MATH 157: Mathematics in the world

Notes 25 (April 30, 2019)

We will play two games involving fictional money. Your wealth at the end of the first game will be your initial wealth for the second. You will know the wealth of all teams after the first game before being required to play the second. Teams will be ranked according to their wealth at the end of the second game, and prizes will be distributed to the top three teams.

Please read through the rules carefully and ask any questions you might have before we begin. Any ambiguities will be adjudicated by the TFs. All such decisions are final. We thank Citadel and Citadel Securities for providing the prizes.

Game 1

You start with \$1000. The game proceeds in 200 turns. In each turn, you decide how much of your wealth to invest. The investment is successful with probability 60%, and the probabilities of success on each turn are independent. If the investment is successful, it is doubled and returned to you. Otherwise, it is lost. For example, if you invest \$100 out of your \$1000 initial capital, your wealth after the turn will be either \$1100 (success) or \$900 (failure).

Your task is to describe a strategy which describes the amount of capital to invest at each turn. You may only invest as much money as you possess. In particular, it is not possible to have negative wealth, and if you lose your entire capital there is no mechanism to recover.

Provide your strategy as a function $f(x, i)$ that returns the amount you want to invest at the turn number i if your total wealth is x. The variable i will be an integer from 1 to 200. The function must be implementable in Python; so, it can include any mathematical expressions, conditional statements, random variables, and so on. You don't need to provide any code, just a reasonable explanation. We will handle the implementation.

The game will be scored based on a single random sequence of successes and failures common to all teams.

Game 2

The game proceeds in 400 turns. There are 5 different stocks $S_1, \dots, S_5$ to invest in. In each turn, you decide which stock to invest in and what amount of your wealth to invest. The success rate for investing in S_i is p_i. The p_i are parameters fixed throughout the 400 turns, but whose values are not known to you.[1] The p_i's will be independently drawn from a Beta$(3, 2)$ distribution at the beginning of the game.

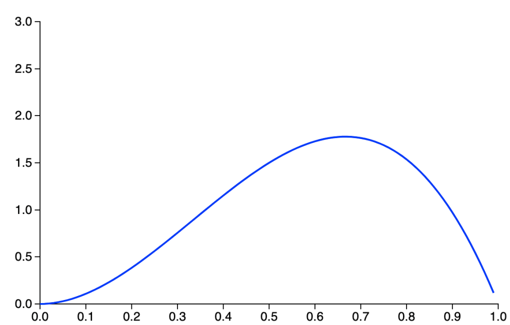
As before, if the investment is successful, your investment will be doubled. Otherwise, you lose the money you put in. The minimal betting increment rule and prohibition against betting more than your wealth still apply.

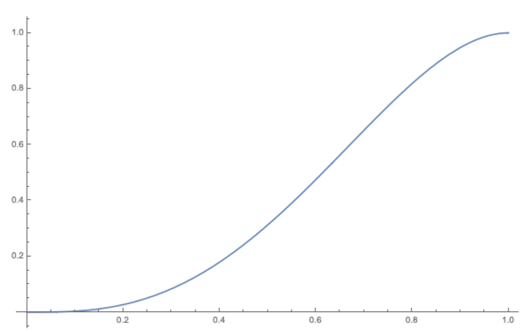
Assume at turn i that you know the number of successful investments for each stock up to turn $i-1$. Provide a strategy as a function $f(x, i, s_1, \dots, s_5)$ that takes your total wealth x, turn number i and the number s_i of successful investments for stock S_i up to turn $i-1$ and returns the stock you want to invest in and the amount you want to invest. Again, the strategies will be implemented (by us) in Python, with the random stock success sequences common to all teams.

We will rank the teams by their ending fortune after this game. In the unlikely case of a tie, we will rank those teams with equal end fortunes based on time of submission of the second strategy, which the tie going to the team who submits first.

Beta distribution

Briefly, Beta$(3, 2)$ is a distribution on $[0, 1]$ with mean $3/5$ and variance $1/25$. A plot of its probability density function $f(x) \sim x^2(1-x)$ (on the left) and the cumulative distribution function (on the right) are shown below





[1]Note that the outcomes of your investments in the various stocks may help you estimate them as the game proceeds, however.