Econ 2010c

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Homework 2

Due Date: November 14th, 23:59 hrs, via canvas

Question 1: Capital Taxation in the Growth Model

Consider an economy with measure one of identical households whose preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \alpha \log(1-n_t)\right]$$

where $c_t \geq 0$ and $0 < n_t < 1$ for all t and where each household is endowed with some initial capital level k_0. Let the production function be $F(K_t, N_t) = AK_t^{\theta} N_t^{1-\theta}$ for $A > 0$, and suppose that the technology is operated by a representative firm that behaves perfectly competitively. Suppose that the government levies a tax τ^k on capital income each period, satisfying $0 < \tau^k < 1$, and spends the proceeds on "foreign aid" which does not affect the economy in any way.

(a) Define a recursive competitive equilibrium for this economy.

(b) Characterize the recursive competitive equilibrium. Solve for the steady-state values of the return on capital, the capital-labor ratio, the wage rate, consumption per unit of labor, and labor input.

(c) Now specify the problem of a social planner who has to spend as much each period on foreign aid as in the competitive equilibrium allocation. You can let g_t represent the expenditure on foreign aid that the planner must make in period t.. Solve for the planner's steady-state values of the marginal product of capital, the capital-labor ratio, the marginal product of labor, consumption per unit of labor, and labor input.

(d) How does the competitive allocation differ from the planner's allocation? Provide intuition for your answer making reference to the differences in your answers to (b) and (c).

(e) Let α=1, A=1, θ=0.33, τ_K = 0.25, β=0.96. Start in steady state and consider two scenarios (1) the planner's allocation each period, and (2) the competitive equilibrium allocation with the marginal capital taxation, but the household "magically" gets consumption of c*(1+x) each period rather than c. Compute the consumption equivalent variation, that is the value x, making the households indifferent between (1) and (2). Interpret the value x in this context.

Question 2: Pollution and Abatement

An economy is populated by measure one of households whose preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t \{\log(C_t) - \alpha P_t^2\},$$

where $0 < \beta < 1$, $C_t \geq 0$ is consumption, $\alpha > 0$ and $P_t \geq 0$ is pollution. The households each supply one unit of labor inelastically. Output is produced using the following production technology:

$$Y_t = A(K_t)^{\theta}(N_t)^{1-\theta}$$

where $0 < \theta < 1$, $A > 0$, and variables Y_t, K_t and N_t represent output, capital and labor. Pollution is a by-product of the production process, but it can be abated (reduced) using "pollution abatement equipment," denoted X_t. Specifically, the pollution produced at time t is given by

$$P_t = \phi \frac{Y_t}{X_t}$$

where $\phi > 0$. The current period's output can be transformed into consumption, abatement equipment or capital for the following period. All the capital and abatement equipment used in the current period depreciates fully after it is used, and the abatement equipment cannot be stored from one period to the next. The resource constraint of the economy is $Y_t = C_t + X_t + K_{t+1}$, where K_{t+1} is capital to be saved for the following period.

(a) Provide a recursive formulation of the social planner's problem.

(b) Characterize the solution to the social planner's problem. Provide a brief interpretation of each optimality condition.

(c) Now assume that resources are allocated in competitive markets, and that the government regulates that each household purchases $\bar{x}$ units of abatement equipment each period. Formulate the household's problem recursively, and define a recursive competitive equilibrium.

(d) Providing a set of conditions that characterize the recursive competitive equilibrium.

(e) What are the steady-state values of capital, output and pollution in the recursive competitive equilibrium? How does the equilibrium capital stock depend on $\bar{x}$? Provide a brief intuition for your answer.

Question 3: "Baumol's Disease" in Services

An economy is populated by measure one of households that have preferences over manufacturing goods, m_t, and services, s_t. Their preferences are:

$$\sum_{t=0}^{\infty} \beta^t \left[\mu m_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\mu)(s_t+\bar{s})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where μ is a constant satisfying $0 < \mu < 1$, $\bar{s}$ is a positive constant, and ε is the elasticity of substitution between manufacturing and services, satisfying $0 < \varepsilon < \infty$. The technology to produce manufacturing output is

$$Y_t^m = A_{m,t} N_{m,t},$$

where $N_{m,t}$ is the labor input in manufacturing, and $A_{m,t}$ represents manufacturing efficiency. The technology to produce services is

$$Y_t^s = A_s N_{s,t},$$

where $N_{s,t}$ is service labor input and A_s is service efficiency. Manufacturing efficiency grows exogenously at rate γ_m each period: $A_{m,t+1} = A_{m,t}(1+\gamma_m)$, where $\gamma_m > 0$. Service production efficiency grows exogenously at rate γ_s each period: $A_{s,t+1} = A_{s,t}(1+\gamma_s)$, for $\gamma_s > 0$.

Neither Y_t^s nor Y_t^m is storable from one period, and both goods are restricted to be non-negative. Employment in the two sectors must satisfy $N_{m,t} + N_{s,t} = 1$ for all t.

1. Define the social planner's problem for this economy. Characterize the solution to the planner's problem.

2. Are both goods consumed in all periods? If so, justify your answer. If not, characterize when both goods are consumed and when just one good is consumed.

3. Now suppose that $\gamma_m > \gamma_s$. What happens to the ratio of manufacturing employment to total employment in the long run? (as $t \to \infty$)? Explain the intuition for your answer.

Question 4: Structural Change in a Roy Model of Sorting

There are measure one of agents in an economy. Half are "strong" types that have a comparative advantage in agricultural tasks. Half are "smart" types that have a comparative advantage in services. In particular, strong types are endowed with z units of agricultural labor units, and one unit of service labor units, where $z > 1$. Smart types are endowed with one unit of agricultural labor units and z units of service labor units. Each agent can work in exactly one sector, and supplies all of her labor units to that sector.

All agents have the following preferences:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t^a - \overline{a}) + \log(c_t^s)],$$

where c_t^a is consumption of agriculture goods, and c_t^s is consumption of services, both of which much be non-negative each period. The parameter $\overline{a}$ is a positive constant that represents a subsistence requirement. The production functions for agriculture and services are:

$$Y_t^a = A_t N_t^a \qquad \text{and} \qquad Y_t^s = A_t N_t^s,$$

where N_t^a is the input of agricultural labor units, N_t^n is the input of service labor units, and A_t is an exogenous efficiency level. The initial efficiency level is $A_0 = 1$, and the law of motion for A_t is $A_{t+1} = A_t(1+g)$ where $g > 0$.

Let services be the numeraire good, and define the relative price of agriculture be p_t^a. Let the wage per agricultural labor unit be w_t^a and the wage per service labor unit be w_t^s. Note that w_t^a needn't equal w_t^s since they are prices for different types of labor inputs. Assume that both technologies are operated by competitive firms with unrestricted entry.

1. Write down the problems for strong types and for smart types, taking prices are given, as sequence problems.

2. Write down the static firms' problems. What must be true of the competitive equilibrium labor prices w_t^a and w_t^s in equilibrium? Briefly explain your answer.

3. Characterize the optimal sorting patterns of the workers in period 0, and the equilibrium relative price of agriculture goods, as a function of A_0.

4. Describe how $p_{a,t}$ and the sorting of workers by sector evolves over time. What happens to the ratio of income for strong types to smart types? Explain your answer.