MATH 157: Mathematics in the world Homework 1 (Due February 12th, 2019 at 1:00PM)

Problem 1

Write a short paragraph (50-100 words) about something you learned about your thinking process recently. How does this insight help you be more efficient, productive, or creative?

Problem 2

Anna and Barry moved into a new house with their three daughters. This is a snippet from a conversation they had with their neighbor Carl.

Carl: Welcome to the neighborhood! How old are your daughters?

Anna: The product of their ages is 36, and the sum is equal to the number of our house.

Carl: I still can't guess their ages.

Barry: Our oldest has blue eyes.

Carl: Thanks! Now, I know the ages.

How old are the daughters?

Problem 3

Express

$$\sqrt{\frac{1}{\sqrt{1+\sqrt{1+\sqrt{1+1}}}}}$$

in a closed finite form.

Problem 4 - Kill Bill - Volume 2

Recall the *Kill Bill* problem from class (see Notes 3 on Canvas). More generally, suppose the initial configuration contains n samural in a circle. Let S(n) denote the the number of the last man standing. 1. Use our first solution from class to define S(n) recursively.¹

If you missed the first class, we discussed computing S(n) by simulating one "cycle" and then reducing to a smaller input. For example, if n = 2m is even, it is possible to express S(n) = S(2m) in terms of S(m). There is a similar reduction for odd n = 2m + 1 expressing S(n) = S(2m + 1) in terms of S(m).

2. Use the second solution to write a closed form expression for S(n).²

As an alternative solution, we observed that it is easy to compute S(n) = 1 when n is a power of two, and that if n is between 2 powers of two, S(n) increases by 2 when n increases by 1. Given an arbitrary value of n, let $m = 2^k$ denote the largest power of 2 satisfying $m \leq n$. We can then start from m and recover the value of S(n) adding 2 the right number of times.

Problem 5 - Kill Bill - Volume 3 - Nikki's Revenge

The aim of this problem is to get hands on experience with Python. There are many introductory resources on the language, so I will abstain from writing a new one.

1. If you are using Mac OS X or Linux, there is a good chance Python is already installed on your computer. To check, open a terminal and type python. If you don't get an error, you don't need to do anything else.

If you are using Windows or got an error in the previous step, you need to download and install Python. While you can obtain the official binaries 3 from

https://www.python.org/downloads/, there are several other distributions which are easier to use and come with additional tools such as an editor, debugger, etc. The following choices look promising.

For example, let F(n) denotes the *n*-th Fibonacci number (where $n \ge 0$ is an integer). We can then characterize F using two base cases

$$F(0) = 0, \qquad F(1) = 1,$$

in addition to the recursion

$$F(n) = F(n-1) + F(n-2)$$

which is valid for all $n \geq 2$.

²Please look up the floor and ceiling functions if you aren't already familiar with them.

³There are two versions of Python, 2 and 3, under current development. At the time of writing, the current releases are 2.7.9 and 3.4.2. The mentioned distributions contain both.

The Python community is actively trying to promote transitioning to the more consistent and fullyfeatured version 3. A number of incompatibilities have made this process cumbersome. If you are a new Python user, it makes sense to opt for version 3.

For the purpose of this class, solutions in both Python 2 and 3 will be accepted. None of the differences will become bothersome to us.

¹A recursive function is one whose value for a given input is defined in terms of its values at other, typically smaller, inputs.

- **Canopy** The most basic version Canopy Express is sufficient for our needs. https://enthought.com/products/canopy/
- **Anaconda** The free version of Anaconda is free but you need to confirm your email before downloading.

https://store.continuum.io/cshop/anaconda/

To answer your question in the homework, please let us know which distribution and tools, if any, you are using. It would be very helpful to your classmates if you share your findings in the discussion board in Canvas before the homework deadline.

2. If you haven't used Python before, find a source that you like and learn about the basic language syntax, arithmetic operations, conditional statements, and functions. Remember the web if your best friend.

I liked the format of http://www.learnpython.org/. Reading all the way up to Functions should be sufficient for the moment.

To answer this question in your homework, please let us know about the sources you found useful. As in the previous part, please post to Canvas so your classmates can benefit from the information.

3. Write two functions S1 and S2 which implement your formulas you derived in the previous problem. Your program should print S1(1000) and S2(1000), so we can verify they agree with our computation from class S(1000) = 977.⁴

 $^{^{4}}$ For example, if the problem asked to implement the Fibonacci sequence as a function and print its value at 10, the following would be a valid solution.