MATH 157: Mathematics in the world

Course description, Spring 2019

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An interactive introduction to problem solving with an emphasis on subjects with comprehensive applications. Each class will be focused around a group of questions with a common topic. Possible topics include: logic, information, number theory, probability, and algorithms.

Prerequisite: Mathematics 21b, 23a, or equivalent. More importantly, students should have a broad mathematical curiosity and be eager to brainstorm during in-class problem solving sessions.

Introduction

Undergraduates choose a concentration in mathematics with a variety of goals in mind. A number of students aim to continue directly to graduate school in the field and plan to pursue an academic career. There are also many concentrators who pursue mathematics because they are fascinated by the modes of thought. Such students tend to be very passionate about finding applications for mathematics in the natural and social sciences, as well as outside the academic world.

We plan to make the classroom experience very interactive by organizing each session around problem solving and discussion activities. Moreover, due to the interdisciplinary nature of the material, the course will be accessible to students interested in computer science, statistics, and logic.

Taking this course is also an ideal way to prepare for a variety of quantitative interviews. Students planning to pursue careers in technology, finance, and engineering should find the material beneficial when approaching the job market.

Syllabus

Taking the course goals into consideration, we find the following topics a natural fit. It is possible to place a stronger emphasis on one or two of these topics, and we plan to take the students' interests into account when making this decision.

1. **Logic and information**

   Simple problems and brainteasers are a great way to practice a structured thinking process. Logic is the bedrock of all quantitative disciplines, so this is a natural place to start. Additionally, the study of information can be seen as a continuation of logic, so it would be impractical to separate the two.

2. **Number theory**

   Number theory is one of the oldest branches of mathematics, and it would be very difficult to avoid it in a course on problem solving. The goal of this section is introduce students to several computational problems in the field such as primality testing, encryption, compression, and error correction. Shannon entropy appears in several of the suggested problems, and could serve as a transition to probability.

3. **Probability**

   Probability is everywhere around us. In fact, most people develop an intuition about the subject in a natural way, in complete isolation from any rigorous theory. Probability also offers a wealth of problems to choose from.

   What is even more interesting is that a number of questions have surprising—or even counter-intuitive—answers. These problems are particularly useful to us since they demonstrate how a rigorous thinking process could be beneficial in analyzing concrete and practical problems.

4. **Algorithms and programming**

   Programming and data analysis is an indispensable part of many research projects. We would like to take a mathematically-inspired point of view, and introduce students to complexity analysis and algorithm design through a series of programming exercises.

   Many problems have no single right solution. Besides showing students how to arrive at solutions, we aim to emphasize that one needs a solid mathematical framework to compare and contrast solutions. In practice, the best approach is often dictated by contextual factors. We hope to teach students how to optimize their solutions based on the environment they have to perform in.

   We realize that some students may not have programming background. To offset some of the difficulties we plan to use a high level language such as *Python*. We can also provide additional materials and one-on-one instruction as necessary.