing and company stock portfolios; 0% against the index portfolios and 100% against the returns-chasing and company stock portfolios

c. Advisers tend to start off differently from the ultimate suggested strategy and reflect more of the sales pitch. The first two rows are the advisers' final recommendations whereas the second two rows are their initial responses. We can see that for all three strategies, advisers were less likely to react negatively at first, compared to the likelihoods of them going against these strategies at the end. For company stock and index funds portfolios, the advisers were more likely to react positively at first, compared to the likelihoods of them supporting the current strategy at the end.

The authors argue that advisers may try to be more positive initially, even through they want to change

the clients' strategies ultimately. It is possible that advisers feel constrained by what they know is the better advice, or that they do not want to undermine their credibility by being outright dismissive of diversification and index funds, assuming that most people have at least heard through the media and other sources that these are good investment strategies. It also reflects that the advisers are aware that they are facing a sales situation and they cannot bluntly criticize what clients might have done in the past.

**d.** The retail advisers studied in this paper are usually paid based on the fees they generate, but not based on the AUM or the performance of the portfolio. The first two rows of the table show that advisers do not serve to de-bias clients. They in fact exaggerate biases that are in the adviser's financial interest while advising against those that do not generate fees. Although they might support the client's existing strategies at first, they have no problem discouraging clients from investing more in their strategies ultimately, especially if it goes against the adviser's interests.

**e.** The fiduciary standard is a legal requirement for an adviser to pursue the client's best interest. It limits the ability of advisors to receive commissions for recommending expensive products. It can help improve the problem. But it cannot eliminate the problem as it still allows some commissions.

## 7 Short questions

**Present bias.** People with present-biased preferences make a decision today that reduces future welfare in a way that the individual later regrets. People who are aware of their present bias demand commitment devices.

**The winner's curse.** In auction bidding, if you bid the amount you believe the item to be worth based only on your private information, you will find you bid too much. The reason is that you only win the auction if your private information is better than everyone else's, in which case it is likely to be at least slightly over-optimistic. The winner's curse also applies to the case where people shop for financial products. If these products are hard to value and people form independent estimates of their value, some people will overvalue the products. If these people don't learn from others but trust their own judgements, then they will buy the products even if they are priced too high. Seller can therefore earn profit by marketing overpriced financial products.

**Base rate neglect.** Bayes' Law says that

$$Pr(Sick|Positive) = Pr(Positive|Sick) \left( \frac{Pr(Sick)}{Pr(Positive)} \right).$$

But people often misform their believes like

$$Pr(Sick|Positive) = Pr(Positive|Sick).$$

Base rate neglect refers to the mistake that people neglect the ratio  $\frac{Pr(Sick)}{Pr(Positive)}$ .

**The breakeven inflation rate.** The difference between the nominal bond yield and the TIPS yield is the breakeven inflation rate. It reflects market participants' expectations of inflation. If you believe future inflation will exceed the breakeven inflation rate, you should buy TIPS.

**The LTV ratio.** The LTV ratio stands for loan to value ratio. No, there is also a limit on the PITI ratio (ratio of mortgage principal, interest, taxes, and insurance to income). This limit also depends on your credit score.

**Stablecoins.** Most cryptocurrencies have unstable USD value so they are not suitable for safe saving, but stablecoins have a fixed USD value. They are similar to unregulated banks in that they are subject to runs. So no confidence.

**CARD Act disclosure.** This act introduced a simplified disclosure as well as some fee regulation. The disclosure highlights the time it will take to pay off the debt by making the minimum payment, and states the payment needed to pay off the debt in 3 years. It has successfully made minimum payments less common and 3-year payments more common.