Worksheet 1 Solutions

Instructions: Find a group of 3-4 students and use the clues and tools provided to measure the quantities listed below. Note that the point of this activity is less about getting the exact answer and doing so quickly, and much more about learning the format of this class and realizing that seemingly ill-posed problems can be solved using a few ingredients, your intuition and some math.

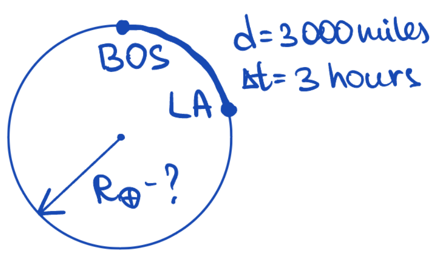
The available clues and tools include:

- Clue: My frequent flyer statement says that the distance between Los Angeles (LAX) and Boston (BOS) is 3000 miles.
- Clue: At the start of class today, it was 10:30am in Boston, and 7:30am in Los Angeles.
- Clue: When I was driving my car the other day, I looked at my speedometer and noticed I was going 100! Oh, wait, that was in km/hr. After setting my digital speedometer back to mph, I was only going 60.
- Clue: Johannes Kepler found that the period of a planet, P, is proportional to its distance, a, from the central mass such that $P^2 \propto a^3$. Newton modified this to be $P^2 = 4\pi^2 a^3/(GM)$ where M is the mass of the central body, $G \approx 7 \times 10^{-8}$ cm^3 g^{-1} s^{-2} and $\pi \approx 3$.
- Tool: A rock
- Tool: A scale
- Tool: A beaker
- Clue: An astronomer's rule of thumb is that the thumb as viewed at arm's length is 1° in angular diameter. The next time the moon is visible, you can verify that it is about half the size of your thumb at arm's length, or 0.5° in angular diameter.

Quantities to estimate. Do all math and express all answers in scientific notation. Express your final answer to **two (2)** significant figures.

1. The radius of the Earth, $R_\oplus =$ _______________ cm

   First, we plot the quantities that are given to us on the plot:



Let's convert miles to cgs units, i.e. centimeters (cm):

$$1 \text{ mile } = 1 \cdot \frac{100}{60} \text{ km} = 1 \cdot \frac{100}{60} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \frac{1}{6} \cdot 10 \cdot 10^5 \text{ cm} = \frac{1}{6} \cdot 10^6 \text{ cm}$$

And then we can calculate distance from Boston to LA

$$d = 3000 \text{ miles } = 3 \cdot 10^3 \cdot \frac{1}{6} \cdot 10^6 \text{ cm} = 0.5 \cdot 10^9 \text{ cm} = 5 \cdot 10^8 \text{ cm}$$

We know that the circumference of a circle with radius R has the equation

$$C = 2\pi R$$

Then for the Earth (assuming that the Earth is a perfect sphere), the circumference of the Earth is

$$C_\oplus = 2\pi R_\oplus$$

Using the clue that time difference is $\Delta t = 3$ hours, we can assume that it takes 3 hours for the Earth to turn, so Sun has the same position on the sky in Boston and LA. Also, it takes $P_\oplus = 24$ hours for the Earth to complete one revolution about its axis, we can deduce the proportionality that

$$\frac{\Delta t}{P_\oplus} = \frac{d}{C_\oplus}$$

Therefore,

$$R_\oplus = \frac{C_\oplus}{2\pi} = \frac{d \cdot P_\oplus \cdot \frac{1}{\Delta t}}{2\pi} = \frac{P_\oplus d}{2\pi \Delta t}$$

Finally, we can plug in all the numbers!

$$R_\oplus = \frac{P_\oplus d}{2\pi \Delta t} = \frac{24 \text{ hrs} \cdot 5 \cdot 10^8 \text{ cm}}{2\pi \cdot 3 \text{ hrs}} \approx 6.4 \cdot 10^8 \text{ cm}$$

2. The mass of the Earth, $M_\oplus =$ ________________ grams

We can assume that density of a rock in the lab is the same as the Earth's density $\rho_r = \rho_\oplus$. Using a beaker, we can find volume of the rock V_r, and we can find it's mass m_r using the scale. Density of any object is defined the following way:

$$\rho = \frac{m}{V}$$

For spherical objects (like the Earth!) volume is related to the radius of sphere this way: $V = \frac{4}{3}\pi R$. Therefore,

$$M_\oplus = V_\oplus \cdot \rho_\oplus = \frac{4}{3}\pi R_\oplus^3 \cdot \rho_\oplus =$$
$$= \frac{4}{3}\pi R_\oplus^3 \cdot \rho_r = \frac{4}{3}\pi R_\oplus^3 \cdot \frac{m_r}{V_r}$$

Now we can plug in numbers...

$$M_\oplus = \frac{4}{3}\pi R_\oplus^3 \cdot \rho_r = \frac{4}{3}\pi(6.4 \cdot 10^8 \text{ cm})^3 \cdot (4.5 \text{ g cm}^{-3}) = 4.9 \cdot 10^{27} \text{ g}$$

3. The distance from the center of the Earth to the center of the Moon, $a_☾ =$ ______________ cm (also express in number of Earth radii)

In order to find the distance from the Earth to the Moon $a_☾$, we will use the Kepler's third law $P^2 = 4\pi^2 a^3/(GM)$. Let's write it down for the Earth-Moon system, assuming Mass of the Earth is waaay higher than the Moon's:

$$P_☾^2 = \frac{4\pi^2 a_☾^3}{G(M_☾ + M_\oplus)} \approx \frac{4\pi^2 a_☾^3}{GM_\oplus}$$

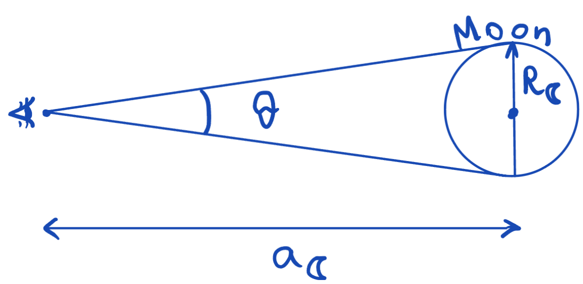
We can estimate $P_☾$ by assuming that there are 13 "Moonths" in a year, therefore $P_☾ = 365/13 \approx 28$ days.

Now we solve the Kepler's law for $a_☾$:

$$a_☾ = \left(\frac{P_☾ \cdot GM_\oplus}{4\pi^2}\right)^{1/3} = 3.6 \cdot 10^{10} \text{ cm} = 32 R_\oplus$$

4. The radius of the Moon, $R_☾ =$ ______________ cm (also express in units of $R_\oplus$)

First, let's draw how we observe the Moon:



By definition of angular diameter of the Moon θ:

$$\sin\left(\frac{\theta}{2}\right) = \frac{R_☾}{a_☾}$$

The angular diameter of the Moon $\theta = 0.5°$ is small angle, therefore, we can assume that $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$ in radians.

Therefore,

$$R_☾ = a_☾ \cdot \theta/2 = 3.6 \cdot 10^{10} \text{ cm} \cdot \frac{0.5}{2} \cdot \frac{\pi}{360°} = 1.6 \cdot 10^8 \text{ cm} = 0.25 R_\oplus$$

5. The mass of the Moon, $M_☾ =$ ________________ grams (also express in terms of $M_\oplus$ [e.g. how many Earth masses?].)

According to the theory, the Moon was created during the Earth's formation because of the huge impact with our planet, therefore we can assume that the Moon is made of the same material, as the Earth, or $\rho_☾ = \rho_\oplus = \rho_r$.

$$M_☾ = \rho_☾ V_☾ = \rho_r \cdot \frac{4}{3}\pi R_☾$$

Amazingly, we know all the values, so we can plug them in:

$$M_☾ = \rho_r \cdot \frac{4}{3}\pi R_☾ = 5.7 \cdot 10^{25} \text{ g} = 1.2 \cdot 10^{-2} M_\oplus$$