PROBLEM SET 6

FALL 2024

Consider a cigar-shaped Bose-Einstein condensate of Na atoms:

$$n(\vec{r}) = n_0\left(1 - \frac{x^2 + y^2 + \epsilon^2 z^2}{a^2}\right)$$

where $n_0 = 10^{14}$ cm^{-3}, $a = 15$ μm, and $\epsilon = \frac{1}{10}$ (note: $n(\vec{r}) = 0$ when $\frac{x^2+y^2+\epsilon^2 z^2}{a^2} > 1$).
The condensate is illuminated with a plane-wave laser beam:

$$\vec{e}(\vec{r}, t) = E_0 \hat{e}_z e^{i(ky-\omega t)}$$

Assume rays through the condensate are straight (but they have phase shifts and are absorbed somewhat, of course).

We want to image the condensate, so please design an imaging system with the following constraints:

Place the lenses and imaging camera outside the vacuum system housing the condensate. The distance from the condensate to the exit window on the vacuum system is 220 mm, and the window diameter is 2.5".

Go on the web and pick lenses (from a vendor, e.g., Thorlabs or CVI) such that the imaging system has the best possible resolution. We want the magnification to be $M = 1$. The imaging system should have a Fourier plane and an image plane. Next:

(a) Find the intensity distribution in the image plane when the imaging system is in focus and the laser is on resonance (hint: you can assume you have infinitely large lenses and then an aperture in the Fourier plane with opening diameter equal to the actual lens diameter).

(b) The laser is tuned to 10 MHz above resonance (+10 MHz detuning), and the imaging system is out of focus by +100 micrometers and -100 micrometers, respectively. Calculate the intensity distribution in the out-of-focus image plane (i.e., calculate the intensity distribution in a plane that is 100 micrometers farther away from the object than the ideal image plane, and in a plane that is 100 micrometers closer to the object than the ideal image plane).

(c) Same as (b) but with -10 MHz detuning.

(d) Consider the same question as in (a), but now place an aperture in the Fourier plane and close it down to an opening diameter of 10 mm. Continued next page....

Hint: These problems will require use of a Fast Fourier Transform (FFT) package in 2D. It should be noted that, while in this example we consider a BEC, these methods are very general and applicable to most any imaging problem in optics.

EXTRA outside of required homework: Once you have the program running, you might want to try playing with increasing the out of focus distance (for the two detunings) beyond the plus and minus 100 microns in (b), and watch the behavior of the image that would be detected by an out-of-focus camera.

You might also want to try closing the aperture even further down than the 10 mm in (d).