Lecture #1

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PROPAGATION OF E&M WAVES

1) HOMOGENEOUS MEDIUM, ISOTROPIC

2) ANISOTROPIC XTALS $\longrightarrow$ BIREFRINGENCE

General:

transport of energy

PROPAGATING WAVES $\longleftarrow$ MAXWELL

Macroscopic Maxwell

(Ampere's law)

from "free" charges

$$\nabla \times \bar{h} = \bar{j} + \frac{\partial \bar{d}}{dt} \qquad \nabla \cdot \bar{d} = \rho$$

MAXWELL!

Coulomb's law

$$\nabla \times \bar{e} = -\frac{\partial \bar{b}}{\partial t} \qquad \nabla \cdot \bar{b} = 0$$

Faraday's law

No magnetic monopoles

$\bar{d} = \varepsilon_0 \bar{e} + \bar{p}$ $\qquad$ ε_0, μ_0 permeabilities

$\bar{b} = \mu_0 (\bar{h} + \bar{m})$ $\qquad$ $\bar{p}, \bar{m}$ dipole densities

$\bar{e}, \bar{h}$ electric, magnetic field

$\bar{d}, \bar{b}$ displacement, magnetic induction

all dependent on $(\bar{r}, t)$

Linear medium:

$$\bar{d} = \varepsilon \bar{e}$$
$$\bar{b} = \mu \bar{h}$$

1 Homogeneous & Isotropic

ε, μ scalars, assume real for now:

$$\nabla \times \bar{h} = \varepsilon \frac{\partial \bar{e}}{\partial t}$$
$$\nabla \times \bar{e} = -\mu \frac{\partial \bar{h}}{\partial t}$$

$\sigma = 0$: non-conducting

$\rho_{free} = 0$

$$\nabla \times \nabla \times \bar{e} = -\nabla \times \left(\mu \frac{\partial \bar{h}}{\partial t}\right) = -\varepsilon\mu \frac{\partial^2 \bar{e}}{\partial t^2} \quad !$$

$$\nabla(\nabla \cdot \bar{e}) - \nabla^2 \bar{e}$$

$\nabla \cdot d = 0$, so $\nabla \cdot \bar{e} = 0$:

$\varepsilon \nabla \cdot e$

$$\boxed{\nabla^2 \bar{e} = \varepsilon\mu \frac{\partial^2 \bar{e}}{\partial t^2}}$$

Wave eq. ! (Maxwell)

Solutions: $e_i \propto e^{\pm i\bar{k}\cdot\bar{r} - i\omega t}$ ← physical "field": real(...)

any component

with $\boxed{k = \sqrt{\mu\varepsilon}\,\omega}$ so

phase velocity $\boxed{v = \frac{\omega}{k} = 1/\sqrt{\mu\varepsilon}}$

Pick $x \parallel \bar{k}$ just for now:

$$e_i = Ae^{i(kx - \omega t)} + Be^{-i(kx + \omega t)}$$
$$= Ae^{ik(x - vt)} + Be^{-ik(x + vt)}$$

<u>Non-dispersive</u> medium: ε, μ indep. of freq.

Fourier transform (in space) $\rightarrow$

$e_i = f(x-vt) + g(x+vt)$ is solution

where f, g arbitrary:

Pulse stays together during propagation

If <u>dispersive</u> medium:

Fourier transform Maxwell eqs; <u>for each</u> ω:

$\uparrow$ in $t \leftrightarrow \omega$

$$e_i = A\, \overset{\longrightarrow}{e^{i(kx-\omega t)}} + B\, \overset{\longleftarrow}{e^{-i(kx+\omega t)}}$$

where $k = \sqrt{\mu(\omega)\varepsilon(\omega)} \cdot \omega$

$\rightarrow$ pulse spreading.

So write travelling plane wave solution for $\bar{e}(\bar{r}, t)$:

$$\bar{e} = \bar{E}\, e^{i(k\bar{n}\cdot\bar{r} - \omega t)}$$

complex

$$\bar{b} = \bar{B}\, e^{i(k\bar{n}\cdot\bar{r} - \omega t)}$$

complex wavevector $k\bar{n} = \bar{k}$

$\bar{n}$ can in general be complex;

$k^2 = \mu\varepsilon\omega^2$; $\bar{n}\cdot\bar{n} = 1$

for $\bar{n}$ real

$$\frac{|E|}{|H|} = \frac{|E|}{|B|}\mu = \sqrt{\frac{\mu}{\varepsilon}} \xrightarrow{\text{vacuum}} 377\,\Omega$$

From $\nabla\cdot$ eqs:

TRANSVERSE WAVES $\rightarrow$ $\bar{k}\cdot\bar{E} = \bar{k}\cdot\bar{B} = 0$

From $\nabla\times\bar{e}$ eq: $\bar{k}\times\bar{E} = +\omega\bar{B}$

$$\Rightarrow \bar{B} = \frac{1}{\omega}\bar{k}\times\bar{E} = \frac{1}{\omega}k\bar{n}\times\bar{E} = \sqrt{\mu\varepsilon}\cdot\bar{n}\times\bar{E}$$

$\boxed{\text{If } \bar{n} \text{ real}}$: pick real, $\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{n}$: orthonormal system

$\left.\begin{matrix}\bar{E}, \bar{B} \\ \text{same phase}\end{matrix}\right\} \longrightarrow \bar{E} = \bar{\varepsilon}_1 E_0 \qquad \bar{B} = \bar{\varepsilon}_2 \sqrt{\mu\varepsilon} \cdot E_0$

complex

OR

$\bar{E} = \bar{\varepsilon}_2 E_0' \qquad \bar{B} = -\bar{\varepsilon}_1 \sqrt{\mu\varepsilon} \cdot E_0'$

TRANSVERSE Plane wave travelling in direction $\bar{n}$

$\boxed{\bar{n} = \bar{n}_r + i\bar{n}_i \text{ has imaginary part}}$: EXP. DECAY OR GROWTH

$e^{i(k\bar{n}\cdot\bar{r}-\omega t)} = e^{-k\bar{n}_i\cdot\bar{r}}\, e^{i(k\bar{n}_r\cdot\bar{r}-\omega t)}$ Total internal reflection

from Imaginary part of RHS and LHS

From : $\bar{n}\cdot\bar{n} = 1$: $\bar{n}_r \perp \bar{n}_i$

<u>Inhomogeneous</u> <u>plane</u> <u>wave</u>

plane of constant amplitude $\neq$

" " " phase

$\boxed{\bar{n} \text{ is real}}$

Now concentrate on travelling plane waves:

Most general travelling wave solution $\xrightarrow{k\bar{n} = \bar{k}}$:

complex

$\bar{e}(\bar{r},t) = \bar{\varepsilon}_1 E_1 e^{i(k\bar{n}\cdot\bar{r}-\omega t)} + \bar{\varepsilon}_2 E_2 e^{i(k\bar{n}\cdot\bar{r}-\omega t)}$

$E_2 = 0$: lin. pol. along $\bar{\varepsilon}_1$

E_1, E_2 same phase: lin. pol.

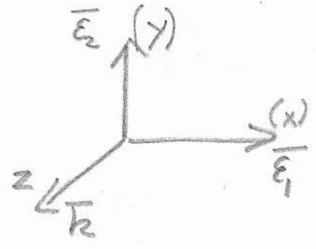
E_1, E_2 different phases:

elliptical polarization

Special case: phase difference $= 90°$ and

$|E_1| = |E_2| \longrightarrow$ Circular polarization:

$$\bar{e}(\bar{r},t) = E_0 (\bar{\varepsilon}_1 \pm i\bar{\varepsilon}_2) e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$



$$\bar{e}(\bar{r},t) = \begin{pmatrix} E_0 \cos(\omega t - kz) \\ E_0 \sin(\omega t - kz) \end{pmatrix} \qquad \bar{\varepsilon}_1 + i\bar{\varepsilon}_2$$

CCW
positive helicity : σ^+

$$\bar{e}(\bar{r},t) = \begin{pmatrix} E_0 \cos(-\omega t + kz) \\ E_0 \sin(-\omega t + kz) \end{pmatrix} \qquad \bar{\varepsilon}_1 - i\bar{\varepsilon}_2$$

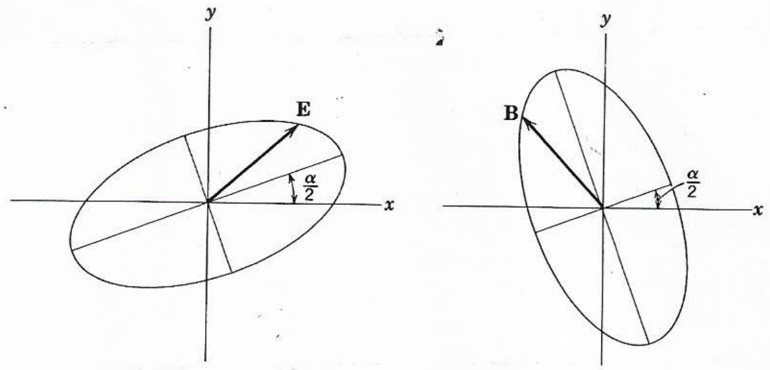
CW
negative helicity : σ^-

General solution:

$$\bar{\varepsilon}_\pm = \frac{1}{\sqrt{2}}(\bar{\varepsilon}_1 \pm i\,\bar{\varepsilon}_2)$$

$$\bar{e}(\bar{r},t) = (E_+ \bar{\varepsilon}_+ + E_- \bar{\varepsilon}_-)\, e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

$$\frac{E_-}{E_+} = r e^{i\alpha} \qquad \text{elliptic polarization}$$



$\frac{E}{E_+} = re^{i\alpha}$: elliptic polarization

$\frac{\text{major axis}}{\text{minor axis}} = \frac{1+r}{1-r}$ rotated by $\alpha/2$ from $(\bar{\varepsilon}_1, \bar{\varepsilon}_2)$

$r = 1$: lin. pol.

Till now: Medium ISOTROPIC

$\longrightarrow$ $\bar{P} = \varepsilon_0 \chi \bar{E}$

scalar

dielectric xtals in general:

$\bar{P}$ dependent on direction of $\bar{E}$

magnitude and direction

$\rightarrow$ $\bar{P} = \varepsilon_0 \underline{\underline{\chi}} \bar{E}$

electric susceptibility tensor

we can always choose 3 cartesian axes x, y, z where $\underline{\underline{\chi}}$ is diagonal (i.e. we can diagonalize $\underline{\underline{\chi}}$) : For principal dielectric axes of xtal :

$$p_x = \varepsilon_0 \chi_{11} e_x$$
$$p_y = \varepsilon_0 \chi_{22} e_y$$
$$p_z = \varepsilon_0 \chi_{33} e_z$$

Similarly :

$$\bar{d} = \varepsilon_0 \bar{e} + \bar{p} = \underline{\underline{\varepsilon}}\, \bar{e}$$

permeability tensor

$$\varepsilon_{11} = \varepsilon_0 (1 + \chi_{11})$$
$$\varepsilon_{22} = \varepsilon_0 (1 + \chi_{22})$$
$$\varepsilon_{33} = \varepsilon_0 (1 + \chi_{33})$$

SUMMARY:

Macroscopic Maxwell eqs

→ Wave eq. for electric field

$\bar{e}(\bar{r},t)$: $\nabla^2 \bar{e}(\bar{r},t) = \varepsilon\mu \frac{\partial^2 \bar{e}}{\partial t^2}$ HOMOGENEOUS MEDIUM

$$\bar{e}(\bar{r},t) = \bar{\varepsilon}_1 E_1 e^{i(\bar{k}\cdot\bar{r}-\omega t)} + \bar{\varepsilon}_2 E_2 e^{i(\bar{k}\cdot\bar{r}-\omega t)}.$$

$\bar{b}(r,t)$ similar and tied to $\bar{e}(\bar{r},t)$ thru Maxwell.

When $\frac{\bar{k}}{k}$ real : $\bar{\varepsilon}_1, \bar{\varepsilon}_2, \frac{\bar{k}}{k}$ form right-handed orthonormal system and $\bar{e} \perp \bar{b}$.

Phase fronts propagate with phase velocity

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$ (Isotropic medium : ε scalar)

Refractive index : $n \equiv \frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_0\mu_0}}$ and

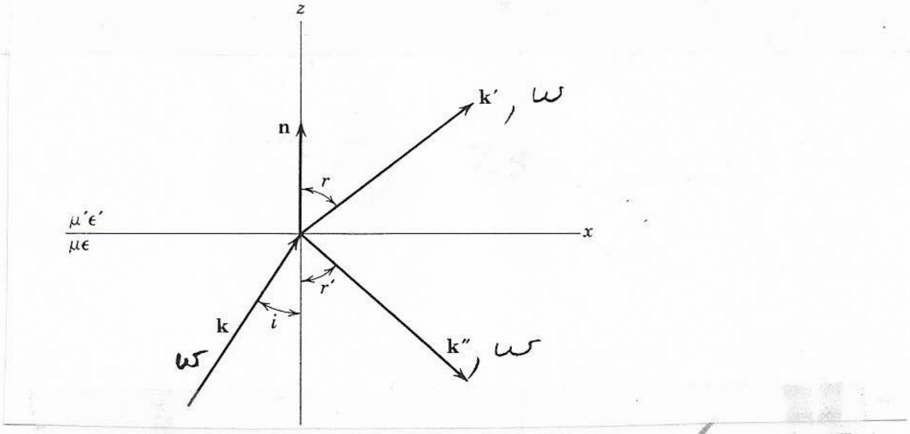
$$v = \frac{\omega}{k} = \frac{c}{n}$$; $c \equiv$ phase velocity in vacuum $= \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

So in other words:

Refractive index causes a speed change of light propagation (typically $n > 1$ and light slows down in medium).

Non-isotropic medium : $\underline{\underline{\varepsilon}}$ is a tensor.

REFLECTION / REFRACTION



PLANE INTERFACE BETWEEN DIELECTRICS

(1) Kinematics ; ⟵ from general boundary conditions

(2) Dynamics ; ⟵ depends on details $\begin{cases} \text{e\&m wave} \\ \text{boundary} \end{cases}$

(1) Goal : show

$r' = i$ and

$$\frac{\sin i}{\sin r} = \frac{n'}{n} \quad :$$

Snell's law

Remember:

$$n = \sqrt{\varepsilon\mu} \,/\, \sqrt{\varepsilon_0\mu_0}$$

$$n' = \sqrt{\varepsilon'\mu'} \,/\, \sqrt{\varepsilon_0\mu_0}$$

Incident : $\quad \bar{e} = \bar{E}_0 \, e^{i(\bar{k}\cdot\bar{r} - \omega t)} \; ; \; \bar{b} = \bar{B}_0 \, e^{i(\bar{k}\cdot\bar{r} - \omega t)}$

$$\bar{B}_0 = \sqrt{\varepsilon\mu} \; \frac{\bar{k}\times\bar{E}_0}{k}$$

Refracted: $\quad \bar{e}' = \bar{E}_0' \, e^{i(\bar{k}'\cdot\bar{r} - \omega t)} \; ; \; \bar{B}_0' = \sqrt{\varepsilon'\mu'} \; \frac{\bar{k}'\times\bar{E}_0'}{k'}$

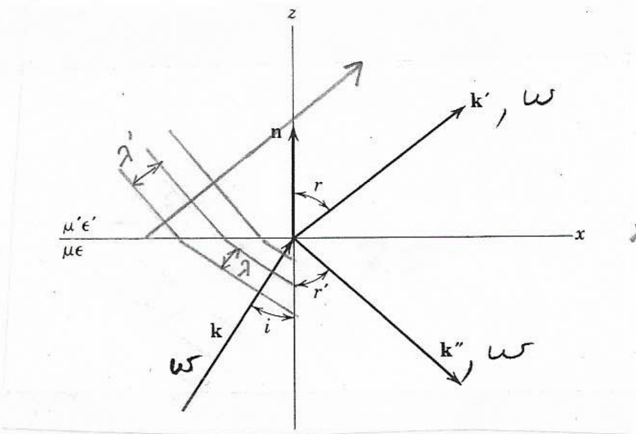
Reflected: $\quad \bar{e}'' = \bar{E}_0'' \, e^{i(\bar{k}''\cdot\bar{r} - \omega t)} \; ; \; \bar{B}_0'' = \sqrt{\varepsilon\mu} \; \frac{\bar{k}''\times\bar{E}_0''}{k''}$

To satisfy boundary conds everywhere at $z=0$:

Reflection plane is defined by $\bar{k}$ and $\bar{n}$.

propagation

Surface normal



x-distance between 2π phase planes is $\lambda/\sin i = \lambda'/\sin r = \lambda''\sin r'$;

$\lambda = \frac{2\pi}{k}$; $\lambda' = \frac{2\pi}{k'}$; $\lambda'' = \lambda$

$k'' = k$

$$\Rightarrow k\sin i = k'\sin r = k''\sin r'$$

$k = \frac{\omega}{v} = \frac{\omega}{c/n}$

$k' = \frac{\omega}{v'} = \frac{\omega}{c/n'}$

$k'' = k$

REFRACTION IS DUE TO PHASE VELOCITY CHANGE

$i = r'$

Snell's law: $\frac{\sin i}{\sin r} = \frac{k'}{k} = \frac{(\omega/c)n'}{\omega/c\ n} = \frac{n'}{n} = \sqrt{\frac{\mu'\epsilon'}{\mu\epsilon}}$

Ratio of refractive indices in the 2 media

Goal: Find

(2) Dynamics: relative amplitudes of reflected, refracted, and incident

Boundary conditions:

D_n, B_n continuous

E_t, H_t —

(phasefactors we know already are the same)

$$D_n \longrightarrow \varepsilon \cdot (\bar{E}_0 + \bar{E}_0'') \cdot \bar{n} = \varepsilon' \bar{E}_0' \cdot \bar{n}$$

$$B_n \longrightarrow \left(\cancel{\sqrt{\mu\varepsilon}} \frac{\bar{k} \times \bar{E}_0}{\cancel{k}} + \cancel{\sqrt{\mu\varepsilon}} \frac{\bar{k}'' \times \bar{E}_0''}{\underset{k}{\cancel{k''}}} \right) \cdot \bar{n} = \cancel{\sqrt{\mu'\varepsilon'}} \frac{\bar{k}' \times \bar{E}_0'}{\cancel{k'}} \cdot \bar{n}$$

note: $$\frac{\sqrt{\mu\varepsilon}}{k} = \frac{1}{\omega} = \frac{\sqrt{\mu'\varepsilon'}}{k'} \quad :$$

$$E_t \Rightarrow (\bar{E}_0 + \bar{E}_0'') \times \bar{n} = \bar{E}_0' \times \bar{n}$$

$$H_t \rightarrow \frac{1}{\mu} (\bar{k} \times \bar{E}_0 + \bar{k}'' \times \bar{E}_0'') \times \bar{n} = \frac{1}{\mu'} (\bar{k}' \times \bar{E}_0') \times \bar{n}$$

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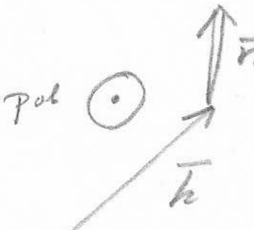
s pol. light



plane of incidence

nr. 1 fulfilled automatically ($\bar{E}$ ∥ interface) or $\bar{E} \perp \bar{n}$

nr. 3 and nr. 2 (w. Snell's law) the same:

$$\frac{E_0''}{E_0} = \frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}$$

$$\frac{E_0'}{E_0} = \frac{2n\cos i}{\text{same denom.}}$$

FRESNEL REFLECTION / REFRACT.

Different eqs for p pol.

If normally incident ($i = 0$): eqs for s and p the same:

$$\left.\begin{aligned} \frac{E_0''}{E_0} &\longrightarrow -\frac{n'-n}{n'+n} \\ \frac{E_0'}{E_0} &\longrightarrow \frac{2n}{n'+n} \end{aligned}\right\} \begin{array}{l} \text{for } \mu = \mu' \\ \text{for both s and p} \end{array}$$

Intensity $\propto |E|^2$: 4% reflection for glass

$n' > n$: 180° phase change

For p polarized light:

$i_B = \tan^{-1}\left(\frac{n'}{n}\right)$ Brewsters angle : no reflection !

$(\mu = \mu')$

56° in glass

Even away from Brewster:

More s than p reflected

Polaroid glasses (linear polarizers) utilize that ?

TOTAL INTERNAL REFLECTION:

$n > n'$

f.ex. confinement of light in a glass cube placed in vacuum or air >< dielectric waveguide (fiber)

Snell's law: $\sin r = \sin i \cdot \frac{n}{n'}$

$r > i$ when $n > n'$

when $i = i_0 = \sin^{-1}\frac{n'}{n}$: $r = 90°$

The refracted wave parallel to surface

Now: For $i > i_0$: $\sin r > 1$: r complex angle :

$$\cos r = \sqrt{1 - \sin^2 r} = i\sqrt{\left(\frac{\sin i}{\sin i_0}\right)^2 - 1}$$

$$\sin r = \sin i \frac{n}{n'} = \frac{\sin i}{\sin i_0}$$

$$e^{i\bar{k}' \cdot \bar{r}} = e^{ik'(\cos r \cdot z + \sin r \cdot x)}$$

$$= e^{-z \cdot k' \sqrt{\left(\frac{\sin i}{\sin i_0}\right)^2 - 1}} \; e^{ik' \sin i / \sin i_0 \cdot x}$$

So $i > i_0$: propagation along x // surface
att. exp. in z $\perp$ surface
within $\sim 1/k' \simeq \lambda$

No energy flow thru surface
and $|E_0'' / E_0| = 1$:
TOTAL REFLECTION

(x rays: total EXTERNAL reflection)

Fresnel rhombus : phase change depends on s or p pol; i; n/n'