Physics 253a Problem set 6

Due Friday December 2, 2022

Consider QED described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\gamma^\mu D_\mu + m)\psi, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi$. Here e is the unit electric charge. We will fix the gauge at the level of path integral by adding to the Lagrangian density

$$\Delta\mathcal{L} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2. \tag{2}$$

(a) Calculate the $2 \to 2$ elastic scattering amplitude of a pair of electrons at tree level, namely at order e^2. Explain why your result is (hopefully) independent of ξ, even though the photon propagator depends explicitly on ξ.

(b) Show that to leading order in the non-relativistic limit (where all spatial momenta are much smaller than the electron mass m), the dependence on the electron spin drops out, and that the scattering amplitude is in agreement with that of Coulomb potential in the Born approximation.