MATH 157: Mathematics in the world
Notes 3 (Feb 5, 2019)

Problem solving techniques

- **Read carefully the statement of the problem,** and don't hesitate to ask questions if something is not clear.
- **Trial and error:** the main mistake to avoid is getting stuck, and stare at a blank page; no idea has to be discarded, it is important to try and put everything on paper. Very often, trying and trying is the only way to get the right idea.
- **Small examples:** the problem involves very big numbers? Just try and see what happens in smaller situations! And then, try to look for patterns; this is much easier than try to blindly guess the correct idea.
- **Write an equation:** you have spent a fair amount of time learning to solve equations; this skill might turn out to be useful once in a while.
- **Draw a picture:** sometimes thinking visually makes things much clearer. Even if the problem doesn't directly involve images, try to think of the way to organize information in the form of a picture (or pictures). Free some space in your brain for verbal thinking by putting data in the visual form.
- **Forget what I just said:** sadly, following the previous points will not magically bring you in front of a solution; in general, the more you understand what's going on, the more likely you are to get to a solution. Experience is another key factor - having thought about many similar problems in the past is certainly a good help to solve the next ones.

Problems

Kill Bill, volume 1

A thousand samurai, numbered 1 to 1000, are standing in a circle. The first one takes his sword and kills the second. Then, the next man in the circle, number 3, kills number 4. The process continues until there is only one samurai standing. What is his number?

The boss's birthday

Alice, a manager at Big-Name-Firm Inc, thought of a riddle to give her colleagues Bryan and Claire. She told them that her birthday is on one of the following dates.

> March 4, March 5, March 8
>
> June 4, June 7
>
> September 1, September 5
>
> December 1, December 2, December 8

Alice also told the Bryan the month of her birthday, and Claire the day. Later on, the following conversation took place.

> Bryan: I don't know Alice's Birthday, but you don't know it either.
>
> Claire: I didn't know her birthday initially, but now I do.
>
> Bryan: Now I know it too.

What is Alice's birthday?

Renaissance security

Leonardo da Vinci would like to send a package to his close friend Niccolò Macchiavelli. He wants the content of the package to stay confidential. He puts the package in a wooden box. The messenger they use can deliver this box from one location to the other. Unfortunately, the service is not secure so anything they place in an unlocked box will be seen or copied. How can they exchange the message securely?

Infinite exponentiation

If

$$x^{x^{x^{\cdots}}} = 2,$$

what is the value of x?

Two numbers

Which one is larger, π^e or e^π? No calculator allowed! (Hint: use what you've learnt in calculus)

A simple game

Two friends, Albert and Barbara, play a game. Starting from 100, they each in turn subtract an integer number between 1 to 10. Albert starts, and the winner is the person who says the number zero. Who will win? And if they start from 99?

Extra

Three computers

You are buying a new computer. There are three options: Lin, Mac and Mic. Lin always tells truth, Mac always lies and Mic sometimes tells truth and sometimes lies. Of course, since Lin and Mac are consistent in their answers, you could work with either of them. However, you want to avoid buying Mic. The seller allows you to ask one computer of your choice a question. How can you end up buying a computer that is not Mic?