MATH 157: Mathematics in the world Notes 5 (February 12, 2019)

Problems

Truth table

Let p, q be propositions, draw the truth tables for

- 1. $\neg (p \lor q)$
- 2. $(\neg p) \land (\neg q)$
- 3. $\neg (p \land q)$
- 4. $(\neg p) \lor (\neg q)$

Knights and Knaves - 1

On the island of Knights and Knaves there are two types of people: knights always tell truth and knaves always lie. You met two people on the island: Abe and Bella Abe tells you that Bella is a knave. Bella says, ?Neither Abe nor Bella are knaves.? So who is a knight and who is a knave?

Knights and Knaves - 2

You kept walking on the island and this time, you met three strangers on the island. You asked each of them: "How many knights are there among your companions?" and got the following answers:

1st: "None".

2nd: "One".

What did the third stranger say?

Knights and Knaves - 3

You entered a castle of the island, and were invited to a dinner. There were twelve knights and knaves sitting at the round table. Each of them made a statement: "Everyone, except may be for me and my neighbors, is a knave". How many knights were there at the table?

Knights and Knaves - 4

Which question would you ask a stranger if you wanted to know whether his friend is a knight or a knave?

Peaceful rooks

What is the maximal number of rooks you can place on a chess board so that no two can attack each other?

Peaceful bishops

What is the maximal number of bishops you can place on a chess board so that no two can attack each other?

Covering with dominoes

- 1. We are given a 8×8 chess board with two diagonal corners missing. Is it possible to cover it completely with 2×1 dominoes without any pieces overlapping?
- 2. We are given a 9×9 board with a corner missing. Is it possible to cover it completely with dominoes?
- 3. Suppose now we are given a 9×9 board with one square adjacent to the corner (sharing one side) missing. Is it possible to cover it completely with dominoes?

Packing bricks

Is it possible to fit 53 bricks of size $1 \times 1 \times 4$ into a $6 \times 6 \times 6$ box?

Packing bricks - 2

Is it possible to cover a 101×101 board with only 2×2 and 3×3 squares?

Extra

Three computers

You are buying a new computer. There are three options: Trubox, Liebox and Mixbox. Trubox always tells truth, Liebox always lies and Mixbox sometimes tells truth and sometimes lies. Of course, since Trubox and Liebox are consistent in their answers, you could work with either of them. However, you want to avoid buying Mixbox. The seller allows you to ask one computer a single "yes or no" question. How can you end up buying a computer that is not Mixbox?

Domino covering again

Invent a connected shape made out of squares on the square grid that cannot be cut into dominoes, but if you add a domino to the shape then you can cut the new bigger shape.