MATH 157: Mathematics in the world

Notes 21 (April 16, 2019)

1 Dynamic Games of Complete Information

By complete information, we mean that the payoff functions are common knowledge.

Stackelberg Model of Duopoly

Consider a dominant firm moving first and a follower moving second.
1. Firm 1 chooses a quantity $q_1 \geq 0$.
2. Firm 2 observes q_1 and then chooses a quantity $q_2 \geq 0$.
3. The payoff to firm i is the profit

$$\pi_i = (P(Q) - c)q_i,$$

where $Q = q_1 + q_2$ and

$$P(Q) = \begin{cases} a - Q & Q \leq a \\ 0 & Q > a \end{cases}$$

What does the backward induction give us?

Observation: Firm 2 has more information in Stackelberg game than in Cournot game.

Question: In which game does firm 2 make more profit?

Ultimatum game Vol 1 / Subgame Perfect Nash Equilibrium

Imagine a robber walks to you, carrying a bomb. The robber tells you that he will set off the bomb, hurting you both, unless you give all your money \$10. Let's assume first the robber is rational and that both you and the robber value that getting hurt by a bomb worth -\$10. How do we analyze this game?

2 Games of Incomplete Information

Robber with bomb Vol 2

Now let's assume the robber can be either rational and insane. If the robber is insane, he will strangely value getting hurt by a bomb worth \$5 if he does not get the money. We also assume that you believe that there is a probability p for the robber to be insane. How do we set this game?

Fish selling problem

Fish being sold at the market is fresh with probability 2/3 and old otherwise, and the customer knows this. The seller knows whether the particular fish on sale now is fresh or old. The customer asks the fish-seller whether the fish is fresh, the seller answers, and then the customer decides to buy the fish or to leave without buying it. The price asked for the fish is \$12. It is worth \$15 to the customer if fresh and nothing if it is old. Thus, if the customer buys a fresh fish, the gain is \$3. The seller bought the fish for \$6, and if it remains unsold, then he can sell it to another seller for the same \$6 if it is fresh, and he has to throw it out if it is old. On the other hand, if the fish is old, the seller claims it to be fresh, and the customer buys it, then the seller loses \$R in reputation.

What is the strategy spaces for seller and buyer?

What are all pure strategy Nash equilibriums?

Prisoner's Dilemma Vol. 2

Two suspects are arrested and charged with a crime. The police lack sufficient evidence to convict the suspects, unless at least one confesses. The suspects are held in separate cells and told 'If only one of you confesses and testifies against your partner, the person who confesses will go free while the person does not confess will surely be convicted and given a 6-year jail sentence. If both of you confess, you will both be convicted and sent to prison for 4 years. Finally, if neither of you confesses, both of you will be convicted of a minor offense and sentenced to 1 year in jail.'

Suspect 1 is a nice person, but suspect 2 has a probability 1/3 of being nice and probability 2/3 of being selfish. Being a nice suspect means that if the other suspect is also nice, then he/she will suffer 4 years equivalence of mental suffering if he/she chooses to confess. In other words, the payoff function is as follows:

	C_1	R_1
C_2	-4, -4	-6, 0
R_2	0, -6	-1, -1

	C_1	R_1
C_2	-8, -8	-6, -4
R_2	-4, -6	-1, -1

What are the strategy spaces for suspect 1 and suspect 2?

How do we analyze this problem?

3 Repeated Games

Prisoner's Dilemma Vol. 3

What happens if we assume the two suspects are playing the game infinitely many times? How do we model their payoffs? What are their strategy spaces? Can you come up with any 'natural' strategies?