Math 157: Lecture 15

Estimating π

You possess an apparatus which generates random numbers uniformly distributed over the unit interval [0, 1]. How can you estimate π ?

Collecting coupons

Coupons in cereal boxes are numbered 1 to 5, and a set of one of each is required for a prize. With one random coupon per box, how many boxes on average are required to make a complete set? What if there are n total coupons to collect?

Bayesian coins

There are five coins in a box; one has heads on both sides, while the other four are normal. You reach into the box, draw a coin at random, and flip the coin three times. If you get three heads in a row, what is the probability you drew the two-headed coin?

Bayesian medicine

A patient goes to see a doctor. The doctor performs a test with 99 percent reliability that is, 99 percent of people who are sick test positive and 99 percent of the healthy people test negative. The doctor knows that only 1 percent of the people in the country are sick. If the patient tests positive, what are the chances the patient is sick?

(Taken from an article by Chris Wiggins in Scientific American: https://www.scientificamerican. com/article/what-is-bayess-theorem-an/)

Yet another dice game

Patrick and Yusheng play a dice game. They take turns rolling a die, with Patrick going first. The first person to roll a 6 wins. What is the probability that Patrick wins?

First-step analysis

You toss a fair coin until two consecutive throws come up heads. What is the expected number of tosses before the game ends?

More first-step analysis

Patrick and Yusheng are playing another game, this time with a fair coin. The coin is tossed repeatedly. Patrick wins when the sequence HHT appears, while Yusheng wins when the sequence HTH appears. What is the probability that Patrick wins? That is, what is the probability the sequence HHT appears before HTH?

Upright table

A carpenter with a good sense of humor attaches three legs to the perimeter of a circular tabletop in random uniformly distributed positions. What is the chance the table will stay upright? (Hint: the table stays upright if the center of mass of the tabletop—its center—lies inside the triangle formed by the legs.)

Stick breaking

Consider a stick of length 1. Pick two points uniformly at random, and break the stick at these points. What is the probability that the resulting three pieces can form a triangle?

Points on a semicircle

If we choose n points randomly on a fixed circle, what is the probability they will lie in a semicircle? (Hint: Try working out small values of n first.)

Airplane boarding

There are 100 people at Boston Logan Airport waiting to board a plane. The first person in line realizes that she has lost her boarding pass, so she decides to take a random seat instead. Every person that boards the plane after her will either take their assigned seat, if it is available, or a random seat if the assigned seat is taken.

What is the probability that the last passenger who boards will end up in their assigned seat?