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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22B

Total:

Unit 14: First Hourly (Practice A)

Problems

Problem 14A.1 (10 points):
The **Fibonacci numbers** are defined recursively as follows: start with $F_0 = 0, F_1 = 1$ then define $F_{n+1} = F_n + F_{n-1}$, so that $F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5$ etc. Prove that

$$F_0 + F_1 + \cdots + F_n = F_{n+2} - 1$$

for every positive integer n.

Problem 14A.2 (10 points):
Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

a) (4 points) Compute AB and $\text{rref}(AB)$.
b) (4 points) Now row reduce both A and B and form $\text{rref}(A)\text{rref}(B)$.
c) (2 points) Is the statement $\text{rref}(AB) = \text{rref}(A)\text{rref}(B)$ true for all A, B?

Problem 14A.3 (10 points):
a) (2 points) Parametrize the line through $(1,1,1)$ and $(4,3,1)$ in $\mathbb{R}^3$.
b) (2 points) Parametrize the ellipse $x^2/16 + y^2/25 = 1$ in $\mathbb{R}^2$.
c) (2 points) Parametrize the graph $y = x^5 + x$ in $\mathbb{R}^2$.
d) (2 points) Parametrize the circle $x^2 + (y-2)^2 = 1, z = 4$ in $\mathbb{R}^3$.
e) (2 points) Parametrize the line $x = y = z$ in $\mathbb{R}^3$.

Problem 14A.4 (10 points):
Find the arc length of the curve

$$r(t) = [t\cos(t^2), t\sin(t^2), t^2]$$

for $0 \le t \le 2$.

Problem 14A.5 (10 points):
a) (2 points) What is the Heine-Cantor theorem?
b) (2 points) Formulate the triangle inequality.
c) (2 points) What is the Al Kashi identity?
d) (2 points) Give the name of a nowhere differentiable function.
e) (2 points) Is it true that a continuous curve $r(t)$ has a finite arc length?

Problem 14A.6 (10 points):
a) (2 points) Find $(3+i)(4+2i)$
b) (2 points) What is $e^{i3\pi/4}$?
c) (2 points) Convert from cylindrical $(r,\theta,z) = (2,\pi/2,1)$ to Cartesian.
d) (2 points) What are the spherical coordinates of $(1,\sqrt{3},2)$?
e) (2 points) What surface is in spherical coordinates given as $\rho\sin(\phi) = 1$?

Problem 14A.7 (10 points):
a) (5 points) You are given $r'''(t) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $r(0) = (7,8,9)$ and $r'(0) = (1,0,0)$ and $r''(0) = (0,1,0)$. Find $r(1)$.
b) (5 points) What is the curvature of $r(t) = [t, t+t^2, t+t^2+t^3]$ at $t = 0$?

Problem 14A.8 (10 points):
a) (5 points) Find a parametrization $r(u,v)$ of the cylinder $x^2+z^2 = 9$.
b) (5 points) Find $r(u,v)$ for the paraboloid $y^2+3z^2 = x$.

Problem 14A.9 (10 points):
Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$. a) (2 points) The image of A is a plane. By using the cross product, write it as $ax+by+cz = d$.
b) (2 points) What is the first fundamental form $g = A^TA$?
c) (2 points) From a) you have $[a,b,c]^T = v \times w$. Find $\sqrt{a^2+b^2+c^2}$.
d) (2 points) Find the distortion factor $||A|| = \sqrt{\det(A^TA)}$ of A.
e) (2 points) What theorem was involved to see $||A|| = |v \times w|$?

Problem 14A.10 (10 points):
a) (5 points) What is the Jacobian matrix df of the map

$$f(x,y,z) = [x^2+y^2+z^2, x+y, -x^2]^T \ ?$$

b) (5 points) Find the distortion factor $\det(df)$.