Econ 2010c

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Homework 1

Due Date: November 5th, 23:59 hrs, via canvas

Question 1: Calibrating the Growth Model

Consider the following economy, the equilibrium of which solves the following social planner's problem. Let N_t is the measure of households at time t. The planner's maximization problem is:

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t N_t[\log(c_t) - Bh_t],$$

subject to

$$N_t(c_t + i_t) = (N_t k_t)^{\theta}(\gamma^t N_t h_t)^{1-\theta}, \quad \gamma \geq 1, \ 0 < \theta < 1$$

$$N_{t+1}k_{t+1} = (1-\delta)N_t k_t + N_t i_t, \quad 0 < \delta < 1$$

where $h_t \geq 0$, population growth is given by $N_{t+1} = \eta N_t$, for $\eta \geq 1$, and k_0 and N_0 are given. Here c_t, i_t, h_t and k_t are consumption per capita, investment per capita, hours worked per capita and capital stock per capita.

(a) Write the Bellman equation for this problem.

(b) Solve for the balanced growth path of this economy.

(c) Show how one can transform this into a stationary (no growth) dynamic programming problem by implementing a change of variables. Show that this leads to the same balanced growth path as obtained in part (b).

(d) Calibrate an annual version of this economy to the following features of the U.S. post-war economy. The number of features should equal the number of parameters to be calibrated.

(i) The average annual growth rate of real output per capita is 2 percent.

(ii) The average annual growth rate of the population is 1 percent.

(iii) The average fraction of total income that is paid to owners of capital is 0.35.

(iv) The average investment-to-output ratio is 0.15

(v) The average capital capital-to-output ratio is 2.5

(vi) Individuals work 24 hours per capita on average.

Question 2: Recursive Competitive Equilibrium

For each of the following economies, do the following:

(i) Specify the dynamic program that would be solved by a social planner in the economy

(ii) Define a recursive competitive equilibrium for the economy

(a) The are measure one of identical households with preferences given by $\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$ where $u(\cdot,\cdot)$ is continuous, increasing, concave and continuously differentiable in both arguments. Variables c_t and ℓ_t represent consumption and leisure. There is a constant-returns-to-scale technology $F(K_t, N_t)$ to produce output, where where K_t is the capital input and N_t is the labor input. Households are endowed with one unit of time that can be allocated to work, n_t, or leisure. They purchase output from the firm and use it for consumption or as capital in the following period. Capital depreciates at rate δ each period. Households are endowed with k_0 units of capital in period 0.

(b) The same economy in Part A except that utility depends not only on current consumption and leisure, but also on consumption and leisure from the previous period. That is, the period utility function is $u(c_t, c_{t-1}, \ell_t, \ell_{t-1})$.

(c) The same economy as in Part A except that there are two firms that operate constant-returns technologies. One produces new capital goods from labor and existing capital supplied by the households. The other produces consumption goods from these same factors of production. Denote the technologies operating in sector i by $f^i(k_i, n_i)$, $i = 1, 2$. Households have one unit of time that can be allocated to leisure, labor in sector 1, $n_{1,t}$, and sector 2, $n_{2,t}$. In addition, households accumulate productive capital that can be allocated to either sector.

Question 3: Postwar Growth in the United States, Germany, and Japan

This problem explores how a simple neoclassical growth model fares in explaining the post-war growth experiences of the US, Germany, and Japan. We will do so using the following model. Consider an economy with measure one of identical households whose preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where c_t is consumption and $\sigma > 0$. Capital depreciates at rate δ, where $0 < \delta < 1$. The initial capital stock is k_0. The production technology is given by

$$Y_t = K_t^{\theta} (\gamma^t H_t)^{1-\theta}$$

where K_t and H_t are the capital and labor inputs, and $\gamma \geq 1$ represents efficiency growth. The household rents its capital to the market at rate r_t, and rents its labor services at wage w_t.

(a) Express the social planner's problem for this economy as a sequence problem.

(b) Solve the planner's problem and derive the optimality conditions.

(c) We want to use the tools learned in the last quarter. Writing code for a value function with two state variables can be awkward (additional loops or using Kronecker products), so we will use a trick. Given the insight from Question 1(c), stationarize the original problem by appropriately detrending the variables. You should then not have any state variable for "technology", or any γ^t-terms. Compare the objective and constraints to the neoclassical growth model without exogenous growth. Argue that you can use a recursive method to computationally solve our original model by transforming some objects, even when not on the balanced growth path.
Hint: The Euler equation holds on any equilibrium path, even if not on the balanced growth path.

(d) Using value function iteration, numerically solve for the planner's policy functions assuming that $\delta = 0.1$, $\beta = 0.96$, $\theta = 0.3$, $\gamma = 1.01$, $\sigma = 2$, and $k_0 = 0.5$. This will represent the United States. Report the competitive equilibrium allocation for the first 30 periods after the initial period, to represent the years 1950 through 1979. On four different plots (or 2x2 subplots in Matlab), plot the following over these 40 periods:

(i) Log GDP per capita

(ii) The rental rate of capital (the marginal product of capital)

(iii) The wage rate (the marginal product of labor)

(iv) The investment-output ratio

(e) Now, using your code for part (b), repeat the graphic by assuming that $k_0 = 0.1$.

(f) Using the data accompanying this Problem Set on canvas, plot postwar growth in the US, Germany, and Japan in one plot. Express it in terms of log GDP, and compare it to the transition path from the model simulations. Does our model do a good job at replicating broad patterns?

(g) Can you use intuition from our growth model to explain the Japanese/German catch-up after World War 2?

(h) Consider the following modification to the model. Household preferences are now given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t - \overline{c})^{1-\sigma}}{1-\sigma}$$

Interpret the parameter $\overline{c}$. Which aspect of the model fit might be improved by it? Describe briefly using economic intuition how you expect this modification work.