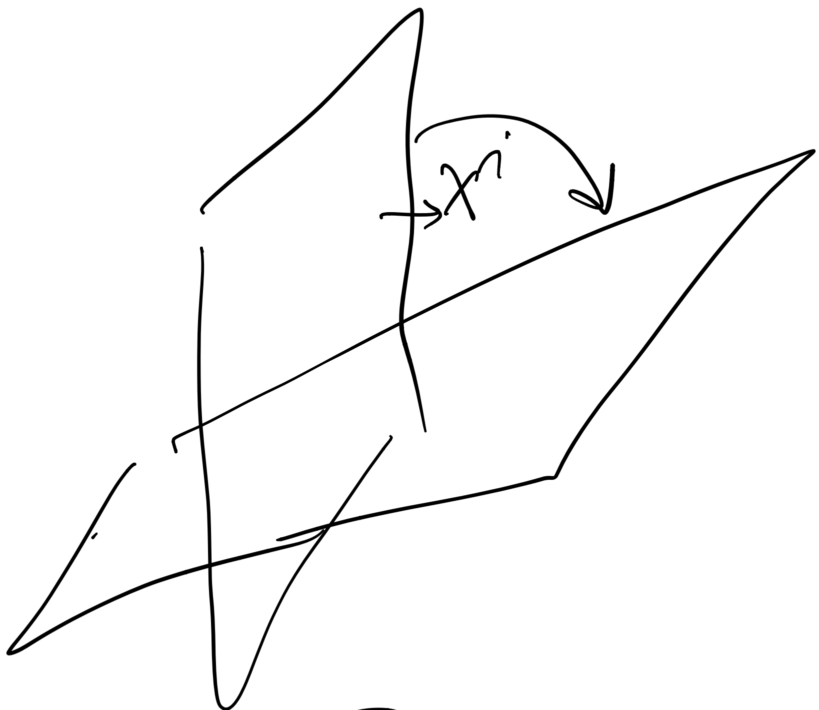
$$S_{\text{brane, bos}} = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\det\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)}$$

$$+ \mu_p \int e^{B + 2\pi\alpha' F} \wedge \sum_q C_q^{RR}$$

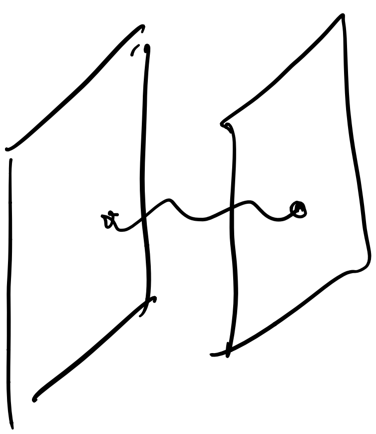
"Born-Infeld"



C^{RR}_{p+1}

$$\boxed{F^{RR}_{p+2} = dC^{RR}_{p+1}}$$

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int \sqrt{-G}\, e^{-2\Phi} R + \cdots$$



$$- \frac{1}{4\kappa^2} \int \sqrt{-G} \, |F^{RR}_{p+2}|^2 + \cdots$$

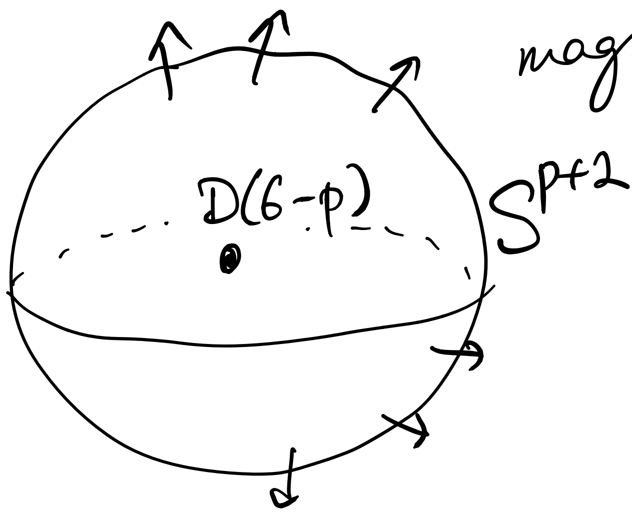
$$\mu_p = T_p = \boxed{\frac{\sqrt{\pi}}{\cdots} \cdot (4\pi^2\alpha')^{\frac{3-p}{2}}}$$

$$(\kappa = \tfrac{\pi}{2} g_s)$$

$$F^{RR}_{8-p} = * F^{RR}_{p+2}$$

$$\begin{array}{ccc} F^{RR}_{8-p} & & F^{RR}_{p+2} \\ \updownarrow & & \updownarrow \\ C^{RR}_{7-p} & & C^{RR}_{p+1} \\ \downarrow & & \downarrow \\ D(6-p) & & Dp \end{array}$$

magnetic source for F^{RR}_{p+2}



$$dC_{7-p} \wedge \overbrace{*dC_{7-p}}^{F_{p+2}}$$

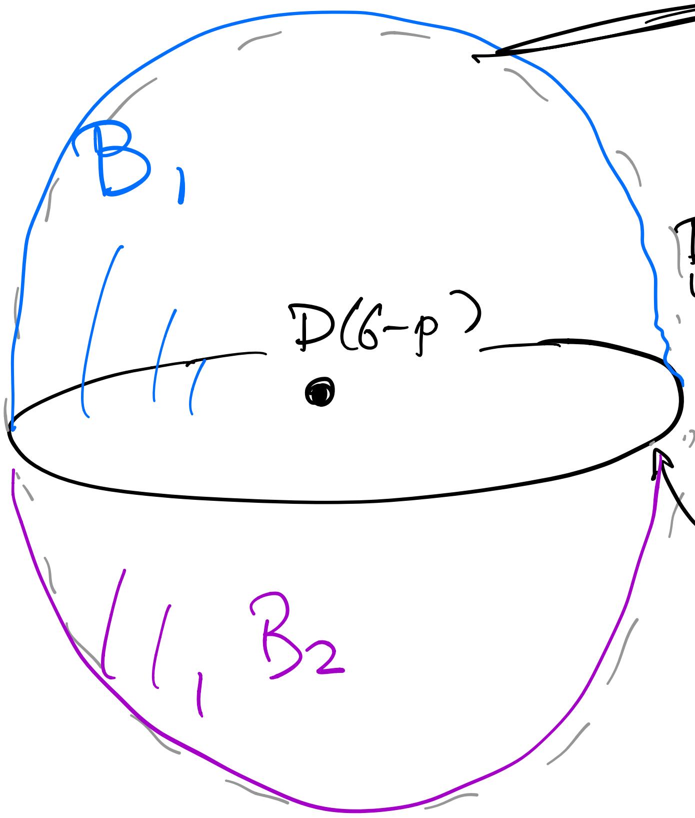
$$\uparrow$$

$$S_{brane+bulk} \supset -\frac{1}{4\kappa^2} \int_{spacetime} |F^{RR}_{p+2}|^2$$

$$+ \mu_{6-p} \int_{brane} C^{RR}_{7-p}$$

ECM : $\frac{1}{2\kappa^2} dF_{p+2} = \mu_{6-p}\,\delta^{p+3}(x_\perp)$

$\Rightarrow \int_{S^{p+2}} F_{p+2} = 2\kappa^2 \mu_{6-p}.$



Dp-brane wrapping S^{p+1}

$S_{eff} \supset \mu_p \int_{S^{p+1}} C_{p+1}$

$= \mu_p \int_{B_1} F_{p+2}$ "=" $-\mu_p \int_{B_2} F_{p+2}$

differ by

$\mu_p \int_{S^{p+2}} F_{p+2}$

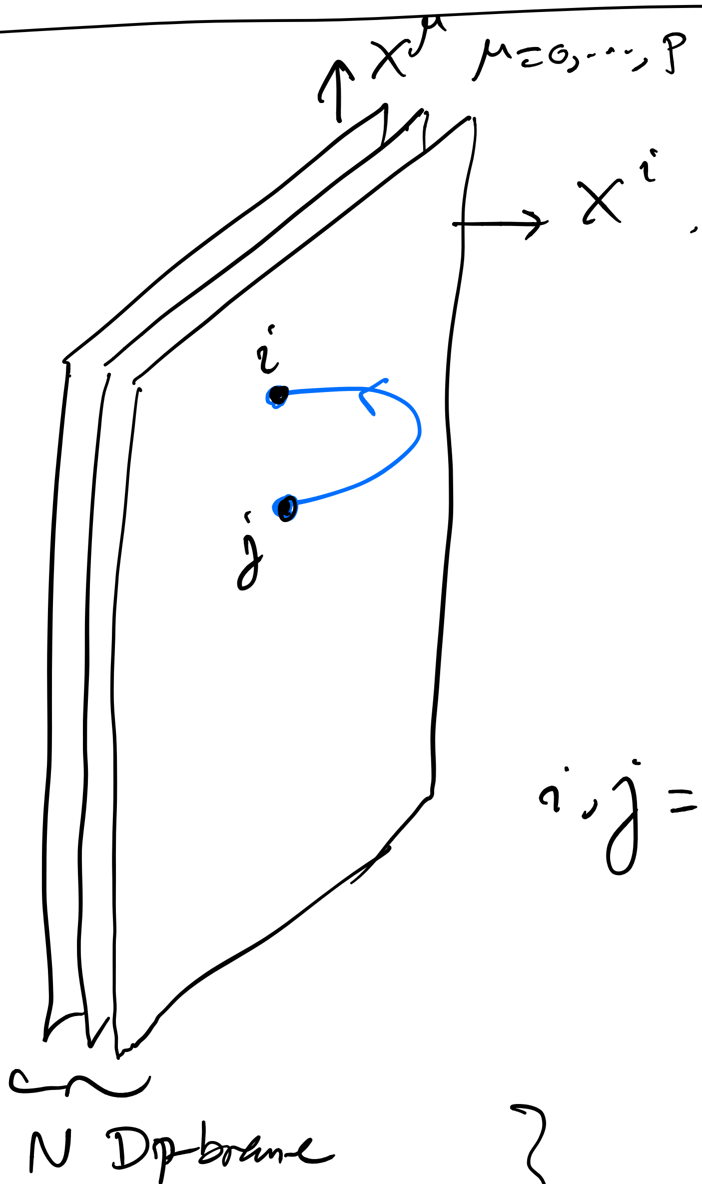
$= 2\kappa^2 \mu_p \mu_{6-p}$

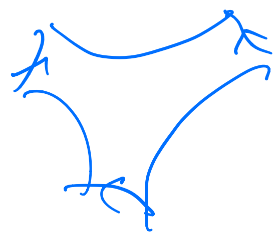
$\in 2\pi\mathbb{Z}.$

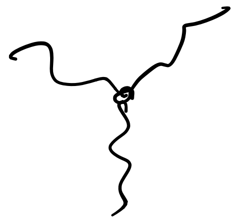
$\int \boxed{e^{iS_{eff}}}$

check:

$$2\kappa^2 \mu_p \mu_{6-p} = 2\pi \quad \checkmark$$



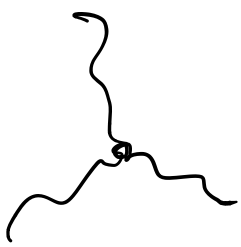


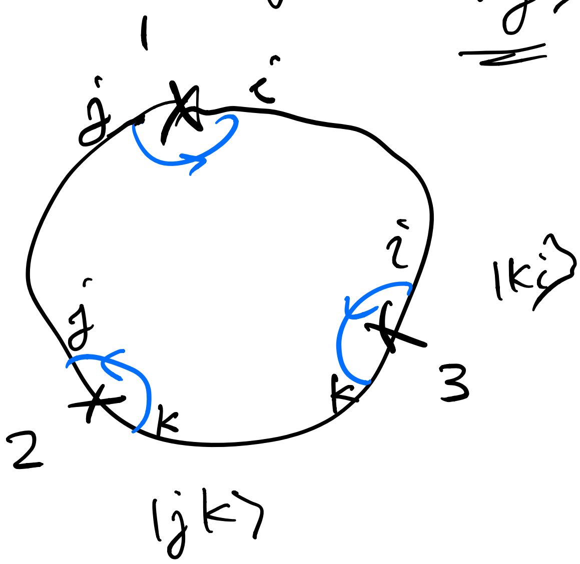


"Chan-Paton factors"

$$i, j = 1, \dots, N$$

$$V = e^{(\dots)} e^{ik\cdot X} \, |ij\rangle$$



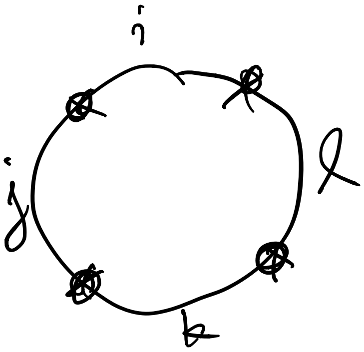


$\to \mathcal{U}(N)$

open strings transform

in adj rep of $\mathcal{U}(N)$

effective theory $\longrightarrow$ $\mathcal{U}(N)$ gauge theory



$A_\mu \longrightarrow$ $\mathcal{U}(N)$ gauge field.

$X^i \longrightarrow$ $N\times N$ matrices.

$\lambda_\alpha \longrightarrow$ --

eff. action for small field strengths.

$(\xi^a = X^a)$ $a=0,\dots,p$.

$$S = -T_p \int d^{p+1}\xi \,\cdot$$

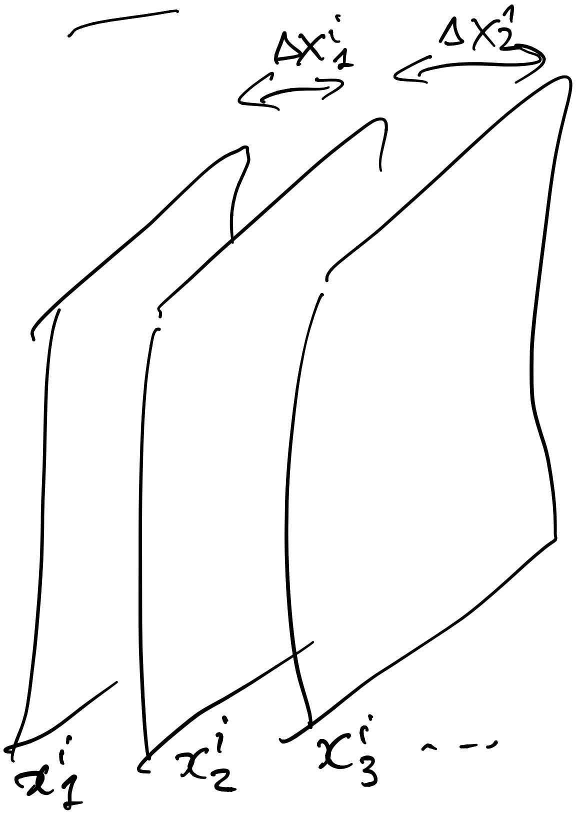
$F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$

$$\times \mathrm{tr} \cdot \Big\{ \frac{1}{4}(2\pi\alpha')^2 F_{ab}F^{ab} + \frac{1}{2}(D_a X^i)^2 - \frac{1}{4}(2\pi\alpha')^{-2}[X^i, X^j][X_i, X_j]$$

$$+ \text{fermions} + \cdots \Big\}$$

$\frac{1}{2}$-BPS. $\rightarrow$ 16 preserved susy.

$$V(X) = -\frac{1}{4} T_p (2\pi\alpha')^{-2} \, \mathrm{tr}\left([X^i, X^j]^2\right)$$



seek vev $\langle X^i \rangle$

$$V(\langle X \rangle) = 0$$

$$\Updownarrow$$

$$[X^i, X^j] = 0$$

$$\Downarrow$$

$$X^i = U \begin{pmatrix} x_1^i & & 0 \\ & x_2^i & \\ 0 & & \ddots \; x_N^i \end{pmatrix} U^{-1}$$

---

e.g. N D3-branes in IIB.

3+1-dim'l eff theory.

U(N) gauge theory

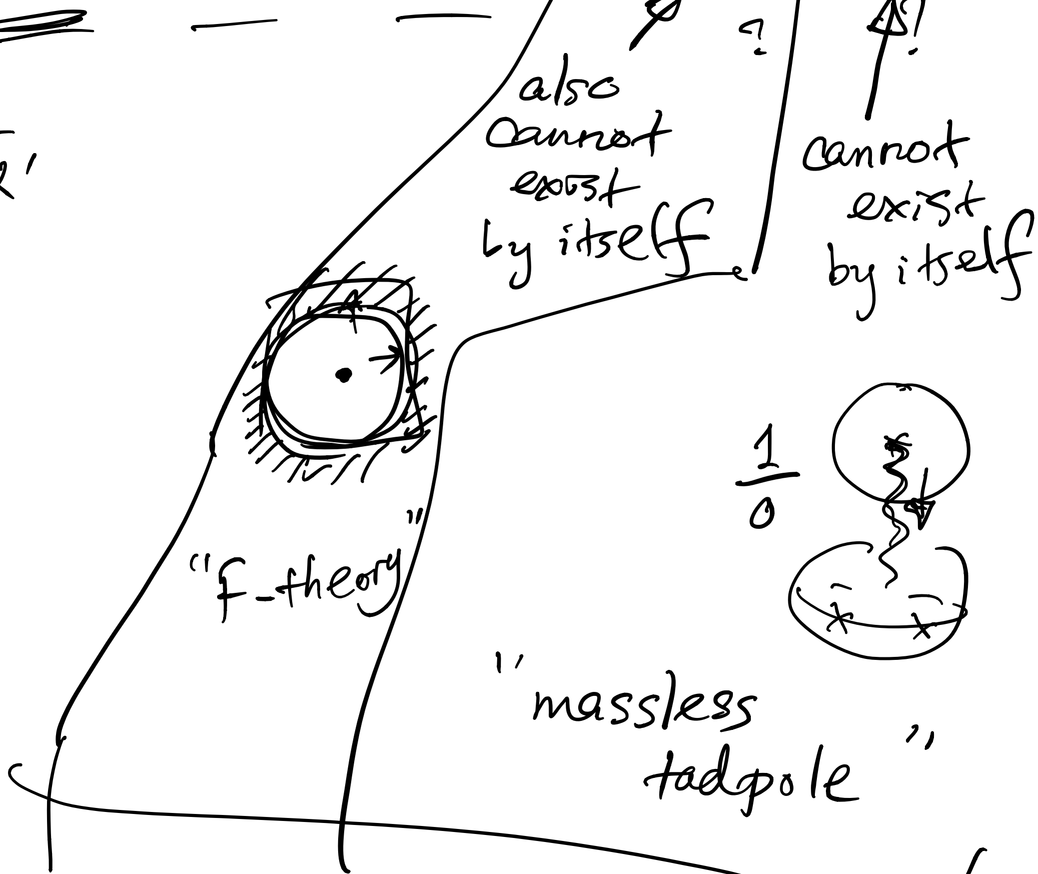
w/ 16 susies.

"$\mathcal{N}=4$ SYM"

IIB. $T \sim \frac{1}{\alpha'}$ $\rightarrow$ F1

D1, D3, D5 / D7 / D9

$$T \sim \frac{1}{g_s \alpha'}$$

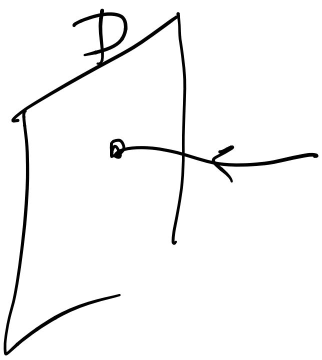


$g_s \ll 1$

$g_s \gg 1$

S-duality

IIB at g_s	F1	D1	D3	D5	NS5
$\wr$					
IIB at $\frac{1}{g_s}$	F1	D1	D3	D5	NS5



$$SL(2,\mathbb{Z}) \subset SL(2,\mathbb{R})$$

$$g_s \sim e^{\phi}$$

$$\tau \equiv \boxed{C_0^{RR}} + i e^{-\phi}$$

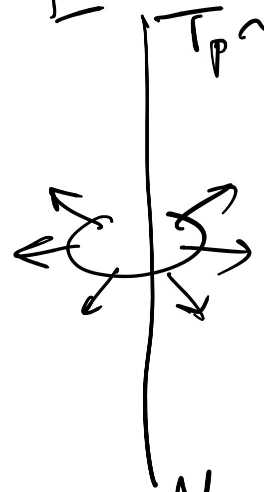
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\tau \mapsto \frac{a\tau+b}{c\tau+d}, \qquad \in SL(2,\mathbb{Z})$$

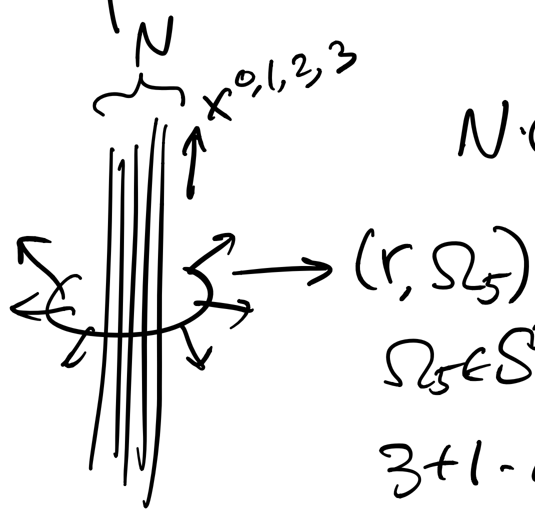
$$\underline{\tau \mapsto \tau+1}, \qquad \boxed{\tau \to -1/\tau}$$

D3 $\quad \underline{p=3}$

$T_p \sim \frac{1}{g_s}$



$$\text{grav} \sim \underset{\substack{\uparrow \\ g_s^2}}{G_N} \cdot T_p \sim g_s$$



$$N \cdot G_N \cdot T_p \sim \underline{\underline{g_s N}}$$

$\Omega_5 \in S^5$

3+1-dim'l Poincaré sym

$SO(6)$ rot^n sym

16 susies.

<u>sol'n to IIB sugra</u>

$$ds^2 = (f(r))^{-\frac{1}{2}}(-dx_0^2+dx_1^2+dx_2^2+dx_3^2) + (f(r))^{\frac{1}{2}}(dr^2+r^2 d\Omega_5^2)$$

$\Phi = \text{const}, \qquad C^{RR} = 0,$

$$\boxed{B_2 = C_2^{RR} = 0}$$

$$F_5^{RR} = (1+*)\, dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge d\frac{1}{f(r)}.$$

$$\boxed{f(r) = 1 + \frac{R^4}{r^4}}$$

$r=0$ horizon

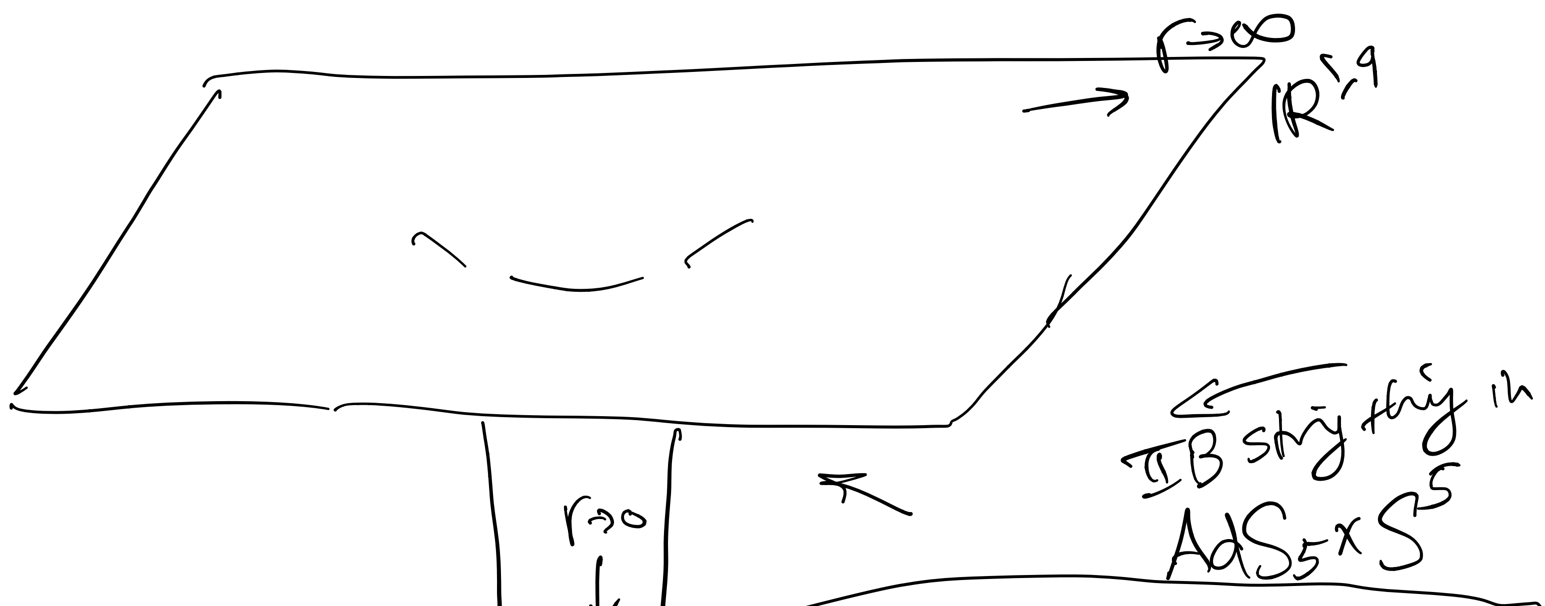
$$R^4 = 4\pi g_s N \alpha'^2 \quad (\gg \alpha'^2)$$

$\Updownarrow$

$g_s N \gg 1$

convention:

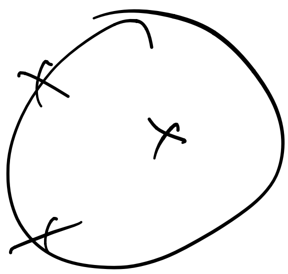
$$g_s = \frac{T_{F1}}{T_{D1}}$$



$$ds^2 \to \frac{r^2}{R^2}\left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2\right)$$

$$+ \frac{R^2}{r^2}\left(dr^2 + r^2 d\Omega_5^2\right)$$

$$+ R^2 \boxed{\left(\frac{dr}{r}\right)^2} + R^2 d\Omega_5^2$$



N D3-brane $\xrightarrow{\text{low energy}}$ 3+1-dim'l

U(N)

$\mathcal{N}=4$ SYM