Physics 253a   Final Project Topics and Problems

You are asked to choose one of the following topics, with a few other students in the class as a team. Treat it as a problem set in which you will formulate your own problem and come up with its solution, guided by the suggested references. You should write up your findings in the form of an expository term paper, and give a 30 minute presentation (per person) on it. The presentation can be done collaboratively, whereas the term paper should be written individually.

The suggested references should serve as a starting point for your exploration of the subject. You should by no means limit yourself to the suggested reference, nor feel obligated to understand/present everything in the reference. The goal of your term paper should be to explain one single nontrivial result on the topic of your choice in a self-contained manner. The presentation should be done in blackboard style (use slides only if you need to show numerical plots and/or computer codes), and you should be prepared to reproduce the derivation of key steps and claims, if asked. AVOID giving broad overviews or merely paraphrasing results without your own derivation/arguments.

The final presentations will be scheduled during read/exam period (precise schedule TBD). The term paper will be due at the end of the exam period. The number in the parenthesis is the cap on the maximal number of people in the team on the topic.

1. (3) Analyze the $2 \to 2$ scattering amplitude of massive ϕ^4-theory in $D = 3$ spacetime dimensions at 2-loop order.

Reference: Peskin and Shroeder section 10.5. *Remark: certain subtleties with the 4-dimensional ϕ^4 theory discussed in P&S are evaded in the 3-dimensional case. In fact, the 3D ϕ^4 theory is known to be a non-perturbatively well-defined QFT, unlike its 4D analog.*

2. (3) The existence of the stress-energy tensor operator in quantum field theory is incompatible with the existence of a massless spin-2 particle (i.e. the graviton).

Reference: S. Weinberg and E. Witten, “Limits on Massless Particles,” Phys. Lett. B96, 59-62 (1980). *Remark: This result indicates that quantum gravity cannot be described within the framework of a local quantum field theory.*

3. (5) Semi-classical scattering and bound states from summing over ladder diagrams.

Reference: Y. Nambu, “Force potentials in quantum field theory,” Prog. Theor. Phys. 5 (1950), 614-633. E. E. Salpeter and H. A. Bethe, “A Relativistic Equation for Bound-

State Problems", Phys. Rev. 84, 1232. Itzykson and Zuber, *Quantum Field Theory* (1980), Chapter 10.

4. (6) Analyze ϕ^4 theory in $D = 2$ non-perturbatively (!) by numerically diagonalizing the Hamiltonian on a truncated Hilbert space.

Reference: N. Salwen and D. Lee, "Modal expansions and nonperturbative quantum field theory in Minkowski space", Phys. Rev. D62, 025006 [arXiv:hep-th/9910103]. *Remark: this is an exercise of numerics but very fun and instructive. There has been a ton of technical improvement on Hamiltonian truncation methods in QFT in recent years that go far beyond this reference.*

5. (5) Froissart-Martin bound on total scattering cross section.

Reference: V.N. Gribov, "The Theory of Complex Angular Momenta", Cambridge University Press 2003. A. Martin, "Rigorous Results from Field Theory and Unitarity",

http://cds.cern.ch/record/944356/files/CM-P00058617.pdf?version=1

Remark: This is a deep result that follows from the analyticity and crossing relations of the S-matrix in local quantum field theories. I do not recommend Froissart's original paper whose argument was more cumbersome than necessary and the result was weaker.

6. (6) Non-perturbative constraints on the S-matrix in 1+1 dimensions

Reference: M. F. Paulos, J. Penedones, J. Toledo, B. C. van Rees and P. Vieira, "The S-matrix bootstrap II: two dimensional amplitudes," JHEP11 (2017), 143 [arXiv:1607.06110 [hep-th]] *Remark: the most important things to understand are the assumptions that go into this analysis.*

7. (6) The Sine-Gordon model.

Reference: S. R. Coleman, "The Quantum Sine-Gordon Equation as the Massive Thirring Model," Phys. Rev. D11, 2088 (1975). *Remark: there is a lot going on here - the sine Gordon model can be analyzed via perturbation theory, but there are also solitons that are not easily accessed through perturbation theory. The model is exactly solvable and has an equivalent descriptions in terms of theory of fermions.*

8. (5) Perturbative (BPHZ) renormalizability.

Reference: K. Hepp, "Proof of the Bogoliubov-Parasiuk theorem on renormalization", Comm. Math. Phys. 2(4): 301-326 (1966). W. Zimmermann, "Convergence of Bogoliubov's method of renormalization in momentum space", Comm. Math. Phys. 15(3): 208-234

(1969). Itzykson and Zuber, *Quantum Field Theory* (1980), Chapter 8. *Remark: the aim is to understand why the so-called "power-counting renormalizable" theories are actually renormalizable in the sense that all UV divergences can be absorbed into a finite set of counter terms.*

9. (5) Infrared divergence in quantum electrodynamics.

Reference: Weinberg QFT chapter 13. *Remark: in particular, the S-matrix elements with charged particles in QED in the conventional asymptotic particle basis are ill-defined, but suitable inclusive scattering probabilities are well-defined.*

10. (5) Non-relativistic quantum field theories.

Reference: Y. Nishida and D. T. Son, "Fermi gas near unitarity around four and two spatial dimensions," Phys. Rev. A **75** (2007), 063617 [arXiv:cond-mat/0607835 [cond-mat]]. *Remark: application of perturbative field theoretic methods in non-relativistic quantum many-body systems.*

11. (3) Measurement of quantum chaos through out-of-time-ordered correlators and a universal bound on the Lyapunov exponent.

Reference: J. Maldacena, S. H. Shenker and D. Stanford, "A bound on chaos," JHEP08, 106 (2016) [arXiv:1503.01409 [hep-th]]. *Remark: it primarily concerns analytic properties of Lorentzian correlators at finite temperature in quantum mechanics.*

12. (6) The SYK model and its solution at large N.

J. Polchinski and V. Rosenhaus, "The Spectrum in the Sachdev-Ye-Kitaev Model," JHEP04, 001 (2016) [arXiv:1601.06768 [hep-th]]. J. Maldacena and D. Stanford, "Remarks on the Sachdev-Ye-Kitaev model," Phys. Rev. D94, no.10, 106002 (2016) [arXiv:1604.07818 [hep-th]]. *Remark: the disordered average of a quantum system with random coupling is solved using path integral methods.*