MATH 157: Mathematics in the world

Notes 4 (February 7, 2019)

Problem solving techniques

- **Read carefully the statement of the problem,** and don't hesitate to ask questions if something is not clear.
- **Trial and error:** the main mistake to avoid is getting stuck, and stare at a blank page; no idea has to be discarded, it is important to try and put everything on paper. Very often, trying and trying is the only way to get the right idea.
- **Small examples:** the problem involves very big numbers? Just try and see what happens in smaller situations! And then, try to look for patterns; this is much easier than try to blindly guess the correct idea.
- **Write an equation:** you have spent a fair amount of time learning to solve equations; this skill might turn out to be useful once in a while.
- **Draw a picture:** sometimes thinking visually makes things much clearer. Even if the problem doesn't directly involve images, try to think of the way to organize information in the form of a picture (or pictures). Free some space in your brain for verbal thinking by putting data in the visual form.
- **Forget what I just said:** sadly, following the previous points will not magically bring you in front of a solution; in general, the more you understand what's going on, the more likely you are to get to a solution. Experience is another key factor - having thought about many similar problems in the past is certainly a good help to solve the next ones.

Problems

Dividing the difference

Given $n+1$ integers, there exists a pair a, b such that $a-b$ is a multiple of n.

Summing to 10

For every 6-element subset S of $\{1, \ldots, 9\}$, there exists a pair of distinct integers $a, b \in S$ such that $a+b=10$.

We have the same number of friends!

Is it possible that at a party all guests have different numbers of friends attending?

The Friendship Theorem

Given a group of six people, show that there always exist three such that either

- all know each other, or
- none of them are acquaintances.

Real number close to an integer

Suppose α is a real number and that N is a positive integer; prove that one of the numbers $\alpha, 2\alpha, 3\alpha, \dots N\alpha$ differs at most $1/N$ from an integer.

The square orchard

You own a small orchard containing 51 trees. The shape of the garden is a square with side length 70 meters. Is it possible to use a circular fence of radius 10 meters to enclose at least 3 of the trees?

Dividing each other

Given $n+1$ integers between 1 and $2n$, there exists a pair a, b such that b is a multiple of a.

n integers problem

Given a set of n integers, one can choose either one number which is divisible by n, or several numbers whose sum is divisible by n.

Monotone sequence

Show that any sequence of integers $a_1, ..., a_{n^2+1}$ contains a monotone subsequence of length $n+1$. Here a sequence is called *monotone* if it is either (non strictly) increasing or decreasing.