IIA sugra $\longleftarrow$ 11D sugra

$d=10$ $d=11$

dimensional reduction

$$ds_{11}^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{10,10} (dx^{10})^2 + 2 g_{\mu,10} dx^\mu dx^{10}$$

$\mu = 0, \cdots, 9.$

depend only on x^μ, independent of x^{10}

$\rightarrow$ IIA sugra in $d=10$

11D sugra

3-form potential

$$g_{MN}, \quad C_{[MNP]}, \quad \psi_{M\alpha}$$

$M, N = 0, 1, \cdots, 10.$

gravitino

11D Majorana spinor

$$\boxed{G_{MNPQ} = \partial_M C_{NPQ} + \text{cyclic perm}}$$

32 Maj. $\quad SO(1,10)$

$\quad\quad\quad\quad\quad\quad\quad\quad \cup$

$\swarrow \quad \searrow \quad SO(1,9)$

16 chiral M-W $\quad$ anti-chiral M-W 16

$$S[g_{MN}, C_{MNP}, \psi_{M\alpha}]$$

$$= \frac{1}{2\kappa^2} \int d^{11}x \sqrt{-g} \Big[R - \frac{1}{24} G_{MNPQ} G^{MNPQ} \Big]$$

$$+ \frac{1}{2\kappa^2} \#. \int \underset{\sim}{C_3} \wedge \underset{\sim}{G_4} \wedge G_4$$

$$+ (\text{fermions})].$$

IIA & IIB string theory

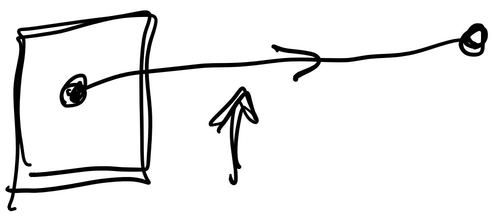
make sense at quantum level

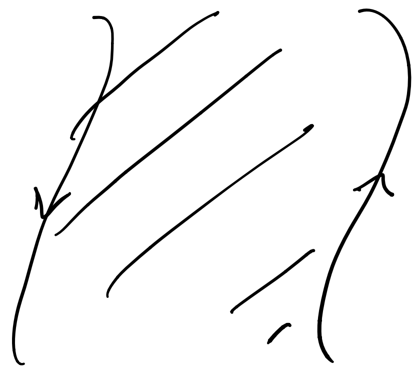
non-perturbatively.

"M-theory"

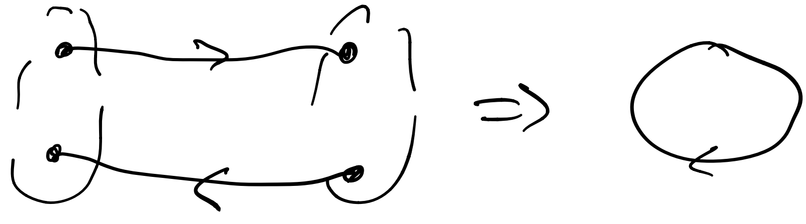
IIA string at finite g_s "$\cong$" an 11-d theory of quantum gravity compactified on S^1

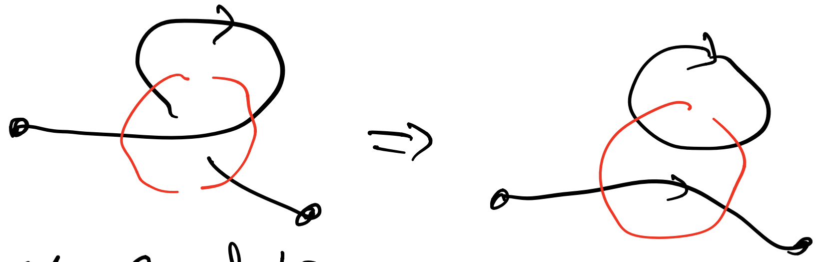
D-branes (open strings)





Q: what are the possible consistent (?) boundary conditions for the worldsheet theory?

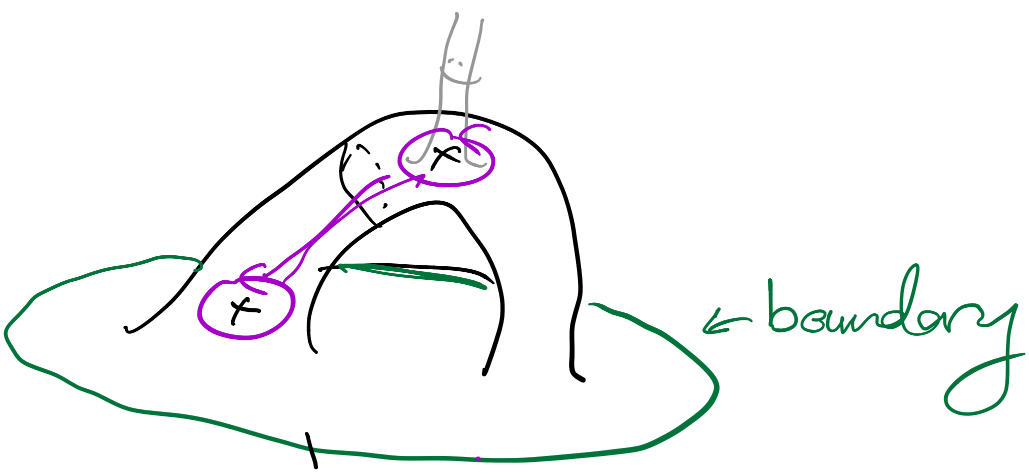


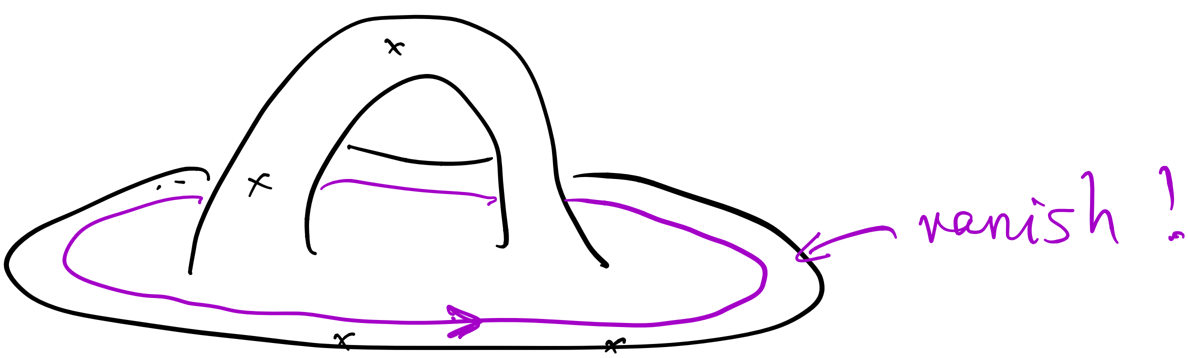


boundary condition

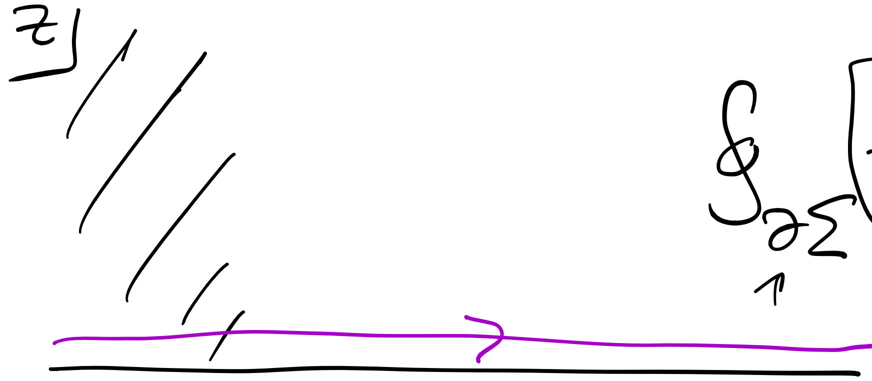
Criteria: should respect BRST-sym

$= 0.$





worldsheet Σ



boundary $\mathrm{Im}\, z = 0$.

$$\oint_{\partial\Sigma} \left[\frac{dz}{2\pi i} j_B(z) - \frac{d\bar{z}}{2\pi i} \tilde{j}_B(\bar{z}) \right] = 0.$$

$$\boxed{j_B(z) = \tilde{j}_B(\bar{z}) \quad \text{at } \mathrm{Im}\, z = 0}$$

$$b(z) = \tilde{b}(\bar{z}) \quad \text{at } \mathrm{Im}\, z = 0$$

$$c(z) = \tilde{c}(\bar{z}) \quad \text{at } \mathrm{Im}\, z = 0.$$

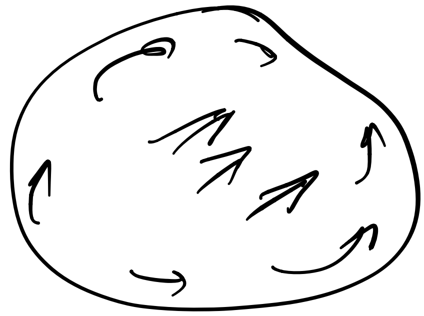
$$\boxed{T(z) = \tilde{T}(\bar{z}) \quad \text{at } \mathrm{Im}\, z = 0.}$$

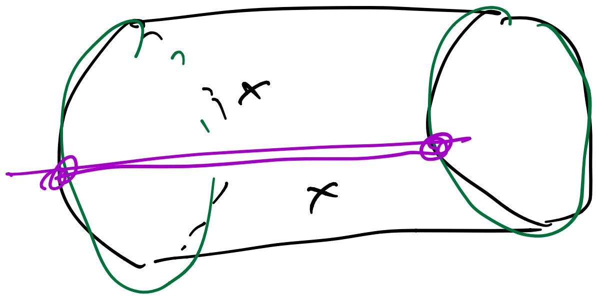
"the boundary condition is conformally invariant"

$$\oint \frac{dz}{2\pi i} v(z) T(z) - \oint \frac{d\bar{z}}{2\pi i} \tilde{v}(\bar{z}) \tilde{T}(\bar{z})$$

if $\mathcal{V}(z) = \mathcal{V}(\bar{z})$ at $\operatorname{Im} z = 0$.

the corresponding conformal sym is preserved





$$T^X = -\frac{1}{\alpha'}\partial X^\mu \partial X_\mu .$$

$$T^X = \tilde{T}^X$$

$$\partial X^\mu \partial X_\mu = \bar{\partial} X^\mu \bar{\partial} X_\mu .$$

can satisfy if e.g.

$$\partial X^\mu = \pm \bar{\partial} X^\mu \quad \text{at} \quad \sigma_2 = 0.$$

sign may be different depending on $\mu = 0, \cdots, 25$

$$\partial = \frac{1}{2}(\partial_1 - i\partial_2)$$

$$\bar{\partial} = \frac{1}{2}(\partial_1 + i\partial_2)$$

(Neumann)

$$+ : \quad \partial_2 X^\mu = 0 \quad \Rightarrow \quad \partial_n X^\mu \big|_{\partial\Sigma} = 0.$$

$$- : \quad \partial_1 X^\mu = 0. \quad \Rightarrow \quad \partial_t X^\mu \big|_{\partial\Sigma} = 0.$$

(Dirichlet)

$$\partial_t X^\mu \big|_{\partial\Sigma} = 0.$$

$$\Downarrow$$

$$X^\mu \big|_{\partial\Sigma} = x_0^\mu$$

Dirichlet : $X^\mu \big|_{\partial\Sigma} = x_0^\mu$ fixed

Neumann: $X^\mu\big|_{\partial\Sigma}$ unfixed.

the sym $X^\mu \to X^\mu + a^\mu$ is preserved

$\Updownarrow$

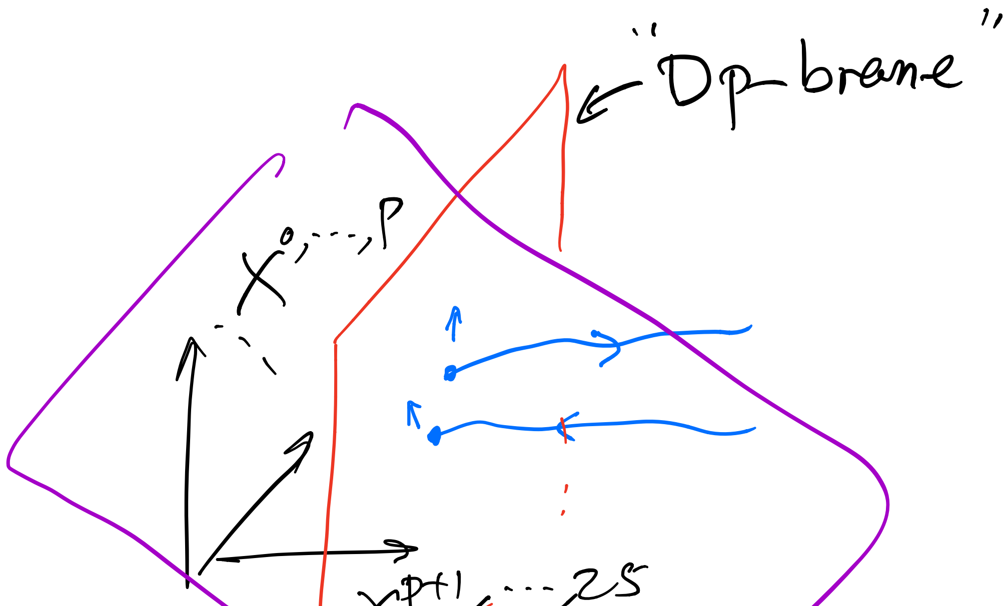
$$(\partial X^\mu, \bar\partial X^\mu)$$

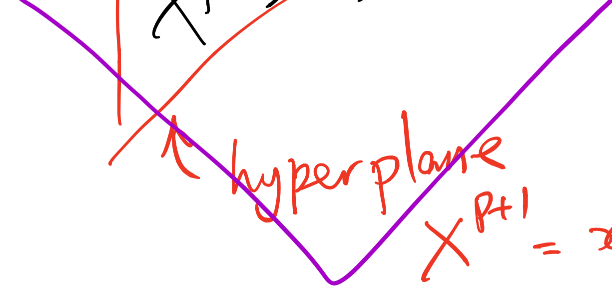
$$\oint_{\partial\Sigma}\left(\frac{dz}{2\pi i}\partial X^\mu - \frac{d\bar z}{2\pi i}\bar\partial X^\mu\right) = 0.$$

Say boundary condition

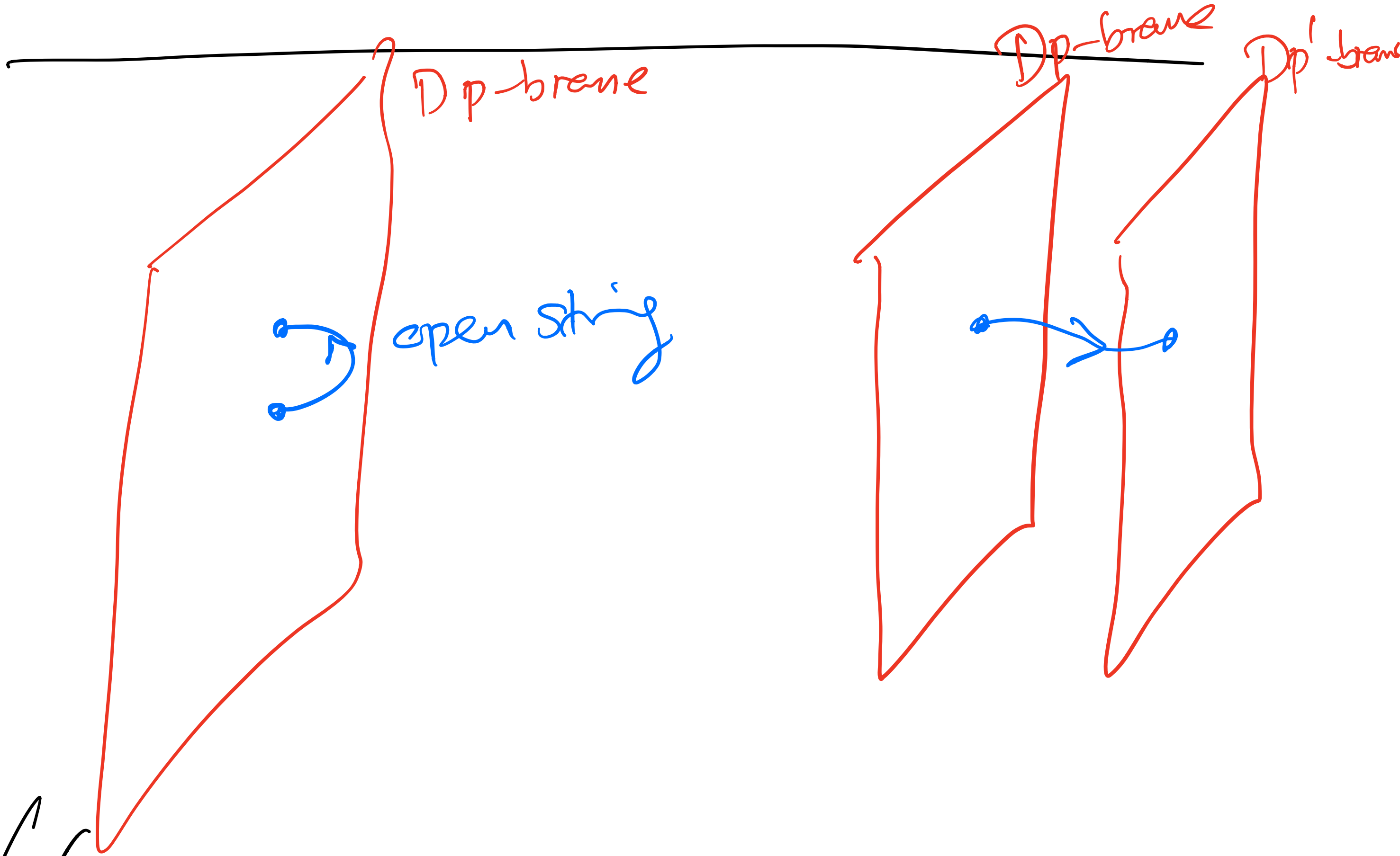
is of N type for X^μ, $\mu = 0, 1, \dots, p$.

of D type for $\mu = p+1, \dots, 25$





$$X^{p+1} = x_0^{p+1}, \ldots, X^{25} = x_0^{25}$$

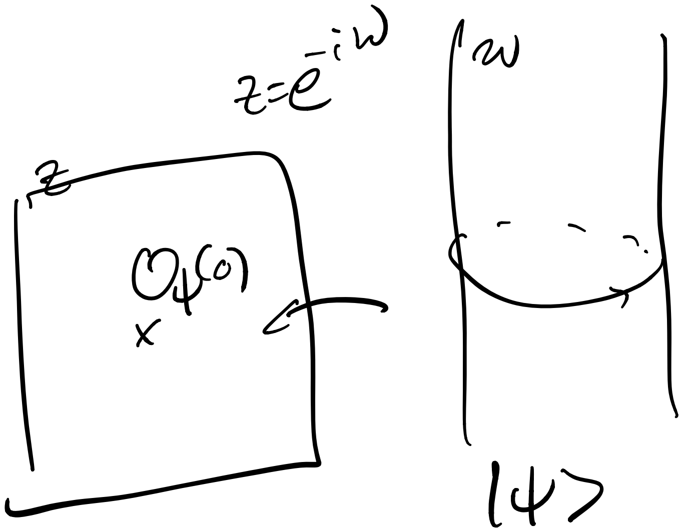


open strings are excitations on the Dp-brane

— . - - - are also . . - - - of the Dp-brane

recall for closed string

$z = e^{-iw}$



$|\psi\rangle$

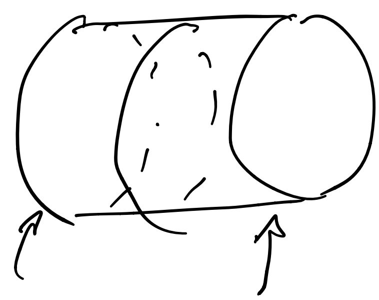
X^μ, b, c CFT

Q_B - coh $\left(b_0|\psi\rangle = \tilde{b}_0|\psi\rangle = 0\right)$

$\Rightarrow$ physical states.

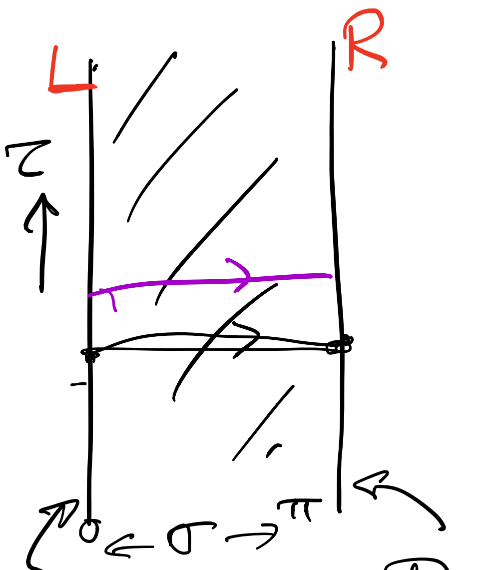
"boundary states"

$\langle\langle B_1| \Rightarrow |B_2\rangle\rangle$



$= \mathrm{Tr}_{\mathcal{H}_0}\, e^{-\beta H}$

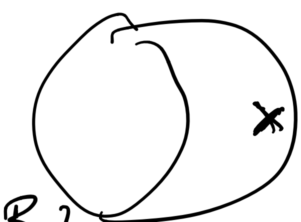
open string:



Dp $|\psi_0\rangle$ Dp

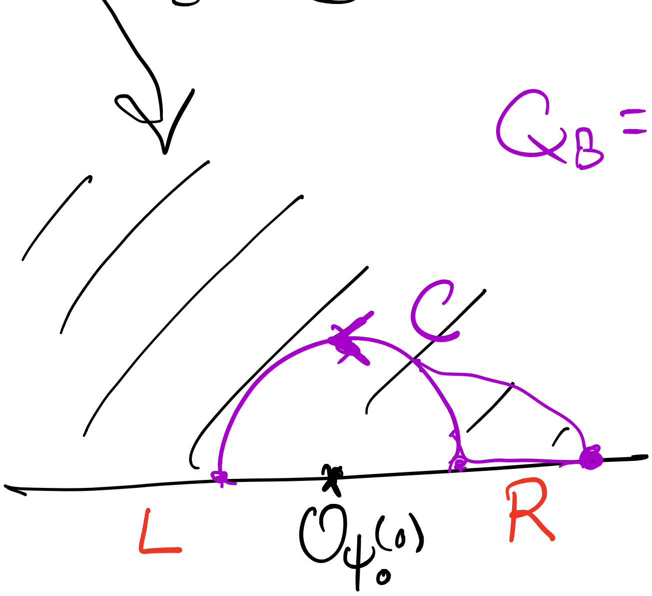
$w = \sigma + i\tau.$

$z = -e^{-iw}$



$\langle\langle B_1|$

$$Q_B = \int_C \frac{dz}{2\pi i} j_B(z) - \frac{d\bar{z}}{2\pi i} \tilde{j}_B(\bar{z})$$



consider a single free boson X.

N: $\partial X = \bar{\partial} X$ at $\mathrm{Im}\, z = 0$.

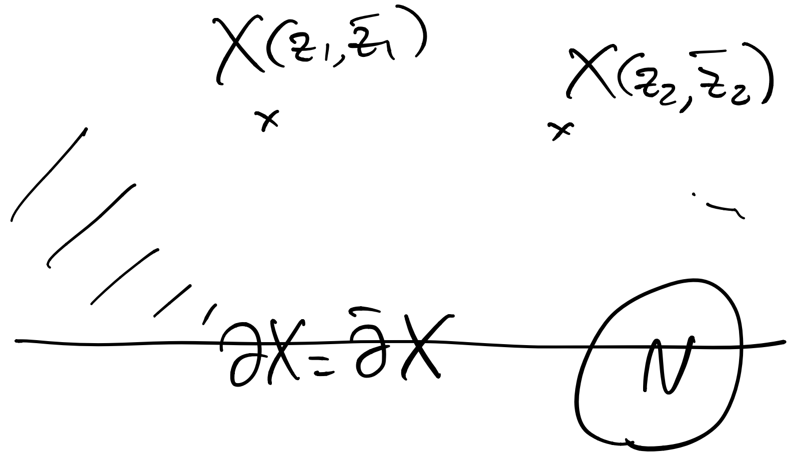
D: $\partial X = -\bar{\partial} X$ at $\mathrm{Im}\, z = 0$.

$$\partial^{n_1} X \partial^{n_2} X \cdots \bar{\partial}^{m_1} X \cdots e^{ikX}$$

boundary.

$\partial^{n_1} X \partial^{n_2} X \cdots e^{ikX}$ if Neuman

$\partial^{n_1} X \partial^{n_2} X \cdots$ if Dirichlet "doubling trick"



$$X(z,\bar z) = X_L(z) + \underline{X_R(\bar z)}$$

$X(z)$ $\quad \mathrm{Im}\, z > 0$

$X_R(\bar z')$ $\quad z'$

$\partial^n X \bar\partial^m X e^{ik\cdot X}$

$\partial X \equiv \partial X_L$

$\quad \| \; \mathrm{Im}\, z = 0$

$\bar\partial X \equiv \bar\partial X_R$

extend X_L to LHP by

Neumann

$$X_L(\bar z') \equiv \pm X_R(\bar z')$$

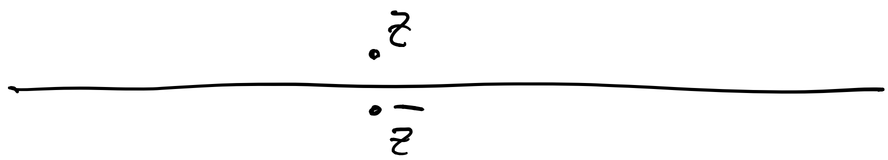
Dirichlet

$\mathrm{Im}\, z' > 0, \quad \mathrm{Im}\, \bar z' < 0.$

∂X_L is now continuous across $\mathrm{Im}\, z = 0.$

$$\partial^n X_L(z)\, \partial^m X_L(\bar z)\, e^{ik(X_L(z)+X_L(\bar z))}$$

singular as $z \to \bar z$



$$\boxed{:\partial^{n_1} X_L\, \partial^{n_2} X_L \cdots e^{2ikX_L(z)}: \quad \text{at } \operatorname{Im} z = 0}$$