

## Worksheet 10 - Transiting Exoplanets

Some exoplanetary systems are aligned just right, such that the planet is seen to eclipse its host star. When this happens, astronomers can observe a periodic dimming of the star's light. This worksheet will guide you through some of the basics of the *transit method* of finding exoplanets.

1. Draw a planet passing in front of its star, with the star on the left and much larger than the planet on the right, with the observer far to the right of the planet. The planet's semi-major axis is  $a$ .
  - (a) Think about the geometry of the situation with the planet's orbital plane having various inclinations  $i$  with respect to the line of sight. Show that the probability that a planet transits its star is  $p_{\text{tr}} = R_{\star}/a$ , assuming  $R_p \ll R_{\star}$ . HINT: Imagine many, many observers arrayed around the star-planet system such that they cover a sphere surrounding the star. Next, imagine the range of solid angles,  $\Omega_{\text{tr}}$ , where some of these observers would see the planet pass in front of the star.  $\Omega_{\text{tr}}$  will form a "band" around the star. The transit probability is  $\Omega_{\text{tr}}$  divided by all solid angles.
  - (b) How does the expression for the transit probability change if  $R_p$  is not much smaller than  $R_{\star}$ ?
  - (c) Look up the semimajor axes of the Solar System planets out to Saturn. What is the probability that an alien civilization would see each planet transit?
  - (d) If 1% of Sun-like stars in the Galaxy have a Jupiter-sized planet in a 3-day orbit, what fraction of Sun-like stars have a transiting planet? How many stars would you need to monitor for transits if you want to detect 10 transiting planets?
  
2. Now draw the star projected on the sky, with a dark planet passing in front of the star along the star's equator.
  - (a) How does the (fractional) depth of the transit depend on the stellar and planetary physical properties?
  - (b) What is the depth of a Jupiter-sized planet transiting a Sun-like star?
  - (c) Compare the transit depth of an Earth-sized planet to the depth of a Jupiter-sized planet.
  - (d) In terms of the physical properties of the planetary system, what is the transit duration,  $T$ , defined as the time for the planet's center to pass from one limb of the star to the other?
  - (e) What is the duration of "ingress" and "egress",  $\tau$ , in terms of the physical parameters of the planetary system?
  
3. Imagine a Saturn-sized planet orbiting 0.5 AU from a red dwarf star with half the mass and radius of the Sun, with an inclination,  $i = \pi/2$  (the planet crosses the star along its equator as viewed from Earth). Sketch the transit light curve with fractional brightness on the y-axis, and time measured in hours on the x-axis. On the same figure, also sketch the light curve for a Jupiter-sized planet in a 10-day orbit.

### Homework Questions:

1. Consider the transit light curve and radial velocity time series on the next page. Assume a 3.1 day orbit and that the host star has the same size as the Sun. on the radial velocity plot, "phase" is just time measured as a fraction of the orbital period.

- (a) What is the qualitative brightness distribution of the star's surface as viewed from the Earth? (HINT: think about the bottom portion of the transit light curve)
- (b) What is the planet's radius compared to the star's radius ( $R_P/R_*$ )? (HINT: Use the average of the portion of the light curve during which the planet is completely in front of the star)
- (c) What is the planet's semimajor axis compared to the star's radius ( $a/R_*$ )?
- (d) What is the size of the planet compared to Jupiter if the star has a radius of  $0.8 R_\odot$ ?
- (e) Show that the scaled semimajor axis,  $a/R_*$ , is related to the average stellar density,  $\rho_*$ .
- (f) What is the density of the star compared to the density of the Sun?
- (g) What is the density of the planet compared to the density of Jupiter?

