Math 153 Final Project Proposal

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We will examine a modified version of the evolutionary language game introduced in "The Evolution of Language" (Nowak and Krakauer, 1999). Specifically, we relax the assumption that all concepts are weighted equally, and introduce a mutant who can communicate a previously 'undefined' concept.

Our modifications were inspired by the idea that small subsets of a larger population may come up with new words for previously unnamed concepts. For example, the scientific community continues to create words to describe new discoveries. Teens invent abbreviations during texting to express different emotions. In the early stages of language development, humans must also have created new words to express increasingly abstract concepts. We did not see a mechanism for this creative process to reach fixation in the existing model.

The Basic Model: An Evolutionary Language Game

We examine Nowak's (1999) language game on a finite population, with a language L defined by sending and receiving matrices $P_{m\times n}$, $Q_{n\times m}$. We have the following criterion for the sending and receiving matrices:

$$P = \begin{pmatrix} p_{1,1} \cdots p_{1,m} \\ \vdots \ddots \vdots \\ p_{n,1} \cdots p_{n,m} \end{pmatrix}, Q = \begin{pmatrix} q_{1,1} \cdots q_{1,n} \\ \vdots \ddots \vdots \\ q_{m,1} \cdots q_{m,n} \end{pmatrix}$$

where $P, Q \in \mathbb{R}^+_{n\times m}$ and $\sum_{j=1}^m p_{ij} = 1\ \forall i$, $\sum_{i=1}^m q_{ji} = 1\ \forall i$. If a sender P interacts with receiver Q, then the probability that they successfully communicate concept i is $\sum_{j=1} p_{ij} q_{ji}$. Since there are n concepts, we define the "communication potential" of a language as

$$\pi(P,Q) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} q_{ji} = \mathrm{tr}(PQ).$$

The payoff function for communication between individuals with two different languages $L1 : (P_1, Q_1)$ and $L2 : (P_2, Q_2)$ is

$$f[(P_1,Q_1),(P_2,Q_2)] = \frac{1}{2}\mathrm{tr}(P_1,Q_2) + \frac{1}{2}\mathrm{tr}(P_2,Q_1) = f[(P_2,Q_2),(P_1,Q_1)],$$

since we assume that each individual has equal probability of being the sender or receiver. This yields a symmetric payoff. The average payoff of the communication game for an individual, given its own type (language) and the languages present in the rest of the population, is equal to its fitness when analyzing replicator dynamics.

Pawlowitsch (2007) finds that, in a finite population, the only evolutionary stable strategies in this game are bidirectional (bijective) languages, where one concept is associated with one signal with probability 1 for both senders and receivers. In other words, $p_{ij}{=}q_{ji}$ for all concepts i and signals j, this value is either 0 or 1, and there is exactly one 1 entry in each column and each row of both the sending and receiving matrices. This is true if and only if a language is "efficient," or, when $P = Q^T$. Intuitively, individuals send concepts with signals that they themselves would understand to have the same meaning.

Our Modification: Concept Weighting

We first relax the assumption that all concepts have equal weights. In our adapted model, the successful communication of certain concepts gives a higher fitness payoff than others. So the payoff of an interaction becomes $\mathrm{tr}(WPQ)$, where W is a $n \times n$ diagonal matrix of concept weights. We retain the assumption that all players have identical priorities, so W is the same for an entire population.

Ex. 1: *Concept weights for non-bijective languages.*

Let

$$\text{L1}: (P_1, Q_1) = (\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix})$$

and

$$\text{L2}: (P_2, Q_2) = (\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix})$$

So the payoff for each protolanguage against itself under equal weights is simply $f[(P_1, Q_1), (P_1, Q_1)] = f[(P_2, Q_2), (P_2, Q_2)] = 2$, since the last two concepts (the last two rows of P_1, the last two columns of Q_1) are mis-communicated under $L1$, and the first two concepts are mis-communicated under $L2$, so each language only has a payoff 2 out of possible total payoff 4. However, suppose we introduce the following weight matrix

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1+\omega \end{bmatrix}$$

where $\omega > 0$ is small. Now, $L2$ has a greater fitness than L1, since it correctly communicates the more 'important' (heavily weighted) final concept.

$$f[(P_2, Q_2), (P_2, Q_2)] = \mathrm{tr}(WP_2Q_2) = \mathrm{tr}(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1+\omega \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}) = 2 + \omega$$

We see that varying the weight distribution can result in unequal fitness levels for languages with the same communicative potential, but different concepts communicated correctly.

However, as shown by Pawlowitsch (2007), efficient languages are the only ESS's in a finite population. But if all languages are one-to-one mappings from all concepts to all signals, then every language will have the same payoff, even with varying concept weights.

We therefore a consider a case where $n \neq m$. Specifically, let $n = m + 1$, so that there must be one concept that is unmatched with a signal in an efficient language. In this case, two unique efficient languages will have a different set of matched concepts. Below, we show that (somewhat obviously), the language with the 'heavier' set of matched concepts will have a fitness advantage.

Ex. 2: *Concept weights with efficient languages, $n = m + 1$*
Consider the following languages $L : (P, Q)$ and $L' : (P', Q')$, which are efficient under our slightly modified formulation of efficiency, where one concept remains unmatched. Here, we have $n = m + 1 = 4$, $m = 3$. (Recall that for all efficient languages, $Q_l = P_l^T$.)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, P' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Suppose we have the same matrix of concept weights, W, as in Example 1. As expected, we have

$$f[(P, Q), (P, Q)] = 3 < 3 + \omega = f[(P', Q'), (P', Q')]$$

since concepts 1, 2, and 4, which L' communicates correctly, have a greater total weight than concepts 1-3, which L communicates correctly.

Further Analysis and Modifications

As described in Nowak et. al (2004), symmetric payoff yields clean results. The introduction of a mutant strategy (P', Q') into a population of (P, Q) will lead to fixation with probability greater than $\frac{1}{N}$, iff $f(L', L) - f(L, L) > f(L', L') - f(L, L)$. We plan to investigate this result on this model with our modifications, and the condition for selection opposing the mutant invasion.

Other modifications to Nowak's model we may explore are:

- Eliminating the assumption that individuals are senders and receivers with equal probabilities. For example, we would have $f(L_1, L_2) = \alpha \mathrm{tr}(P_1 Q_2) + (1 - \alpha)\mathrm{tr}(P_2 Q_1)$, where $\alpha \neq \frac{1}{2}$. If $\alpha > \frac{1}{2}$, then individual 1 is the sender more often than the receiver when interacting with individual 2. In real life, α translates to the idea of "chattiness", i.e. how much we talk instead of listen.
- Introducing a cost to communication, for all senders, or for senders of a certain language.
- Introducing different concept-weight matrices for the non-mutant and mutant as senders. For example, creating W_1 and W_2 with different values along the diagonal, so that $f(L_1, L_2) = \frac{1}{2}\mathrm{tr}(W_1 P_1 Q_2) + \frac{1}{2}\mathrm{tr}(W_2 P_2 Q_1)$. In real life, this is reflected by the idea that we experience a benefit, happiness hormones, from talking about ourselves or our favorite subjects.

We plan to graph frequencies of the mutant and non-mutant over time using Mathematica.

Works Cited

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