PRINCIPLE OF FERMAT (Principle of Least Time)

$$\int_{P_1}^{P_2} n\, ds$$ = OPTICAL LENGTH
"PATH LENGTH"

RAY = Shortest path between two points (not necessarily straight line)

COMPARE TO GRAVITATIONAL FIELD

FOURIER OPTICS

Goodman: Introduction to Fourier Optics

we treated refraction/reflection

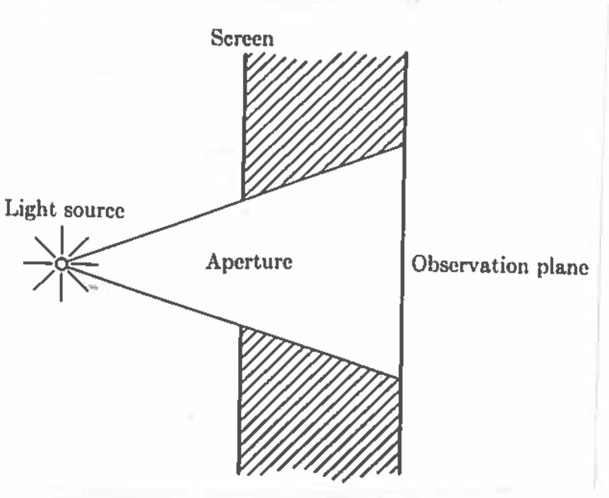
Now:

DIFFRACTION: Sommerfeld "Optics" (1954)

"any deviation of light rays from rectilinear paths which cannot be interpreted as reflection or refraction"

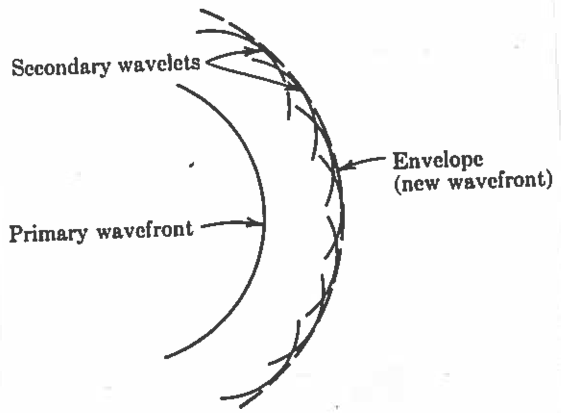
Wave nature ⟺ diffraction

Grimaldi 1665:



result not explained by corpuscular theory of light propagation

Huygens : first proposed wave theory of light 1678



Time t_1 :
Each point on the wave front new source of a secondary wave

Time t_2 later:
Wavefront is envelope of secondary wavelets

Fresnel 1818:
Huygens idea combined with Young's idea of interference :
· calculated diffraction patterns

↓
1882 - Kirchhoff mathematical foundation
Got it - by far - mostly right
- however did use two boundary conditions proven inconsistent by Poincaré 1892, Sommerfeld 1894

Follow Sommerfeld's corrective approach: 1896

Scalar theory here (one component)

Sommerfeld ⟺ Feynman diagrams
spatial analysis, Green's functions

contributions to electric field amplitude in one spatial point propagated thru Green's functions from other space points

ALTERNATIVE (not widely used in traditional physical optics)

: FOURIER OPTICS

linear systems theory used in

(Spatial) FREQUENCY ANALYSIS of imaging system

electric circuit theory

communication theory:

e-m light field decomposed

typically in plane waves

= (SPATIAL) Fourier transformation

TRANSFER FUNCTION

basis functions

OUTPUT = $\sum$ TRANSFORMED INPUT COMPONENTS

LINEARITY

Spatial FREQUENCY ANALYSIS of Imaging system
(TRANSFER FUNCTION)

Like

(temporal) FREQUENCY ANALYSIS for audio amplifier characterization

Analysis

AND

Synthesis ← phase-contrast imaging

: frequency plane processing

filtering

LENS acts as FOURIER TRANSFORMER

FOURIER TRANSFORMATION SUMMARY:

Light field (e-m wave) = { spatial distribution of complex valued field amplitude = $U(x, y, z)$

for simple (lin. polarized) plane wave it would be $E_{0x}\, e^{ikz}$

Pick z as propagation direction (like time in QM)

Input field = $U(x, y, z=0)$

Output field = $U(x, y, z=L)$

So generally: $g(x,y)$

Complex valued field in 2 variables

. Fourier transform $FT\{g\} \equiv G$:

$$G(f_x, f_y) = \iint_{-\infty}^{\infty} dx\, dy\; g(x,y)\, e^{-i2\pi(f_x x + f_y y)}$$

spatial frequencies

$$FT^{-1}\{G\}(x,y) = \iint_{-\infty}^{\infty} df_x\, df_y\, G(f_x, f_y)\, e^{i2\pi(f_x x + f_y y)}$$

$$= g(x,y)$$

So need to find response of system to

$e^{i 2\pi (f_x x + f_y y)}$ for each (f_x, f_y)

* Fourier transform is *linear*

* $FT\{g(ax, by)\}(f_x, f_y) = \frac{1}{|ab|} FT\{g\}(\frac{f_x}{a}, \frac{f_y}{b})$

contraction in space domain ⟷
stretching in frequency domain

* $FT\{g(x-a, y-b)\}(f_x, f_y) = FT\{g\}(f_x, f_y) e^{-i 2\pi (f_x a + f_y b)}$

translation in space ⟷
phase shift in frequency domain

* Parseval's theorem: $G \equiv FT\{g\}$:

$$\iint dx\, dy\, |g(x,y)|^2 = \iint df_x\, df_y\, |G(f_x, f_y)|^2$$

* Separable function : $g(x,y) = g_x(x)\, g_y(y)$:

$FT\{g\}(f_x, f_y) = FT_x\{g_x\}(f_x) \cdot FT_y\{g_y\}(f_y)$

* CONVOLUTION THEOREM :

$$FT\{g\} = G \quad ; \quad FT\{h\} = H \ :$$

$$FT\left\{ \iint_{-\infty}^{\infty} dx'dy' \, g(x', y') \, h(x-x', y-y') \right\}(f_x, f_y)$$

$$= \quad G(f_x, f_y) \cdot H(f_x, f_y)$$

! !

SEPARABLE IN POLAR COORDINATES :

More Complicated ; simplest case :

IMPORTANT case $\longrightarrow$ circular symmetry

$$g(r,\theta) = g(r)$$

$$r = \sqrt{x^2+y^2} \; ; \; \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

FT$\{g\} = G \Leftarrow$ function of f_x, f_y or ρ, ϕ :

$$\rho = \sqrt{f_x^2+f_y^2} \; ; \; \phi = \tan^{-1}\left(\frac{f_y}{f_x}\right)$$

$$G(\rho,\phi) = \int_0^\infty dr \; r\, g(r) \int_0^{2\pi} d\theta\, e^{-i2\pi r\rho\cos(\theta-\cancel{\phi})}$$

$$= \underline{G(\rho)} \leftarrow \text{Circular symmetry also}$$

Use:

J_0 : Bessel function of the first kind, zero order :

$$J_0(a) = \frac{1}{2\pi}\int_0^{2\pi} e^{-ia\cos\theta} \cdot d\theta$$

$$G(\rho) = 2\pi \int_0^\infty dr \cdot r\, g(r)\, J_0(2\pi r\rho)$$

one-dim. transform : Fourier-Bessel transform

Idealized point source of light :

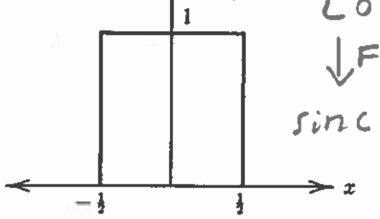
Dirac δ function

$$\delta(x,y) = \lim_{N\to\infty} \underbrace{N^2 e^{-N^2\pi(x^2+y^2)}}$$

$$FT\{\delta\} : = \lim_{N\to\infty} FT\{\downarrow\} = \lim_{N\to\infty} e^{-\pi(f_x^2+f_y^2)/N^2}$$

$$= e^{-0} = \underline{\underline{1}}$$

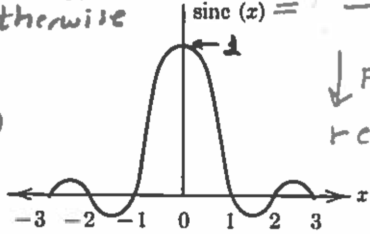
$\text{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$



$\downarrow$ FT

sinc (f_x)

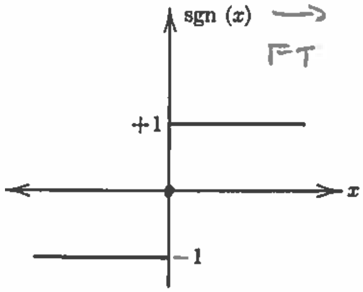
$\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$



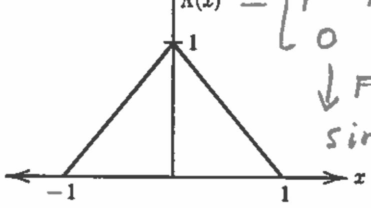
$\downarrow$ FT

rect (f_x)

$\text{sgn}(x) \xrightarrow[FT]{} \frac{1}{i\pi f_x}$



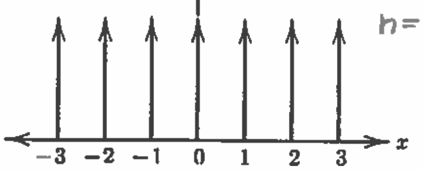
$\Lambda(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0 & \text{otherw.} \end{cases}$



$\downarrow$ FT

$\text{sinc}^2(f_x)$

$\text{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$

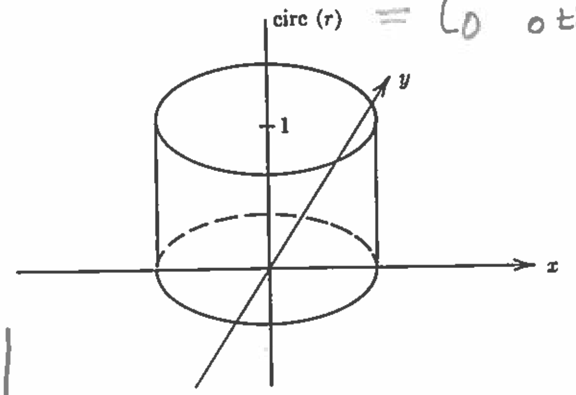


$\downarrow$ FT

comb (f_x)

2 DIM. FT :

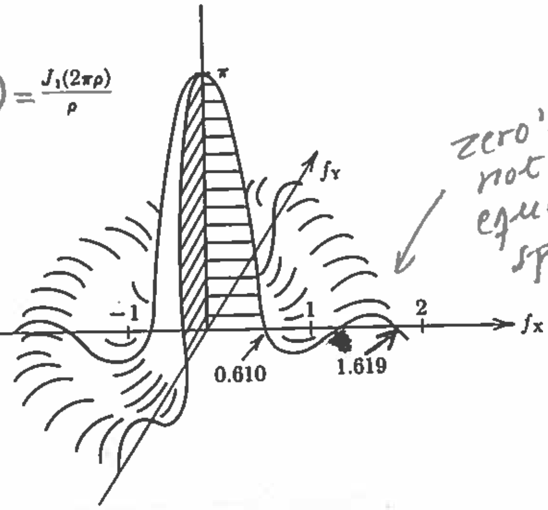
$$= \begin{cases} 1 & r=\sqrt{x^2+y^2}^{\,2} \le 1 \\ 0 & \text{otherwise} \end{cases}$$



← aperture !

FT ↓

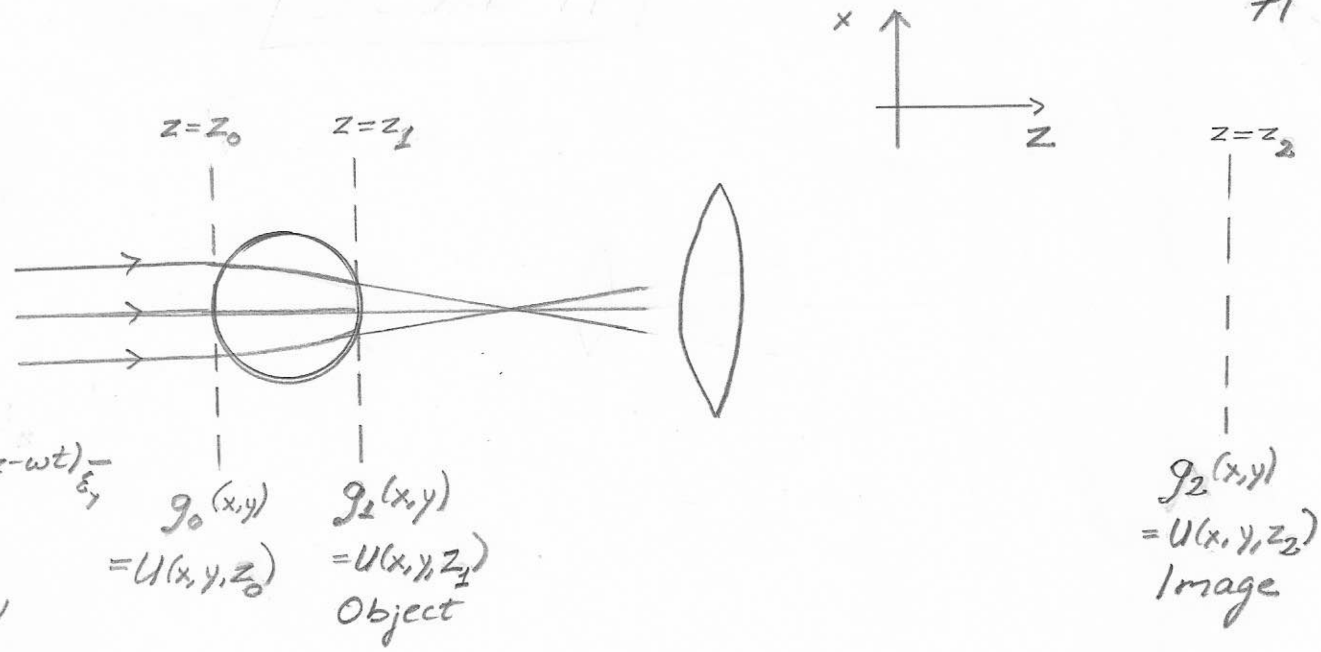
$$2\pi \int_0^{\infty \to 1} dr \cdot r \underset{\substack{\downarrow \\ 1}}{g(r)} J_0(2\pi r \rho) = \frac{J_1(2\pi\rho)}{\rho}$$



zero's not equally spaced

$$\int_0^x dy\, y \cdot J_0(y) = x \underset{9}{J_1}(x)$$

Bessel func.
1st kind
order 1



$z<z_0$:

$\bar{e}(\bar{r},t)$

$= E_y e^{i(kz-\omega t)}\hat{\epsilon}_y$

↓

Spatial part of E field (y compont - scalar field)

$= U(x,y,z) = E_y e^{ikz}$

MORE GENERALLY :

$U(x,y,z)$ = complex field (representing $E(x,y,z)$ or $B(x,y,z)$: both single components in scalar theory).

Spatial part of solutions to Maxwell eqs. Temporal part: $e^{-i\omega t}$.

PROPAGATION IN SPACE:

$$\begin{array}{ccc} U(x,y,z=0) & \longrightarrow & U(x,y,z=L) \\ ||| & & ||| \\ g_1(x,y) & & g_2(x,y) \end{array}$$

Mapping S: $g_2 = S\{g_1\}$

linear systems: S linear

One possible way of decomposing input into elementary input functions:

Use δ functions:

$$g_1(x,y) = \iint\limits_{-\infty}^{\infty} dx'\,dy'\, \underbrace{g_1}_{\substack{\text{weighting}\\ \text{factor}}}(x',y')\,\delta(x-x',y-y')$$

Linearity – we just need to find

$$S\{\delta(x-x',y-y')\}\Big|_{x_2,y_2} \equiv h(x_2,y_2\,;\,x',y')$$

$=$ IMPULSE response of system at x_2,y_2 in $z=L$ plane to δ-function input at x',y' in $z=0$ plane

$$g_2(x_2,y_2) = \iint dx'\,dy'\,g_1(x',y')\,h(x_2,y_2;x',y')$$

so just need to know response of all point sources in object plane

$h(x_2,y_2;x',y')$

exactly equivalent to Feynman path integral propagator in 2D quantum mechanics with z acting as time coordinate (Ref.: Feynman & Hibbs Quantum Mechanics and Integrals)

Space invariance:

$$h(x_2,y_2;x',y') = h(x_2-x';y_2-y')$$

$$\therefore\quad g_2(x_2,y_2) = \iint dx'\,dy'\,g_1(x',y')\,h(x_2-x';y_2-y')$$

Convolution integral

$$FT\{g_2\}(f_x,f_y) = \underbrace{FT\{g_1\}(f_x,f_y)}_{\text{Fourier transform of input field in object plane}} \cdot \underbrace{FT\{h\}(f_x,f_y)}_{\text{Fourier transform of impulse response}}$$

$\equiv$

$H(f_x,f_y)$

$=$

TRANSFER FUNCTION for imaging syst

effects in freq. domain →

So: CONVOLUTION INTEGRAL complicated

↓

PRODUCT simple

BEST BASIS for linear, space invariant syst.

PLANE WAVES

$$g_1(x,y) = \iint df_x\, df_y\, \underset{\overset{\uparrow}{FT\{g_1\}}}{G_1}(f_x, f_y)\, e^{i2\pi(f_x x + f_y y)}$$

$$g_2(x,y) = \iint df_x\, df_y\, G_1(f_x, f_y) \cdot \underbrace{H(f_x, f_y)}_{\uparrow}\, e^{i2\pi(f_x x + f_y y)}$$

simple multiplication w/ complex number.

gives response to basis function input.

FOURIER OPTICS view of DIFFRACTION

1) Free space propagation: find transfer function $H(f_x, f_y)$

(see notes p. 73)

$$U(x, y, z=0) \longrightarrow U(x,y, z=L)$$

$$U(x, y, 0) = \iint A_0(f_x, f_y)\, e^{i2\pi(f_x x + f_y y)}\, df_x\, df_y$$

$$A_0 = \iint U(x, y, z=0)\, e^{-i2\pi(f_x x + f_y y)}\, dx\, dy$$

AND (generally)

$$U(x,y,z) = \iint A(f_x, f_y; z)\, e^{i2\pi(f_x x + f_y y)}\, df_x\, df_y$$

U satisfies Helmholtz' wave equation IN FREE SPACE:

$$\nabla^2 U + \underbrace{\varepsilon\mu\omega^2}_{k^2} U = 0 \qquad \text{so}$$

$$0 = \frac{d^2}{dz^2} A(f_x, f_y; z) + \left(k^2 - (2\pi f_x)^2 - (2\pi f_y)^2\right) A(f_x, f_y; z)$$

$$\Rightarrow A(f_x, f_y; z) = A_0(f_x, f_y)\, e^{i \underbrace{\sqrt{k^2 - (2\pi f_x)^2 - (2\pi f_y)^2}}_{\overset{\shortparallel}{k_z}}\, z}$$

Note:

Remember : wave equation predicted by Maxwell's eqs :

$$\nabla^2 \bar{e} = \varepsilon\mu \frac{\partial^2 \bar{e}}{\partial t^2} \qquad \bar{e}(\bar{r},t) = e^{-i\omega t}\bar{E}(\bar{r})$$

$$\nabla^2 \bar{E} = -\underbrace{\varepsilon\mu}_{1/c^2 \text{ in free space}}\omega^2 \bar{E}$$

Helmholtz eq.

Here scalar theory of diffraction:

consider f.ex. $E_x(\bar{r}) \equiv U(\bar{r})$

TRANSFER FUNCTION

for propagation in free space thru distance z:

$$H(f_x, f_y) = \frac{A(f_x, f_y; z)}{A_0(f_x, f_y)} = e^{i\sqrt{k^2-(2\pi f_x)^2-(2\pi f_y)^2}\cdot z}$$

$$\equiv e^{ik_z z}$$

$\uparrow$ phase change dependent on f_x, f_y AND z

So: Like 2D QUANTUM MECHANICS

$$\boxed{\begin{aligned} \text{time} &= (-)\, z/c \\ \text{energy} &= \hbar\omega\sqrt{1-k_\perp^2/k^2} \end{aligned}} :$$

$$U(x,y;z) =$$

$$= \iint df_x\, df_y\, A_0(f_x, f_y)\cdot e^{i2\pi(f_x x + f_y y)}.\, e^{ik_z z}$$

$$k_z z \equiv \sqrt{k^2-(2\pi f_x)^2-(2\pi f_y)^2}\, z = z\cdot \underbrace{\overbrace{\omega\sqrt{\varepsilon\mu}}^{k}}_{\omega\sqrt{\varepsilon_0\mu_0} = \omega/c} \cdot\sqrt{1-\frac{k_\perp^2}{k^2}}$$

$$\uparrow$$

$$e^{ik_z z} = e^{i\frac{z}{c}\cdot\omega\sqrt{1-k_\perp^2/k^2}} \leftarrow \boxed{k_\perp = (2\pi f_x, 2\pi f_y)}$$

2) <u>Diffraction</u>: $\left\{ \text{Just need to find transfer function } H(f_x, f_y) \text{ thru aperture and multiply with free-space transfer function} \right.$

Sommerfeld boundary condition :

Screen placed at $z = 0$:

$$U(x, y; z = 0^+)$$

$$= U(x, y, z = 0^-) \cdot t(x, y)$$

t = transmittance function of aperture Σ (with arbitrary shape):

$$t(x,y) = \begin{cases} 1 & (x,y) \text{ in } \Sigma \\ 0 & \text{otherwise} \end{cases}$$

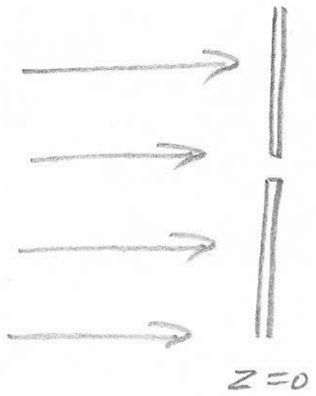
Use convolution theorem :

$T = FT\{t\}$:

$FT\{U(x,y;z=0^+\} =$ CONVOLUTION OF $FT\{U(x,y;z=0^-\}$ and T

FOR EXAMPLE:

PLANE WAVE hitting aperture at normal incidence:



$U(x,y;z=0^-)$ is uniform, say 1.

Then:

$FT(U(x,y;z=0^-)$

$= \delta(f_x, f_y)$, and

$$FT\{U(x,y:z=0^+)\}(f_x,f_y) =$$

$$= \iint df_x' df_y' \, \overbrace{FT\{U(x,y;z=0^-)\}(f_x';f_y')}^{\delta(f_x';f_y')} \cdot$$

$$\cdot \; T(f_x - f_x', f_y - f_y')$$

$= \quad T(f_x, f_y) =$ Fourier transform of aperture transmission function

If circle aperture radius a } : $t(r) = \mathrm{circ}(r/a)$

$$T(\rho) = \frac{J_1(2\pi\rho a)}{\rho a} \cdot a^2$$

see notes p. (70)

DIFFRACTION RESULT :

So : APERTURE

BROADENS MOMENTUM SPREAD $\hbar(2\pi f_x, 2\pi f_y)$

OF LIGHT FIELD AS

$\Delta p \propto \frac{\hbar}{a}$: The smaller the aperture the bigger the momentum spread.

First node in 1st order Bessel function J_1:

at $pa = 0.610 \Rightarrow \Delta p = \underbrace{\frac{0.610}{a}}_{p} \cdot 2\pi \cdot \hbar = 0.610 \frac{h}{a}$

$\boxed{\text{Uncertainty principle}}$

FRESNEL & FRAUNHOFER DIFFRACTION

We found transfer function for propagation of distance z in free space:

$$\boxed{H(f_x, f_y) = e^{i\sqrt{k^2-(2\pi f_x)^2-(2\pi f_y)^2}\cdot z}}$$

Exact (p. 77)

$k = \frac{\omega}{c}$

Fresnel approximation: $2\pi\sqrt{f_x^2+f_y^2} \ll k$:

$$H(f_x,f_y) \simeq e^{ikz}\, e^{-i\pi\lambda z(f_x^2+f_y^2)} \quad ; \lambda = \frac{2\pi}{k}$$

$$FT\{U(x,y,z)\} = FT\{U(x,y,z=0)\}\cdot H(f_x,f_y) :$$

$$(*) \quad U(x_1,y_1,z) \simeq \frac{e^{ikz}}{i\lambda z}\int dx_0 \int dy_0\, e^{i\frac{k}{2z}((x_1-x_0)^2+(y_1-y_0)^2)}\, U(x_0,y_0,z=0)$$

↗ spatial version of Fresnel approximation

Rewrite eq. (*):

$$U(x_1,y_1,z) = \frac{e^{ikz}}{i\lambda z}\, e^{i\frac{k}{2z}(x_1^2+y_1^2)} \iint dx_0\, dy_0\, e^{i\frac{k}{2z}(x_0^2+y_0^2)}\, U(x_0,y_0,z=0) \cdot e^{-i\frac{k}{z}(x_0x_1+y_0y_1)}$$

$$= \frac{e^{ikz}}{i\lambda z}\, e^{i\frac{k}{2z}(x_1^2+y_1^2)}\, FT\{e^{i\frac{k}{2z}(x^2+y^2)}\, U(x,y,z=0)\}(f_x,f_y)$$

$$f_x = \frac{x_1}{z\lambda} \ ; \ f_y = \frac{y_1}{z\lambda}$$

Fraunhofer Approx. is:

$$FT\left\{U(x,y,z=0)\underbrace{e^{i\frac{k}{2z}(x^2+y^2)}}\right\}$$

neglect

$\frac{k}{2z}(x^2+y^2) \ll 1$

1mm aperture

6000 Å red light

$z \gg 2.5\,m$

So:

$$U(x_1,y_1,z)=\frac{e^{ikz}}{i\lambda z}\,e^{i\frac{k}{2z}(x_1^2+y_1^2)}\,FT\{U(x,y,z=0)\}(f_x,f_y)$$

$$(f_x,f_y)=\frac{k}{2\pi z}(x_1,y_1).$$

$U(x,y,z)$

So diffraction pattern at distance z is obtained from Fourier transform of field distribution right after aperture.

$U(x,y,z=0^+)$

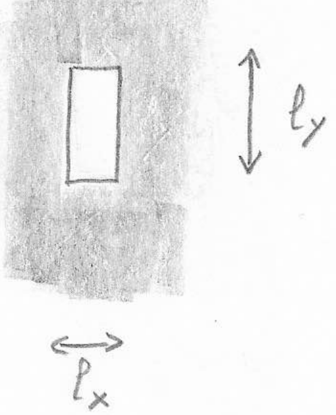
So apart from phase factor $\frac{e^{ikz}}{i}e^{i\frac{k}{2z}(x_1^2+y_1^2)}$ and amplitude factor $\frac{1}{\lambda z}$

free space propagation in Fraunhofer regime acts as Fourier transformer.

Rectangular aperture:

$$t(x,y) = \mathrm{rect}\left(\frac{x}{l_x}\right)\cdot \mathrm{rect}\left(\frac{y}{l_y}\right)$$

widths



Irradiate with orthogonal plane wave:

$$U(x_1,y_1;z) = \frac{e^{ikz}}{i\lambda z}\, e^{i\frac{k}{2z}(x_1^2+y_1^2)}\, FT\{t(x,y)\}(f_x,f_y)$$

Fraunhofer

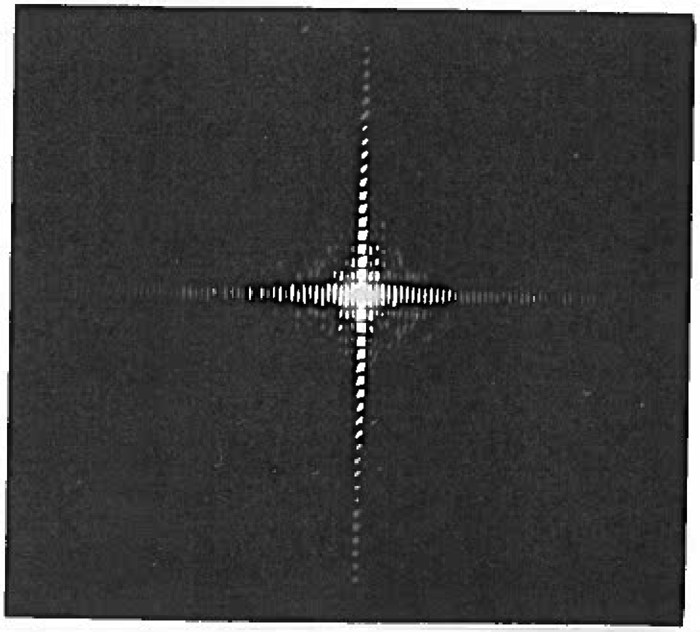
$$(f_x,f_y) = \frac{k}{2\pi z}(x_1,y_1)$$

$$FT\{t(x,y)\} = l_x l_y \,\mathrm{sinc}(l_x f_x)\,\mathrm{sinc}(l_y f_y)$$

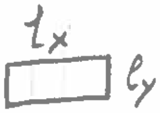
$$\mathrm{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

Eye: Intensity $\propto |U(x_1,y_1;z)|^2$

$$= \frac{l_x^2 l_y^2}{\lambda^2 z^2}\,\mathrm{sinc}^2\frac{l_x x_1}{\lambda z}\,\mathrm{sinc}^2\frac{l_y y_1}{\lambda z}$$



Fraunhofer of square slit



$l_x = 2l_y$

$$\text{Width of central spot} = \begin{cases} \dfrac{\lambda z}{l_x}\cdot 2 & \text{along } x \\ \dfrac{\lambda z}{l_y}\cdot 2 & \text{along } y \end{cases}$$

Circular aperture

Radius a : $t(x,y) = t(r) = \text{circ}\left(\frac{r}{a}\right)$

Irradiate with orthogonal plane wave :

$U(x_1, y_1; z) = U(r_1; z) =$

$$= \frac{e^{ikz}}{i\lambda z}\, e^{i\frac{k}{2z}r_1^2}\, FT\left\{ \overset{t(r)=\text{circ}(r/a)}{\overset{\parallel}{U(r;z=0)}} \right\}(f_x, f_y)$$

$(f_x, f_y) = \frac{1}{\lambda z}(x_1, y_1)$

Bessel transform (circle symmetry)

$$FT\left\{\text{circ}(r/a)\right\}_{\rho=\sqrt{f_x^2+f_y^2}} = a^2\,\frac{J_1(2\pi a\rho)}{a\rho}$$

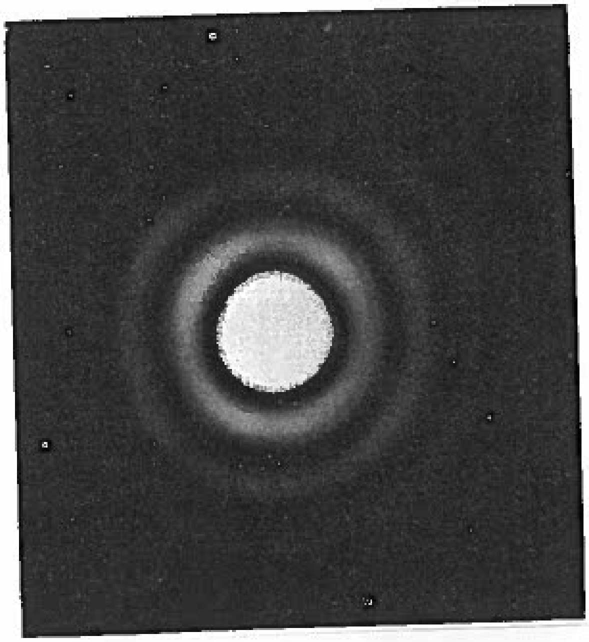
$$\rho = \frac{1}{\lambda z}\cdot r_1$$

$$\text{Fraunhofer} = \frac{e^{ikz}}{i\lambda z}\cdot e^{i\frac{k}{2z}r_1^2}\cdot a^2\,\frac{J_1(2\pi a r_1/(\lambda z))}{a\cdot r_1/(\lambda z)}$$

$$\text{Intensity} = |U(x_1, y_1; z)|^2 = \left(\frac{k\,a^2}{z}\right)^2 \left(\frac{J_1\left(\frac{k a r_1}{z}\right)}{k a r_1/z}\right)^2$$

= AIRY PATTERN

FRAUNHOFER
OF
ROUND
APERTURE
radius=a



AIRY
PATTERN

Diameter of central spot =

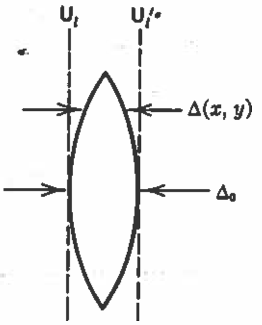
$$= 1.22 \cdot \frac{\lambda z}{a}$$

LENSES

1) FOURIER TRANSFORMER

2) IMAGING

what are they



GLASS
with index
$n \sim 1.5$

Thin lens if position does not change:
$(x,y)_i = (x,y)_o$ in ray sense.

· Phase shifter $t_\ell(x,y)$:

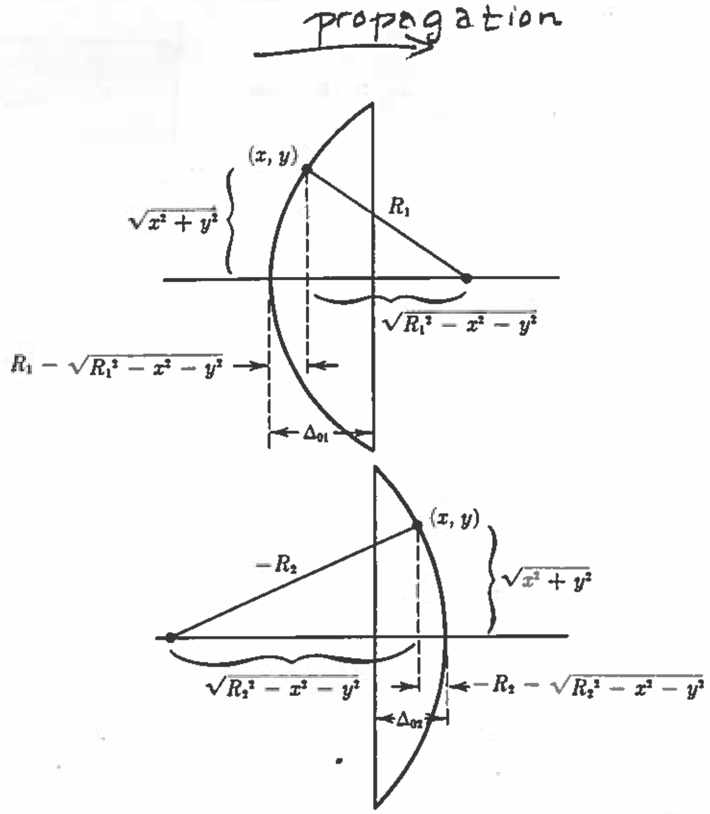
$$U_{\ell'}(x,y) = t_\ell(x,y)U_\ell(x,y)$$

$t_\ell(x,y) =$:

thickness function $\Delta(x,y)$

$$\text{phase delay} = \phi(x,y) = \overset{\text{in lens}}{kn\Delta(x,y)} + \overset{\text{in free space}}{k(\Delta_0 - \Delta(x,y))}$$

FIND $\Delta(x,y)$ = thickness function of Lens



PART 1

POS RAD. OF CURVATURE R_1

PART 2

NEG RAD. OF CURVATURE R_2

$$\Delta(x,y) = \Delta_1(x,y) + \Delta_2(x,y)$$

$$\Delta_1(x,y) = \Delta_{01} - \left(R_1 - \sqrt{R_1^2 - x^2 - y^2}\right)$$

$$\Delta_2(x,y) = \Delta_{02} - \left(\underbrace{-R_2}_{POS} - \sqrt{R_2^2 - x^2 - y^2}\right)$$

$$\Delta(x,y) = \underbrace{\Delta_{01} + \Delta_{02}}_{\overset{\parallel}{\Delta_0}} - R_1\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_1^2}}\right) + R_2\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_2^2}}\right)$$

$\boxed{\text{PARAXIAL APPROX}}$:

$$\sqrt{1 - \frac{x^2+y^2}{R_i^2}} \simeq 1 - \frac{x^2+y^2}{2R_i^2} \quad ; \; i = 1, 2$$

So : $\Delta(x,y) = \Delta_0 - \frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Lens action

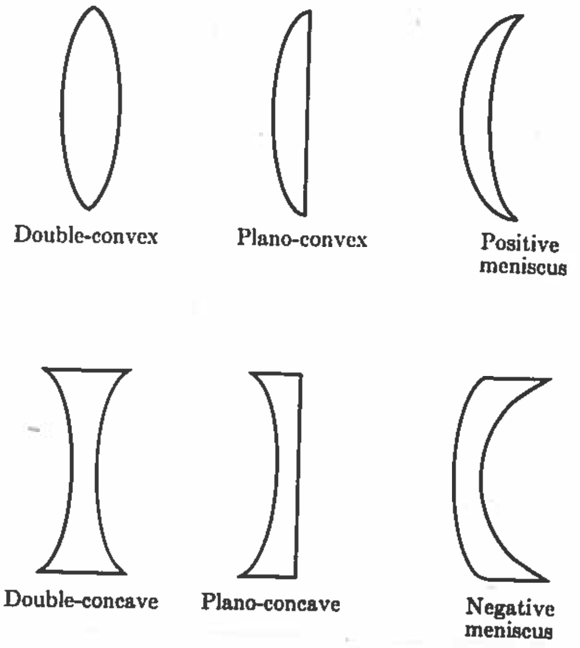
$$t_\ell(x,y) = e^{i\phi(x,y)}$$

$$= e^{ikn\Delta_0} \, e^{-ik(n-1)\frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

lens shape defined by $\underbrace{R_1, R_2, n}_{\text{combined in focal length } f}$:

$$\frac{1}{f} \equiv (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) :$$

$$t_\ell(x,y) = e^{ikn\Delta_0} \, e^{-i\frac{k}{2f}(x^2+y^2)}$$



Pos. f

NEG f

What is lens actually doing:

Plane wave, normal incidence: $U_l(x,y)=A$:

$$U_l'(x,y) = \underset{\uparrow\ \varphi}{e^{ikn\Delta_0}}\ \ e^{-i\frac{k}{2f}(x^2+y^2)}\uparrow A$$

phase delay

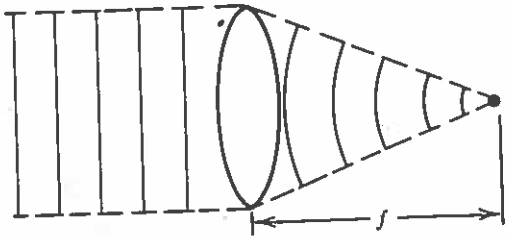
spherical wave

Taylor expanded to 1st ord.

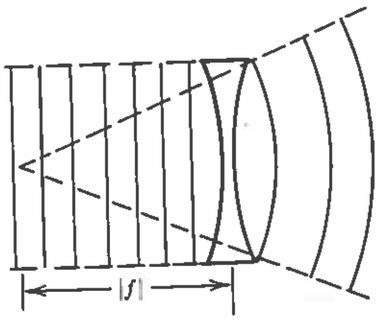
$$\left(e^{\mp ik\sqrt{f^2+x^2+y^2}} \underset{\uparrow\, x^2+y^2 \ll f^2}{\simeq} e^{\mp ikf} e^{\mp i\frac{k}{2f}(x^2+y^2)}\right)$$

f pos : converging

f neg : diverging



f > 0



f < 0

Spherical surfaces → Spherical waves

→ focus = δ function

Non paraxial conditions:

Spherical surface ⟶ Non spherical waves

//

aberration

⟶ corrected for aberrations by grinding surface aspherical