MATH 157: Mathematics in the world
Sample problems

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Problems

- At a party of six people either there are three mutual acquaintances or there are three mutual strangers. Why?

- How many trailing zeros does 100! have?

- Consider the Fibonacci sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots.$$

  Does the sequence of last digits

$$0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, \ldots$$

  eventually repeat itself? If so, how long can the cycle be?

- How many people do we need, so that the probability of two having the same birthday is at least $1/2$?

- How thick should a coin be to have $1/3$ chance landing on its edge?

- Three friends play the following game: Each chooses an integer among 1, 2, and 3. The person who chose the smallest unique integer wins. For example, if the choices are $1, 1, 2$, then the third person wins.

  What is a good strategy for this game? Is it deterministic?

  What if each person can choose among all positive integers? What if there are four friends instead of three?

- Given a list of numbers, what is the smallest number of comparisons necessary to find both the minimum and maximum?

- Suppose we are given a list of matrices $A_1, \ldots, A_n$ such that the product $A = A_1 \cdots A_n$ is well-defined. What is the most efficient way to compute A?