MATH 157: Mathematics in the world Notes 1 (January 29, 2019)

Announcements

- My name is Yusheng Luo, and my email address is yushengluo@g.harvard.edu . The CAs of this class will be
 - Stefan Spataru, stefanspataru@college.harvard.edu .
 - Vinh-Kha Le, le@college.harvard.edu .
 - Patrick Lopatto, lopatto@math.harvard.edu .
- The main source of score (70%) will come from homework assignments (discarding the lowest); because of interactive nature of the class, 5% of the final score will come from attendance (detailed rules will be made); there are then two options on the remaining 25% (the decision on which one of the two will be made in class):
 - a final project on some topic related to this class (a paper around 10 pages long)
 - a final exam including only problems that have been discussed in class or assigned as homework.
- My office hours, as well as the CAs' problem sessions, will be determined in the next weeks based on your availability; I will set up a doodle and let you know through a Canvas announcement; *please stay tuned on Canvas announcements*. In the meantime, please write me an email if you want to meet.
- The first homework assignment will be available on Canvas next Tuesday (Feb. 5th) and will be due in a week (Feb. 12th) half an hour before the start of the class (at 1:00pm). Late homework is *never* accepted. Please submit the work you have done by the deadline; we will be generous on assigning partial credit in every exercise. Homework will have to be submitted in PDF format through Canvas; we *strongly* advice to use LATEX, and we will provide LATEX (and programming) tutorials.
- Please fill the sign-up sheet.

Problem solving techniques

There is no exact recipe to solve problems in Mathematics. Here is some advice:

- Read carefully the statement of the problem, and don't hesitate to ask questions if something is not clear.
- **Trial and error:** the main mistake to avoid is getting stuck, and stare at a blank page; no idea has to be discarded, it is important to try and put everything on paper. Very often, trying and trying is the only way to get the right idea.
- Small examples: the problem involves very big numbers? Just try and see what happens in smaller situations! And then, try to look for patterns; this is much easier than try to blindly guess the correct idea.
- Write an equation: you have spent a fair amount of time learning to solve equations; this skill might turn out to be useful once in a while.
- Draw a picture: sometimes thinking visually makes things much clearer. Even if the problem doesn't directly involve images, try to think of the way to organize information in the form of a picture (or pictures). Free some space in your brain for verbal thinking by putting data in the visual form.
- Forget what I just said: sadly, following the previous points will not magically bring you in front of a solution; in general, the more you understand what's going on, the more likely you are to get to a solution. Experience is another key factor having thought about many similar problems in the past is certainly a good help to solve the next ones.

Problems

Calendar dice

You are planning to order two custom-made dice so they can show the current day of the month (01 to 31). Each face can be engraved with a single digit. You want to use both dice every day, but the order in which you arrange them doesn't matter. Is this possible? If so, provide a sample arrangement of the faces.

Racing horses

Imagine we have 25 horses and a track with 5 lanes. What is the minimum number of races necessary to find the top three horses? You can assume all horses perform consistently (e.g., if horse A is faster than horse B, horse A will always beat horse B).

A magic island

On a magic island, there were a hundred tigers. They didn't like very much eating the local vegetation, but they weren't cannibals either.

One day, a sheep appeared on the island. Every tiger was craving for a proper meal, but they knew that the magic on the island would turn the tiger eating a sheep into a sheep as well, and then risk to be eaten as consequence. The tigers on that island were very smart and rational, and their first priority was to stay alive.

What happened then? Did the most hungry tiger eat the sheep?

A simple card game

Someone stops you on the street asking to play the following game. You start with a standard shuffled deck of 52 cards, and take two cards at a time. If both cards are red, they go to your pile. If both cards are black, they go to the opponent's pile. If the cards have different colors, they are discarded. Once the deck is exhausted, the game ends. If you have more cards in your pile, you win \$100, and otherwise you don't get anything.

How much are you willing to pay for the game?

The monochromatic rectangle

Imagine that each point with integer coordinates in the plane is colored either black or white. Show that there always exists a rectangle with sides parallel to the axes and vertices of the same color.

Extra: Strategy

Round table

You and your friend are playing the following game. Each turn a player puts a quarter on the round table, but the coins are not allowed to overlap. The one who can't put a quarter loses. You start first. Who wins the game, you or your friend? What is the right strategy?

Round tower

You are alone on the top floor of the round tower that consists of a number of rooms that go in a circle, and you do not know how many rooms there are in the palace. Some of the rooms have the lights on, some have the lights off. Your job is to turn all the lights off and then report when you've finished. How would you do it?