ENERGY DENSITY and ENERGY FLOW IN ELECTRO-MAGNETIC FIELDS

Let's figure out what the energy density is in the <u>electro-magnetic</u> <u>fields</u> :

Let's go back to <u>microscopic</u> Maxwell:

$\bar{e} \underset{\sim}{\overset{-qN\bar{v}}{\overset{\shortparallel}{\bar{J}_{total}}}}$ work per vol per unit time done by E-M field on medium (no contribution to work from $\bar{b}$); here $\bar{J}_{total}$ = <u>total</u> current density

$$\bar{J}_{total} = \frac{1}{\mu_0} \nabla \times \bar{b} - \varepsilon_0 \frac{\partial \bar{e}}{\partial t} \qquad \text{(see p. 19)}:$$

$$\bar{e} \cdot \bar{J}_{total} = \frac{1}{\mu_0} \bar{e} \cdot (\nabla \times \bar{b}) \underbrace{- \varepsilon_0 \bar{e} \frac{\partial \bar{e}}{\partial t}}_{= -\frac{1}{2} \varepsilon_0 \frac{\partial}{\partial t} (\bar{e} \cdot \bar{e})}$$

$$\bar{e} \cdot (\nabla \times \bar{b}) = \nabla \cdot (\bar{b} \times \bar{e}) + \bar{b} \cdot (\nabla \times \bar{e})$$

$$\nabla \times \bar{e} = -\frac{\partial \bar{b}}{\partial t}$$

$$\bar{e} \cdot \bar{J}_{total} = \frac{1}{\mu_0} \left(\nabla \cdot (\bar{b} \times \bar{e}) - b \frac{\partial \bar{b}}{\partial t} \right) - \frac{1}{2} \varepsilon_0 \frac{\partial}{\partial t} (\bar{e} \cdot \bar{e})$$

$$= -\frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon_0 \bar{e} \cdot \bar{e} + \frac{1}{2} \frac{1}{\mu_0} \bar{b} \cdot \bar{b} \right)$$

$$+ \frac{1}{\mu_0} \nabla \cdot (\bar{b} \times \bar{e})$$

$\int_V dv \ldots$:

$$\frac{1}{\mu_0}\int_S (\bar{e}\times\bar{b})\cdot\bar{n}\,da + \int_V dv\,\bar{e}\cdot\bar{J}_{total} =$$

$$= -\frac{d}{dt}\int_V dv\left(\frac{1}{2}\varepsilon_0\,\bar{e}\cdot\bar{e} + \frac{1}{2}\frac{1}{\mu_0}\bar{b}\cdot\bar{b}\right)$$

$\frac{\text{Power}}{\text{unit area}}$ in <u>E-M field</u> (intensity) =

$\bar{S} = \frac{1}{\mu_0}\,\bar{e}\times\bar{b}$ POYNTING VECTOR

Energy density in <u>E-M field</u> =

$$u = \frac{1}{2}\varepsilon_0\,\bar{e}\cdot\bar{e} + \frac{1}{2}\frac{1}{\mu_0}\bar{b}\cdot\bar{b}$$

INTERPRETATION !

NOW :

USE THESE RESUTS FOR

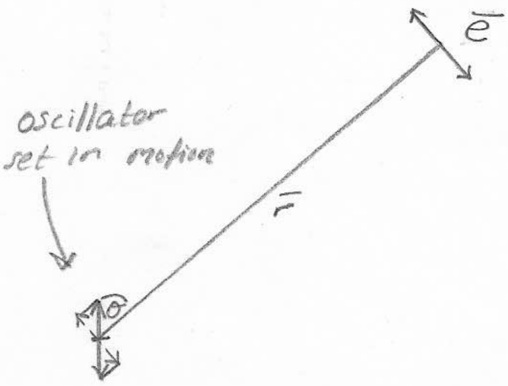
CALCULATION OF

RADIATION DAMPING

RADIATION DAMPING IN LORENTZ OSCILLATOR

$\overline{\underbrace{\bar{e}\cdot\frac{\partial\bar{P}}{\partial t}}} \neq 0$ only if we have damping of Lorentz oscillator $(\gamma \neq 0)$.

= work on bound charges : heat in medium or radiated power!



acceleration & e 180° out of phase (for positive charge)

$$e(t) = \frac{-(-q)a(t-r/c)\sin\theta}{4\pi\varepsilon_0 c^2 r}$$

oscillating charge $-q$

$x(t) = x_0 e^{-i\omega_0 t}$

Radiated power per unit area in direction $\bar{r}$ is

$$S = \frac{1}{\mu_0}|\bar{e}\times\bar{b}|\,;\quad \bar{b} = \sqrt{\mu_0\varepsilon_0}\,\frac{\bar{r}}{r}\times\bar{e}\,;$$

$$S = c\varepsilon_0|\bar{e}|^2 = c\varepsilon_0\frac{q^2a^2(t-r/c)\sin^2\theta}{16\pi^2\varepsilon_0^2c^4r^2}$$

$$= \frac{q^2a^2(t-r/c)\sin^2\theta}{16\pi^2\varepsilon_0c^3r^2}$$

Total radiated power is

$$-\frac{dW}{dt} = \int_0^\pi 2\pi r\sin\theta\, r\,d\theta\cdot S(\theta) = \frac{q^2a^2(t-r/c)}{6\pi\varepsilon_0c^3}\quad ;\ \text{note: } \langle a^2\rangle = \tfrac{1}{2}\omega_0^4|x_0|^2:$$

$$\left\langle -\frac{dW}{dt}\right\rangle = \frac{q^2\omega_0^4|x_0|^2}{12\pi\varepsilon_0c^3}\,.$$

This radiated power corresponds to a damping of the osillator: the oscillator – once set in motion – does not oscillate for ever: W = energy of osillator:

$$Q = 2\pi\frac{\langle W\rangle}{\langle -dW/dt\rangle\cdot T}$$ ← denominator = energy loss per period

Energy ring-down:

$$\langle W \rangle = W_0 e^{-\omega_0 t/Q}$$

$$\langle W \rangle = \tfrac{1}{2} m \omega_0^2 |x_0|^2 \quad ; \quad \left\langle -\frac{dW}{dt} \right\rangle = \frac{q^2 \omega_0^4 |x_0|^2}{12\pi \epsilon_0 c^3} \propto \langle W \rangle$$

$$\frac{1}{Q} = \frac{q^2 \omega_0}{6\pi \epsilon_0 c^3 m} \underset{\uparrow}{=} \frac{q^2}{3 \cdot \epsilon_0 m c^2 \lambda}$$

$$\lambda = \frac{c}{\omega_0} 2\pi$$

For Na: $\lambda = 589\,nm$ (D lines)

$$\frac{1}{Q} = \underbrace{\frac{1}{4\pi\epsilon_0} \frac{q^2}{\hbar c}}_{\alpha} \overbrace{\hbar c}^{1973\,eV\,\text{Å}} \underbrace{\frac{1}{mc^2}}_{511000\,eV} \frac{1}{5890\text{Å}} \cdot \frac{4\pi}{3} \sim \frac{1}{5 \times 10^7}$$

α = Fine-structure constant $\approx 1/137$

$$\frac{Q}{\omega_0} = \text{life-time (ring-down time)} \sim \frac{5 \times 10^7}{2\pi \cdot 10^{15}/s}$$

$\sim 10^{-8} s$ (spontaneous emission time is $16\,ns$)

We can relate Q directly to damping coefficient of course:

$$-\frac{dW}{dt} = -\underbrace{F_{damp}}_{\gamma m v} \cdot v = \gamma m v^2 \quad \text{so} \quad (\langle W \rangle = 2 \cdot \langle \tfrac{1}{2} m v^2 \rangle)$$

$$Q = \frac{\omega_0}{\gamma} \quad \text{or} \quad \gamma = \frac{\omega_0}{Q} \quad \left(\simeq \frac{1}{10\,ns} \text{ for Na}\right).$$

Note that

$$\alpha(\omega)\epsilon_0 = \frac{q^2}{m} \cdot \frac{1}{(\omega_0^2 - \omega^2) - i\omega\gamma} = \frac{q^2}{m} \cdot \frac{\omega_0^2 - \omega^2 + i\omega\gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2}$$

$$\varepsilon = \varepsilon_0 (1+\chi) \quad ; \quad \chi = N\alpha \quad \begin{cases} \bar{d} = \varepsilon_0 \bar{e} + \bar{P} = \varepsilon \bar{e} \\ \bar{P} = N\alpha \varepsilon_0 \bar{e} \end{cases}$$

$$\alpha(\omega)\varepsilon_0 \simeq \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2} \; ; \; |\omega - \omega_0| \gg \gamma \; : \; \boxed{\text{FAR FROM ATOMIC RESONANCE}}$$

$$\mathrm{Re}\big(\alpha(\omega)\,\varepsilon_0\big) \simeq \frac{1}{2\omega_0} \frac{q^2}{m} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \; ; \; \frac{|\omega - \omega_0|}{\omega_0} \ll 1 \; \boxed{\text{CLOSE TO RESONANCE}}$$

$$\omega_0^2 - \omega^2 = (\omega_0 - \omega)(\omega_0 + \omega) \simeq 2\omega_0(\omega_0 - \omega)$$

$$\mathrm{Im}\big(\alpha(\omega)\,\varepsilon_0\big) \simeq \frac{1}{4\omega_0} \gamma \frac{q^2}{m} \frac{1}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \; ; \; \frac{|\omega - \omega_0|}{\omega_0} \ll 1 \; \boxed{\text{CLOSE TO RESONANCE}}$$

Lorentzian shape

$$n(\omega) = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$ (non-magnetic material)

$n_r(\omega) = \mathrm{Re}(n(\omega))$ ⟵ gives phase velocity

$n_i(\omega) = \mathrm{Im}(n(\omega))$ ⟵ gives decay

$$\boxed{\begin{aligned} k &= \frac{\omega}{c} n \\ k &= \sqrt{\mu_0 \varepsilon} \cdot \omega \end{aligned}}$$

$$e^{i(\bar{k}\cdot\bar{r} - \omega t)} = e^{i(k\frac{\bar{k}}{k}\cdot\bar{r} - \omega t)} = e^{i(\frac{\bar{k}}{k}\cdot\bar{r}\,\frac{\omega}{c} n - \omega t)}$$

$$= e^{i(z\overbrace{\sqrt{\mu_0\varepsilon}}^{n/c}\omega - \omega t)} = e^{i(z n_r/c - t)\omega} \, e^{-z n_i \omega/c}$$

$\bar{k}$ along $\bar{e}_z$ say

phase velocity $= c/n_r$

decay length $= \frac{c}{\omega} \cdot \frac{1}{n_i}$

BORN & WOLF
"PRINCIPLES OF OPTICS"
Ch. 3

RAY OPTICS
GEOMETRICAL OPTICS
EIKONAL APPROXIMATION

CLASSICAL LIMIT OF DIFFRACTION THEORY

$$\boxed{\lambda \to 0}$$

MAXWELL'S EQS:

$$\nabla \times \bar{h} = \bar{j} + \frac{\partial \bar{d}}{\partial t}$$

$$\nabla \times \bar{e} = -\frac{\partial \bar{b}}{\partial t}$$

$$\nabla \cdot \bar{d} = \rho$$

$$\nabla \cdot \bar{b} = 0$$

non-conducting:
$\bar{j}$ and $\rho = 0$

$$\bar{d} = \varepsilon \bar{e}$$

$$\bar{b} = \mu \bar{h}$$

Isotropic medium
ε, μ scalars

BUT MEDIUM IS CERTAINLY ALLOWED TO BE NON-HOMOGENEOUS (HETEROGENEOUS)

INSPIRED BY:
IN A HOMOGENEOUS MEDIUM:
TRAVELLING PLANE WAVE SOLUTION:

$$\bar{e}(\bar{r},t) = \bar{E}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)} \; ; \; \bar{h}(\bar{r},t) = \bar{H}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

$$k^2 = \omega^2 \varepsilon\mu \; ; \; \bar{E}_0, \bar{H}_0 \text{ complex constants}$$

$$e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

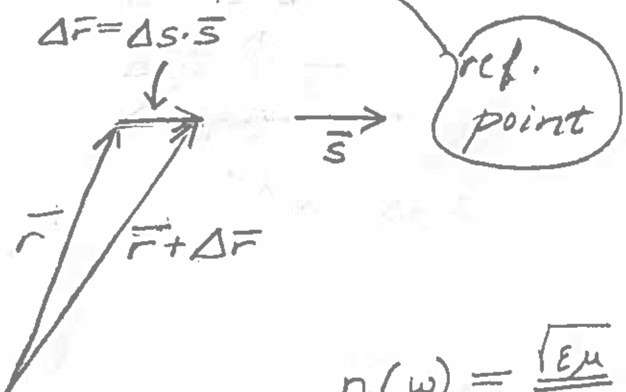
spatially dependent phase:

Let $\bar{k} = k\cdot\bar{s}$

unit vector determines propagation direction

Phase development from $\bar{r}_0$ to $\bar{r}$:

$$\bar{k}\cdot(\bar{r}-\bar{r}_0) = k\cdot\bar{s}\cdot(\bar{r}-\bar{r}_0) = \int_{\substack{\text{along}\\\text{ray}}} k\,ds = k_0\int_{\substack{\text{along}\\\text{ray}}} n(\omega)\,ds = \underline{\underline{k_0 S(\bar{r})}}$$



$$d\bar{r} = ds\cdot\bar{s}$$

$$k_0 = \frac{\omega}{c}$$

$$k = k_0\frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_0\mu_0}} = k_0\cdot n(\omega)$$

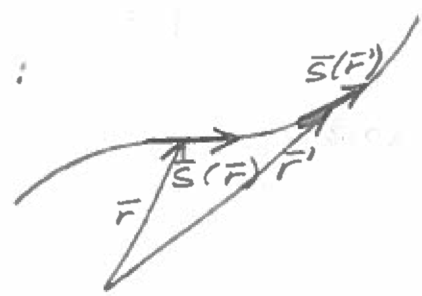
$$n(\omega) = \frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_0\mu_0}} = n_r(\omega) + i\,n_i(\omega)$$

In non-homogeneous medium:

ANSATZ:

$$\bar{e}(\bar{r},t) = \bar{E}_0(\bar{r})\,e^{ik_0 S(\bar{r})}\,e^{-i\omega t}$$

amplitude vector is spatially varying



Picture: Locally: plane wave

Insert ansatz into Maxwell:

$$\bar{e}(\bar{r},t) = \bar{E}_0(\bar{r})\, e^{ik_0 S(\bar{r})}\, e^{-i\omega t}$$

$$\bar{h}(\bar{r},t) = \bar{H}_0(\bar{r})\, e^{ik_0 S'(\bar{r})}\, e^{-i\omega t}$$

Maxwell # 1

$$\nabla \times \bar{h} = \left(\nabla \times \bar{H}_0 + ik_0 \nabla S \times \bar{H}_0\right) e^{ik_0 S(\bar{r})}\, e^{-i\omega t}$$

$$= \frac{\partial \bar{d}}{\partial t} = \varepsilon \frac{\partial \bar{e}}{\partial t} = -i\omega\varepsilon\, \bar{e}$$

$$\Rightarrow \boxed{\nabla \times \bar{H}_0 + ik_0 \nabla S \times \bar{H}_0 = -i\omega\varepsilon\, \bar{E}_0}$$

Maxwell # 2

$$\nabla \times \bar{e} = \left(\nabla \times \bar{E}_0 + ik_0 \nabla S \times \bar{E}_0\right) e^{ik_0 S(\bar{r})}\, e^{-i\omega t}$$

$$= -\frac{\partial \bar{b}}{\partial t} = i\omega\mu\, \bar{h}$$

$$\Rightarrow \boxed{\nabla \times \bar{E}_0 + ik_0 \nabla S \times \bar{E}_0 = i\omega\mu\, \bar{H}_0}$$

Maxwell # 3

$$\nabla \cdot \bar{d} = \nabla \cdot (\varepsilon \bar{e}) = \nabla\varepsilon \cdot \bar{e} + \varepsilon\, \nabla \cdot \bar{e}$$

$$= \nabla\varepsilon \cdot \bar{e} + \varepsilon\left(\nabla \cdot \bar{E}_0 + \bar{E}_0 \cdot ik_0 \nabla S\right) e^{ik_0 S}\, e^{-i\omega t}$$

$$= 0$$

$$\Rightarrow \boxed{\nabla\varepsilon \cdot \bar{E}_0 + \varepsilon\, \nabla \cdot \bar{E}_0 + ik_0 \varepsilon\, \bar{E}_0 \cdot \nabla S = 0}$$

Maxwell #4 $\quad \nabla \cdot \overline{B} = 0$

$$\Rightarrow \boxed{\nabla\mu \cdot \overline{H}_0 + \mu \nabla \cdot \overline{H}_0 + i k_0 \mu \overline{H}_0 \cdot \nabla S = 0}$$

Prepare to make expansion in $\frac{1}{k_0}$:

$$\left(\frac{1}{k_0} = \frac{\lambda_0}{2\pi} \to 0 \right)$$

$$(1) \qquad \nabla S \times \overline{H}_0 + \varepsilon \cdot c \overline{E}_0 = -\frac{1}{ik_0} \nabla \times \overline{H}_0 \qquad \boxed{\frac{\omega}{c} = k_0}$$

$$(2) \qquad \nabla S \times \overline{E}_0 - \mu \cdot c \overline{H}_0 = -\frac{1}{ik_0} \nabla \times \overline{E}_0$$

$$(3) \qquad \overline{E}_0 \cdot \nabla S = -\frac{1}{ik_0} \left[E_0 \cdot \frac{\nabla \varepsilon}{\varepsilon} + \nabla \cdot \overline{E}_0 \right]$$

$$(4) \qquad \overline{H}_0 \cdot \nabla S = -\frac{1}{ik_0} \left[\overline{H}_0 \cdot \frac{\nabla \mu}{\mu} + \nabla \cdot \overline{H}_0 \right]$$

Get $\overline{H}_0$ from (2) and insert into (1):

$$\overline{H}_0 = \frac{1}{\mu c} \nabla S \times \overline{E}_0 + \frac{1}{\mu c} \frac{1}{ik_0} \nabla \times \overline{E}_0 \qquad \leftarrow (2)$$

$\leftarrow$ Into (1)

(1) and (2) $\rightarrow$

$$\boxed{\nabla S \times (\nabla S \times \bar{E}_0) + \nabla S \times (\nabla \times \bar{E}_0) \cdot \frac{1}{ik_0} + \varepsilon\mu c^2 \bar{E}_0 = -\frac{1}{ik_0} \nabla \times (\nabla S \times \bar{E}_0) + \frac{1}{k_0^2} \nabla \times \nabla \times \bar{E}_0}$$

assume $\mu(\bar{r}) = \mu$

LHS
1. term : $\nabla S \times (\nabla S \times \bar{E}_0) = (\nabla S \cdot \bar{E}_0) \nabla S - (\nabla S)^2 \bar{E}_0$
2. term : $\nabla S \times (\nabla \times \bar{E}_0) = \nabla(\nabla S \cdot \bar{E}_0) - (\bar{E}_0 \cdot \bar{\nabla}) \nabla S - (\nabla S \cdot \nabla) \bar{E}_0 - \bar{E}_0 \times \underbrace{(\nabla \times \nabla S)}_{= 0}$

$\nabla(\bar{a} \cdot \bar{b})$
$\bar{a} = \nabla S$, $\bar{b} = \bar{E}_0$
Jackson inside front cover

RHS
1. term : $\nabla \times (\nabla S \times \bar{E}_0) = \nabla S (\nabla \cdot \bar{E}_0) - \bar{E}_0 \nabla^2 S + (\bar{E}_0 \cdot \bar{\nabla}) \nabla S - (\nabla S \cdot \nabla) \bar{E}_0$

2. term : $\nabla \times \nabla \times \bar{E}_0 = \nabla(\nabla \cdot \bar{E}_0) - \nabla^2 \bar{E}_0$

Insert expressions :

$$(\nabla S \cdot \bar{E}_0) \nabla S - (\nabla S)^2 \bar{E}_0 + \frac{1}{ik_0} \cdot \left[\nabla(\nabla S \cdot \bar{E}_0) - (\bar{E}_0 \cdot \nabla) \nabla S - (\nabla S \cdot \nabla) \bar{E}_0 \right] + \varepsilon\mu c^2 \bar{E}_0$$

$$= -\frac{1}{ik_0} \left[\nabla S (\nabla \cdot \bar{E}_0) - \bar{E}_0 \nabla^2 S + (\bar{E}_0 \cdot \bar{\nabla}) \nabla S - (\nabla S \cdot \nabla) \bar{E}_0 \right] + \frac{1}{k_0^2} \left[\nabla(\nabla \cdot \bar{E}_0) - \nabla^2 \bar{E}_0 \right]$$

(3) $\rightarrow \nabla\cdot\bar{E}_0 = -ik_0\,\bar{E}_0\cdot\nabla S - \bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}$; Insert :

$$\cancelto{a}{(\nabla S\cdot\bar{E}_0)\nabla S} - (\nabla S)^2\bar{E}_0 + \frac{1}{ik_0}\Big[\cancelto{b}{\nabla(\nabla S\cdot\bar{E}_0)} - \cancelto{d}{(\bar{E}_0\cdot\nabla)\nabla S}$$
$$- (\nabla S\cdot\nabla)\bar{E}_0\Big] + \varepsilon\mu c^2\bar{E}_0$$

$$= -\frac{1}{ik_0}\Big[\nabla S\big(\cancelto{a}{-ik_0\bar{E}_0\cdot\nabla S} - \bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}\big) - \bar{E}_0\nabla^2 S$$
$$+\cancelto{d}{(\bar{E}_0\cdot\nabla)\nabla S} - (\nabla S\cdot\nabla)\bar{E}_0\Big]$$
$$+\frac{1}{k_0^2}\Big[\nabla\big(\cancelto{b}{-ik_0\bar{E}_0\cdot\nabla S} - \bar{E}_0\frac{\nabla\varepsilon}{\varepsilon}\big) - \nabla^2\bar{E}_0\Big]$$

$$\Rightarrow -(\nabla S)^2\bar{E}_0 + \varepsilon\mu c^2\bar{E}_0 =$$
$$= \frac{1}{ik_0}\Big[2(\nabla S\cdot\nabla)\bar{E}_0 + \nabla S\big(\bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}\big) + \bar{E}_0\nabla^2 S + \cancel{(\nabla S\cdot\nabla)\bar{E}_0}\Big]$$
$$-\frac{1}{k_0^2}\Big[\nabla\big(\bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}\big) + \nabla^2\bar{E}_0\Big]$$

$$\Rightarrow \boxed{\begin{aligned} &\big[(\nabla S)^2 - \varepsilon\mu c^2\big]\bar{E}_0 = \\ &= \frac{2i}{k_0}\Big[(\nabla S\cdot\nabla) + \frac{1}{2}\nabla^2 S\Big]\bar{E}_0 + \frac{i}{k_0}\big(\bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}\big)\nabla S \\ &+ \frac{1}{k_0^2}\nabla\big(\bar{E}_0\cdot\frac{\nabla\varepsilon}{\varepsilon}\big) + \frac{1}{k_0^2}\nabla^2\bar{E}_0 \end{aligned}}$$

$\boxed{\varepsilon = \varepsilon_1 + i\,\varepsilon_2}$; assume: $\boxed{\mu \text{ real}}$

Assume $\boxed{S,\ \bar{E}_0 \text{ real}}$:

all phase changes in $e^{ik_0 S(\bar{r})}$

all amplitude changes in $\bar{E}_0(\bar{r})$ (decays etc)

REAL PART:

$$\left[(\nabla S)^2 - \varepsilon_1 \mu c^2\right]\bar{E}_0 =$$

$$= \frac{1}{k_0^2}\nabla^2 \bar{E}_0 + \mathrm{Re}\left[\frac{1}{k_0^2}\nabla\left(\bar{E}_0 \cdot \frac{\nabla\varepsilon}{\varepsilon}\right) + \frac{i}{k_0}\left(\bar{E}_0 \cdot \frac{\nabla\varepsilon}{\varepsilon}\right)\nabla S\right]$$

IMAGINARY PART:

$$-\varepsilon_2 \mu c^2 \bar{E}_0 = \frac{2}{k_0}\left[(\nabla S \cdot \nabla) + \frac{1}{2}\nabla^2 S\right]\bar{E}_0$$

$$+ \mathrm{Im}\left[\frac{1}{k_0^2}\nabla\left(\bar{E}_0 \cdot \frac{\nabla\varepsilon}{\varepsilon}\right) + \frac{i}{k_0}\left(\bar{E}_0 \cdot \frac{\nabla\varepsilon}{\varepsilon}\right)\nabla S\right]$$

EXAMPLE:
Note: for plane wave in homogeneous medium: plane wave propagating along z

ε real : $S(\bar{r}) = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\, z = \sqrt{\varepsilon\mu} \cdot c\, z$; $\nabla S = \sqrt{\varepsilon\mu} \cdot c$

$(\nabla S)^2 = \varepsilon\mu \cdot c^2 \rightarrow \nabla^2 \bar{E}_0 = 0$

Find Real, Imaginary parts:

$$\frac{\nabla\varepsilon}{\varepsilon} = \frac{\nabla\varepsilon_1 + i\nabla\varepsilon_2}{\varepsilon_1 + i\varepsilon_2} = \frac{(\nabla\varepsilon_1 + i\nabla\varepsilon_2)(\varepsilon_1 - i\varepsilon_2)}{\varepsilon_1^2 + \varepsilon_2^2}$$

$$= \frac{\varepsilon_1\nabla\varepsilon_1 + \varepsilon_2\nabla\varepsilon_2}{\varepsilon_1^2 + \varepsilon_2^2} + i\,\frac{\varepsilon_1\nabla\varepsilon_2 - \varepsilon_2\nabla\varepsilon_1}{\varepsilon_1^2 + \varepsilon_2^2}$$

REAL PART:

$$\left[(\nabla S)^2 - \varepsilon_1\mu c^2\right]\bar{E}_0 =$$

$$= -\frac{1}{k_0}\left(\bar{E}_0 \cdot \frac{\varepsilon_1\nabla\varepsilon_2 - \varepsilon_2\nabla\varepsilon_1}{\varepsilon_1^2 + \varepsilon_2^2}\right)\nabla S + \frac{1}{k_0^2}\left[\nabla^2\bar{E}_0 + \nabla\left(\bar{E}_0 \cdot \frac{\varepsilon_1\nabla\varepsilon_1 + \varepsilon_2\nabla\varepsilon_2}{\varepsilon_1^2 + \varepsilon_2^2}\right)\right]$$

IMAGINARY PART:

$$-\varepsilon_2\mu c^2\bar{E}_0 = \frac{2}{k_0}\left[(\nabla S\cdot\nabla) + \tfrac{1}{2}\nabla^2 S\right]\bar{E}_0$$

$$+ \frac{1}{k_0}\left(\bar{E}_0 \cdot \frac{\varepsilon_1\nabla\varepsilon_1 + \varepsilon_2\nabla\varepsilon_2}{\varepsilon_1^2 + \varepsilon_2^2}\right)\nabla S$$

$$+ \frac{1}{k_0^2}\nabla\left(\bar{E}_0 \cdot \frac{\varepsilon_1\nabla\varepsilon_2 - \varepsilon_2\nabla\varepsilon_1}{\varepsilon_1^2 + \varepsilon_2^2}\right)$$

EXACT TILL NOW (assumed: $\mu(\bar{r}) = \mu$ real; S, $\bar{E}_0$ real).