Final Project Proposal

MATH 153

1 Group members

I plan to work individually and check in with Professor Nowak regularly.

2 Field of interest

My interests lie at the intersection of language and evolutionary graph theory, where I want to investigate how the structure of a population can affect the speed and success of memoryless language learning. In this particular project, I will focus on how "stubbornness" affects learning on different population structures by introducing a parameter p_k, which represents an individual's probability of changing languages, given two events: (1) that it hears its neighbors speak a different language, and (2) exactly k of its neighbors speak its current language.

Of course, this framework is not limited to the learning of natural language; it can also be interpreted as the exchange of ideas or cultures over a social network, for example.

3 Specific question

Given this setup, we can ask the following questions: What combination of population structure and p_k would lead to the formation of language "cliques"? How much time would it take for the entire population to converge to the same language? What happens if p_k is not a global parameter, but instead depends on each individual? For example, p_k might not correspond to the situation where exactly k neighbors speak an individual's language, but instead some proportion of its neighbors.

For the scope of this project, I will most likely focus on the first two questions.

4 List of references

Ibsen-Jensen et al. (2017) explore the dynamics of learners receiving linguistic input from a teacher, focusing on how the time to convergence depends on the structure of the "classroom." In my approach, however, I do not formulate a teacher, and instead assume that individuals only learn from each other.

I also plan to consult Lieberman et al. (2007), Nowak et al. (2001), and Nowak et al. (2002) for basic background on learning theory and evolution of language. Finally, I will also draw from our textbook Nowak (2006) as a general resource.

5 Mathematical approach

My mathematical approach will draw from evolutionary graph theory, learning theory, and formal language theory.

I use the following formal approach. Consider a search space of grammars $\mathcal{G} = \{G_1, \ldots, G_n\}$ and a population of N memoryless learners. Let the population be represented by a graph with N nodes, where every individual speaks a language from $\mathcal{G}$ and updates its beliefs by communicating with its neighbors. Consider an individual x. There is some probability p_1 that x does not switch its language, even after hearing its neighbors speak a different language, if exactly one neighbor of x speaks x's current language. If $p_1 = 1$, then x will never switch languages as long as it has one neighbor which speaks its current language. Similarly, there is some probability p_2 that x does not switch its language, even after hearing its neighbors speak a different language, if exactly two neighbors of x speak x's current language.

I will simulate memoryless learning without any "stubbornness" parameter p_k for various types of graphs (complete graph, cycle, star, etc.). Next, I will observe how learning changes for various values of p_1. Depending on what I observe, I will then introduce the parameter p_2. I plan to use Python to run the simulations.

References

Ibsen-Jensen, Rasmus, Josef Tkadlec, Krishnendu Chatterjee, and Martin A. Nowak (2017). "Language acquisition with communication between learners".

Lieberman, Erez, Jean-Baptiste Michel, Joe Jackson, Tina Tang, and Martin A. Nowak (2007). "Quantifying the evolutionary dynamics of language". In: *Nature* 449.

Nowak, Martin A. (2006). *Evolutionary Dynamics: Exploring the Equations of Life*. Cambridge: Belknap Press.

Nowak, Martin A., Natalia L. Komarova, and Partha Niyogi (2001). "Evolution of universal grammar". In: *Science* 291.

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