MATH 157: Mathematics in the world
Notes 18 (April 3, 2019)

1 Random walks

A bug's life

A small flea Alice lives on an 1-dimensional island of width m. n inches to her left is the Cliff of Doom, which drops to a raging sea filled with flea-eating monsters. $m - n$ inches to her right is the Pit of Disaster, which falls to gap with a stream of molten lava.

Each second, Alice hops 1 inch to the right or the left with equal probability, independent of the direction of all previous hops. Our job is to analyze the life of Alice. Does she have any chance of avoiding a fatal plunge? If not, what is the probability of her falling into the Cliff or Pit respectively? What is her life expectancy?

What happens if Alice hops 1 inch to the right with probability p and to the left with probability $q = 1 - p$?

A bug's new life

A sudden plate movement has changed the geography: the gap of Pit of Disaster has been filled. To Alice's right, there is an endless flat plateau. Does Alice have a chance to survive now? How about her life expectancy now?

Gambler's ruin

This summer, I am planning to go to Las Vegas and try to earn some cash. The game I am going to play is Roulette. To summarize the game, I will win \$1 with probability

$$\frac{18}{38} \approx 0.474,$$

and lose \$1 with probability

$$\frac{20}{38} \approx 0.526.$$

As a mathematician, I am very rational. So I will choose to leave either if I win \$100 or if I lose all my money. To improve my chance of going home a winner, I can bring more money. What will be the probability of wining if I bring

1. $100
2. $1,000
3. $1,000,000,000

Random Walk in Circles

We are going to play a game of passing pomelo. Suppose there are $n+1$ people numbered by $0, 1, ..., n$ standing in a circle. I, being person 0, have a pomelo to start passing around. Every time, the person who is holding the pomelo has equal probability of passing it to the left or right. The game ends when all but one have touched the pomelo, in which case the final person who never touches it will win the pomelo. Where do you want to stand at the beginning?

2 Strategy games

Finding the perfect fit

Alice (not the flea) is using a dating app on her phone. She receives 100 requests for the date. The app randomly displays the information of the dating candidate one by one, and Alice can choose to accept the request or pass it. Developed by a philosopher who wants to teach a lesson about regret in life to common people, the app does not allow you to go back and accept the candidates that you have passed. Assume that Alice can assign a fitting score for each candidate based on their information, and there is no tie. How to maximize the probability of finding the best fit?

Saving hostages

100 hostages are being held by a bunch of pirates who like to play math games. The pirates set up the following game.

They label the 100 hostages with number 1 - 100, and ask them to go into a secret room one by one. The pirates randomly put 100 pieces of paper labeled by 1 - 100 into the 100 boxes labeled by 1 -100. These boxes are put in the secrete room, and each hostage can open 50 of them. If the hostage opens a box containing his/her number, he/she will get a *pass*. He/she will then leave the room, and a pirate will close all the boxes that he/she opened. The next person in line will enter the room and repeat the process.

If *every hostage* gets a pass, they will be saved. Otherwise, all of them will be thrown into the ocean. The hostages are allowed to discuss the strategy before hand, but no more communication when the game starts.

Can you give some suggestions on the strategy?