Physics 253c Problem set 2

Due Tuesday October 10, 2023

Problem 1. Show that in a d-dimensional unitarity conformal field theory, a conformal primary operator that transforms in the rank-s symmetric traceless representation of the Lorentz group (commonly referred to as a "spin-s" primary) $\phi_{\mu_1 \cdots \mu_s}(x)$ obeys the unitarity bound

$$\Delta_\phi \geq d - 2 + s, \tag{1}$$

for $s \geq 1$.

A special case of this is the stress-energy tensor $T_{\mu\nu}(x)$, which in addition to the symmetric traceless condition also obeys the conservation law $\partial_\mu T^{\mu\nu} = 0$. From this deduce that the scaling dimension of the stress-energy tensor Δ_T is exactly equal to d.

Problem 2. Using conformal invariance or equivalently the conformal Ward identities, determine the Euclidean two-point function of the stress-energy tensor

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle \equiv G_{\mu\nu,\rho\sigma}(x) \tag{2}$$

as a function of x up to an overall constant normalization factor.

Next, calculate (2) explicitly, particularly its normalization factor, for (i) the free massless scalar field theory, (ii) the free massless Dirac fermion field theory, and (iii) the free Maxwell theory, in $d = 4$ spacetime dimensions.

Problem 3. Consider a d-dimensional free massless scalar field theory, with the elementary field operator $\phi(x)$.

(a) With the product of a pair of ϕ's or their derivatives, we can construct local operators of the form $: \partial_{\mu_1} \partial_{\mu_2} \cdots \phi \, \partial_{\nu_1} \partial_{\nu_2} \cdots \phi :$. Find all linear combinations of such operators that are conformal primaries.

(b) Express the operator product expansion $\phi(x)\phi(0)$ as a sum over the aforementioned primaries and their conformal descendants.