MATH 157: Mathematics in the world
Notes 2 (January 31, 2019)

Problem solving techniques

- **Read carefully the statement of the problem,** and don't hesitate to ask questions if something is not clear.
- **Trial and error:** the main mistake to avoid is getting stuck, and stare at a blank page; no idea has to be discarded, it is important to try and put everything on paper. Very often, trying and trying is the only way to get the right idea.
- **Small examples:** the problem involves very big numbers? Just try and see what happens in smaller situations! And then, try to look for patterns; this is much easier than try to blindly guess the correct idea.
- **Write an equation:** you have spent a fair amount of time learning to solve equations; this skill might turn out to be useful once in a while.
- **Draw a picture:** sometimes thinking visually makes things much clearer. Even if the problem doesn't directly involve images, try to think of the way to organize information in the form of a picture (or pictures). Free some space in your brain for verbal thinking by putting data in the visual form.
- **Forget what I just said:** sadly, following the previous points will not magically bring you in front of a solution; in general, the more you understand what's going on, the more likely you are to get to a solution. Experience is another key factor - having thought about many similar problems in the past is certainly a good help to solve the next ones.

Problems

The monochromatic rectangle (From Notes 1)

Imagine that each point with integer coordinates in the plane is colored either black or white. Show that there always exists a rectangle with sides parallel to the axes and vertices of the same color.

Apples and oranges

A stall at the farmer's market sells three products: boxes of apples, boxes of oranges, and a mix of the two. All boxes are fully *opaque* and *mislabeled*. The seller offers his customers a discount if they can guess the the contents of all boxes by opening a minimal number of them. What is this number, and how can one get the discount?
Imagine that the seller becomes more strict with his discount policy. He has redefined "opening" a box to mean reaching in and picking a single piece of fruit. How does the problem change?

Democratic pirates

Ten democratic pirates looted 100 gold coins. Seeing that they need to divide the treasure, they agreed on the following method.
First, the most senior pirate proposes a distribution. All pirates, including the most senior one, proceed to vote. If at least half of them accept distribution, the loot is divided as proposed. If not, the most senior pirate will be thrown overboard. The process continues with the next most senior pirate, and so on.
Assume the pirates are rational: they prefer to stay alive first, and earn more gold second. Also, given two otherwise equal outcomes, they prefer to have fewer pirates on the ship.
Assume also that the pirates do not trust each other, and will neither make nor honor any promises between pirates apart from a proposed distribution plan that gives a whole number of gold coins to each pirate.
How will the pirates divide the gold?

Separating coins

There is a room with 100 coins on the floor, 10 of them are heads up, and the remaining are tails up. If you enter blindfolded, can you produce two piles such that the number of heads up coins in each is equal?
Assume you cannot distinguish the two sides of a coin by touching it, but you are allowed to turn coins.

The average offer

A number of Harvard seniors plan to meet and celebrate their job offers. Being naturally curious, they would like to know the average salary in the group. On the other hand, no single student would like to disclose his offer to anyone else. How can they learn the group average?

Extra

Camel

A camel starts with 900 gallons of water on a 200 miles route on a desert. He can carry at most 300 gallons at a time, and for every mile he walks he needs to drink a gallon of water. How much water at most he will be able to carry to the end of the route?

Glass balls

You have 2 identical glass balls. You have a 100-story building. You are trying to determine at which floor these glass balls break. What is the most efficient way to determine the lowest floor at which the balls break?
(The most efficient way is the one for which the worst case scenario has the smallest number of attempts. We also assume that non-breaking impacts have no effect on the integrity of the balls.)