Econ70 Section 4

2023 Fall

Review on probability

Exercise: roll a dice!

Suppose that somebody secretly rolls a six-sided dice.

We are interested in the "moments" that characterize this event:

- What is the average?
  - **First moment**: "Mean"
- How risky is this? How far are the values spread out around the average?
  - **Second moment**: Variance, standard deviation.

Let's first define the problem in a precise, mathematical way.

Exercise: roll a dice!

A **random variable** formalizes a random event, and is a variable that takes values according to a certain probability distribution.

- Here the outcome of the roll is a random variable; the possible values it can take are $\{1, 2, 3, 4, 5, 6\}$, the probability of each outcome is $\frac{1}{6}$.

Let $\mathcal{X}$ denote the random variable and let x be the actual outcome.

Exercise: roll a dice!

We can rewrite the situation using a **probability distribution** function $f(x)$. $f(x)$ gives the probability that our roll is equal to x:

$$f(x) = Prob(\mathcal{X} = x)$$

Note that $0 \leq Prob(\mathcal{X} = x) \leq 1$ and $\sum_x f(x) = 1$.

- In our situation, $f(x) = \frac{1}{6}$, where $x \in \{1, 2, 3, 4, 5, 6\}$

Exercise: roll a dice!

The **mean** is the average of possible values, weighted by probabilities

- $1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{21}{6} = 3.5$

The **variance** measures whether the possible values concentrate around the mean, or they vary around and take vastly different values.

- $\sum_{i=1}^{6}(i - 3.5)^2 \times \frac{1}{6}$

The **standard deviation** is the square root of the variance. (Sometimes this is more useful than variance)

- $\sqrt{\sum_{i=1}^{6}(i - 3.5)^2 \times \frac{1}{6}}$

Exercise: roll a dice!

More generally, the **mean** for a random variable is calculated by multiplying each value by its probability, and then summing everything up

- $\mu = \sum_x xf(x)$

The **variance** is calculated by squaring each value and multiplying by its probability, summing them up, and then subtrating the square of the mean

- $Var(x) = \sum_x (x - \mu)^2 f(x)$

The **standard deviation** is the square root of the variance, and is expressed in the same unit of the original values of the random variable

- $\sigma = \sqrt{Var(x)}$

Exercise: roll two dice!

Suppose that somebody secretly rolls two fair six-sided dice.

- What is the probability that the first one is 2?
- What is the probability that the sum of the two is no greater than 5?
- What is the probability that the first one is 2 and the sum of the two is no greater than 5?
- Given the sum of the two is no greater than 5, what is the probability that the first one is 2?

Exercise: roll two dice!

What is the probability that the first one is 2?

Table 1

+		D_2					
		1	2	3	4	5	6
D_1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- There are 36 combinations of rolled values of the two dices, each of which occurs with probability $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.
- $Prob(D_1 = 2) = \frac{6}{36} = \frac{1}{6}$
- This is the unconditional probability of the first dice is 2.

Unconditional probability is the probability of one event without any other conditions.

Exercise: roll two dice!

What is the probability that the sum of the two is no greater than 5?

Table 2

+		D_2					
		1	**2**	**3**	**4**	**5**	**6**
D_1	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- There are 10 combination that sum to no greater than 5
- $Prob(D_1 + D_2 \leq 5) = \frac{10}{36}$

Exercise: roll two dice!

What is the probability that the first one is 2 and the sum of the two is no greater than 5?

Table 3

+		D_2					
		1	2	3	4	5	6
D_1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- There are three combinations where, the first is 2 and the outcomes sum to no greater than 5
- $Prob(D_1 = 2, D_1 + D_2 \leq 5) = \frac{3}{36} = \frac{1}{12}$

Joint probability is the probability of two events occurring together.

Exercise: roll two dice!

Given the sum of the two is no greater than 5, what is the probability that the first one is 2?

Table 3

+		D_2					
		1	2	3	4	5	6
D_1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- There are 10 combination that sum to no greater than 5 and 3 of them have the first dice being 2.
- $Prob(D_1 = 2 | D_1 + D_2 \leq 5) = \frac{3}{10}$

Conditional probability is the probability of an event occurring, given that another event has already occurred.

Exercise: roll two dice!

Given the first one is 2, what is the probability that the sum of the two is no greater than 5?

Table 3

+		D_2					
		1	2	3	4	5	6
D_1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- $Prob(D_1 + D_2 \leq 5 | D_1 = 2) = \frac{3}{10} \times \frac{10}{36} / (\frac{1}{6}) = \frac{1}{2}$.

Bayes' Rule relates the probability of an event occurring with the probability of an related event occurring through coniditional probability.

Summary of concepts

- **Unconditional probability**: the probability of one event happening irrespective of any other event
- **Joint Probability**: the probability of two independent events (A and B) happening together

$$P(A \cap B) = P(A) \times P(B)$$

- **Conditional probability**: the probability of an event (A) happening, conditional on another event (B) already happend

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Bayes' Rule**:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$