Dp-branes in IIB: $p = \text{odd}$ (p = −1 D-instanton)

IIA: $p = \text{even}$

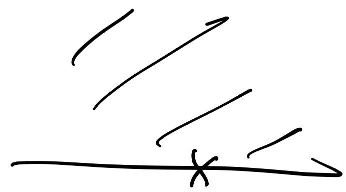
open strings on Dp-brane

$$Q_B \circlearrowright \mathcal{H}^{o}_{k} \simeq \mathcal{H}^{L}_{2k}$$

$$\uparrow$$

$$\left(\mathcal{H}^{c}_{k} = \mathcal{H}^{L}_{k} \otimes \mathcal{H}^{R}_{k}\right)$$

NS ⊕ R



massless open string states:

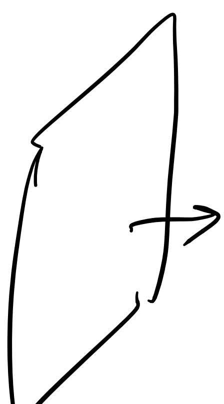
$$c e^{-\phi} e_\mu \psi^\mu e^{ik_{\parallel} X} \simeq e^{2ik_{\parallel}\cdot X_L}$$

NS:

$\mu = 0, \ldots, p.$ $\quad e \cdot k_{\parallel} = 0$

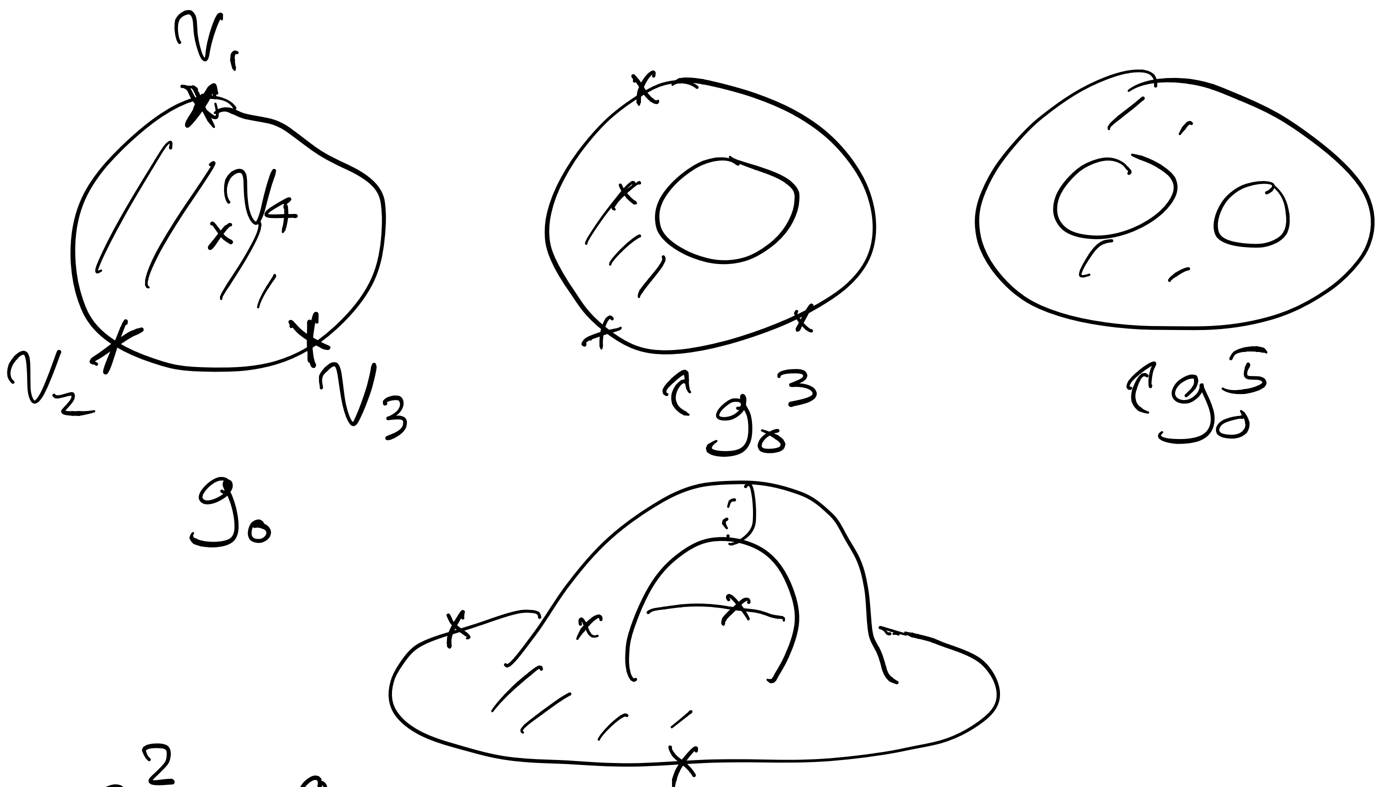
$$\boxed{c e^{-\phi} \psi^i e^{ik_{\parallel}\cdot X}}$$

$i = p+1, \ldots, 9$



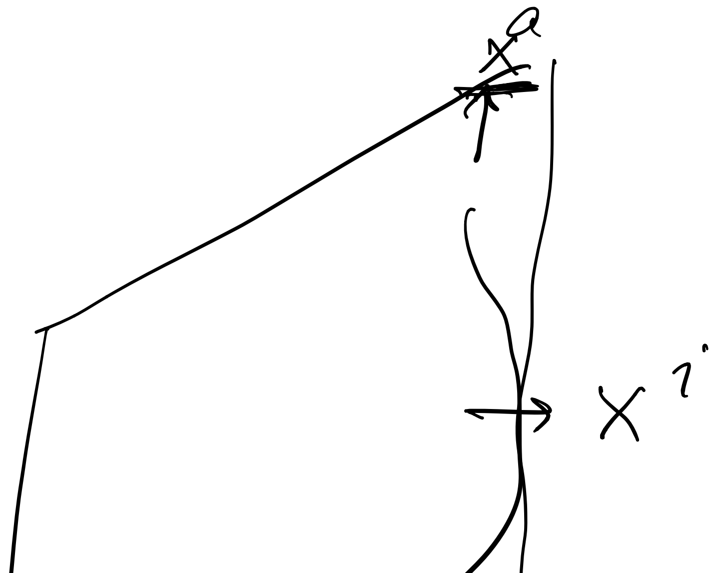
R $\quad c\, e^{-\frac{\phi}{2}} u^\alpha S_\alpha\, e^{i k_{\parallel} X}$ ← Goldstino

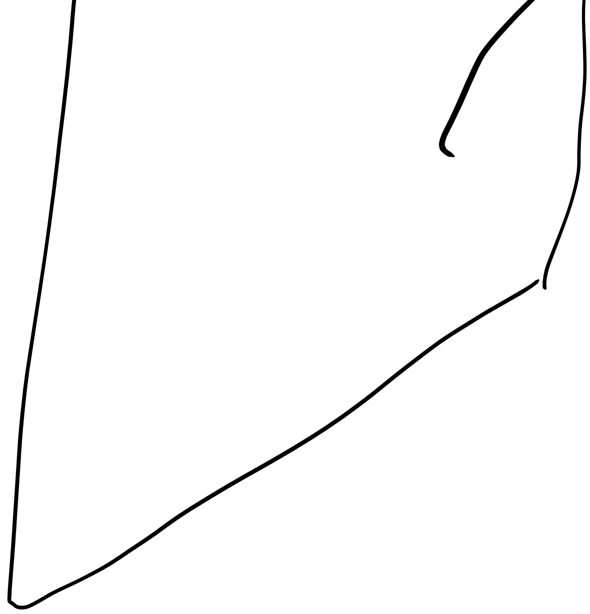
$$k_\mu (\Gamma^\mu u) = 0.$$



$\# \; \underline{g_0^2} = \underline{g_s}$

D-brane effective action





$X^{\mu=0,\dots,9}(\xi^a)$

$a=0,\dots,p.$

long distance fluctuations:

$$S_{\text{brane}} = -T_p \int d^{p+1}\xi \, \sqrt{-\det G_{ab}} + \dots$$

tension of Dp-brane

induced metric

$$G_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu}(X)$$

$$(B_{\mu\nu},\ \Phi,\ C^{RR}) = 0.$$

static gauge : $\xi^a = X^a$

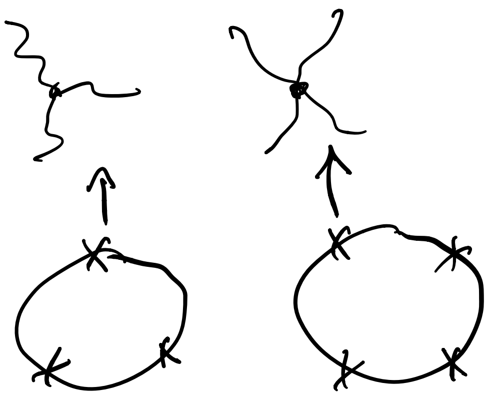
$a = 0, \dots, p$.

$X^{i = p+1, \dots, 9}(X^a)$

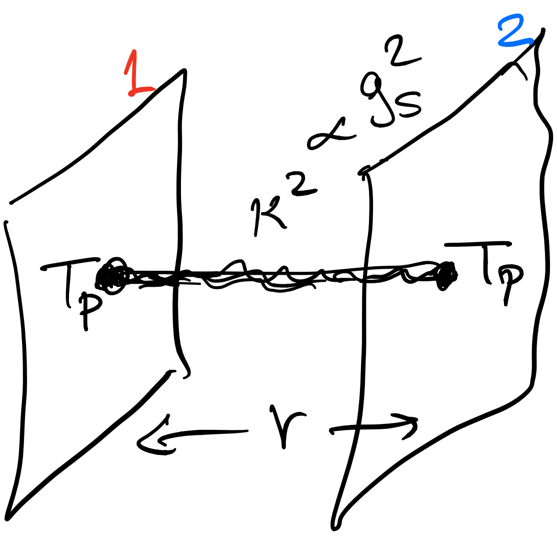
$$S = \underline{S_{brane}} + \boxed{\underline{S_{bulk}}}$$

10D sugra.

expand in X^i in static gauge.



$$T_p = \frac{\#}{g_o^2} \propto \frac{1}{g_s}$$



$$S_{bulk} \sim \frac{1}{2\kappa^2}\int \sqrt{G} R + \dots$$

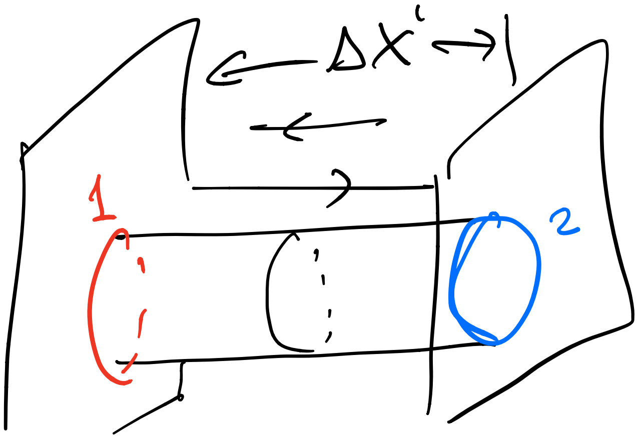
$$G_{\mu\nu} = \eta_{\mu\nu} + \boxed{h_{\mu\nu}}$$

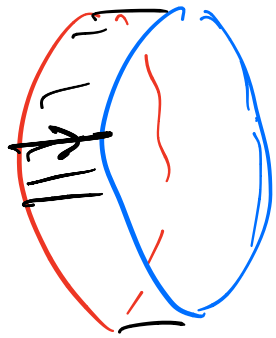
potential energy b/t a pair of Dp-brane

vol

$$= \boxed{\boxed{\# \, \kappa^2 \cdot T_p^2} \, f(r)} \qquad f(r) \sim \frac{1}{r^{23-p}}$$

$$T_p = \frac{\#}{\kappa}$$



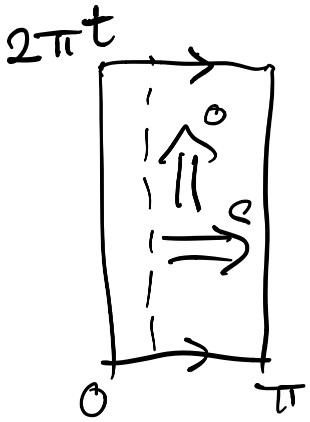


illustrate in critical bosonic string.

two orientation of open string

$$A_{cyl} = 2 \times \int_0^\infty \frac{dt}{2t} \, \mathrm{Tr}_{\mathcal{H}_0} (-)^{N_{bc}} b_0 c_0 \, e^{-2\pi t L_0}$$

$\mathbb{Z}_2$ vol(CKG)

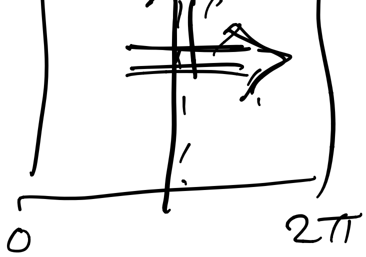


$$i V_{p+1} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \frac{1}{\eta(it)^{24}}$$

doubling trick

$\tau = it$

$$\times \, e^{-2\pi t \left(\alpha' k^2 + \alpha' \frac{(\Delta X)^2}{(2\pi\alpha')^2}\right)}$$



$\times e$

$$\eta(-\tfrac{1}{\tau}) = \sqrt{-i\tau}\cdot\eta(\tau)$$

$$\eta(\tfrac{i}{t}) = \sqrt{t}\cdot\eta(it)$$

$$A_{cyl} = \frac{i V_{p+1}}{(8\pi^2\alpha')^{\frac{p+1}{2}}}\int_0^\infty dt\cdot t^{\frac{21-p}{2}}\cdot e^{-t\frac{(\Delta X)^2}{2\pi\alpha'}}$$

$$\times\, \underbrace{\eta(\tfrac{i}{t})^{-24}}$$

$$(\eta(\tau))^{-24} = q^{-1}\prod_{n=1}^{\infty}(1-q^n)^{-24}$$

$$= \underbrace{q^{-1}}_{\text{tachyon}} + \underbrace{24}_{\text{massless}} + \underbrace{\# q + \# q^2 + \cdots}_{\text{massive}}$$

closed string exchange

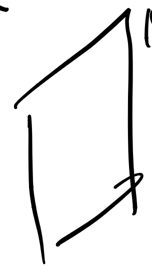
$\tau = i/t,\quad q = e^{-\frac{2\pi}{t}}$

$$\int_0^\infty dt\cdot t^{\frac{21-p}{2}}\, e^{-t\frac{(\Delta X)^2}{2\pi\alpha'}} \times 24$$

$$= \frac{\#}{|\Delta x|^{23-p}}$$

$\mathbb{R}^{1,25}$

$\mathbb{R}^{1,p}$



$25-p$ transverse dimensions

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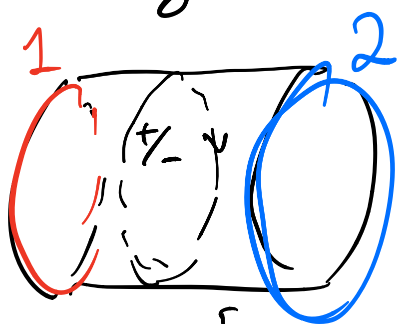
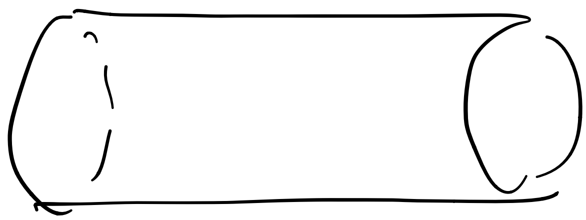
superstring

$$\frac{(R,R)}{(+,+)} \qquad \frac{(NS,NS)}{(-,-)}$$

$\Longrightarrow$



$$\mathrm{Tr}\,\frac{1+(-)^F}{2}\cdots$$



$= 0.$

$G_{\mu\nu}, \Phi.$

(NS,NS)

$\neq 0$

(R,R)

$\neq 0$

$$S_{brane} = -T_p \int d^{p+1}\xi \sqrt{-\det G_{ab}}$$

$$\Phi,\ B_{\mu\nu},\ C^{RR}$$

$$(F_{ab} = \partial_a A_b - \partial_b A_a)\ \underline{A_a(\xi)},\ (\text{fermions})$$

in type II string theory

$$S_{brane,\,bosonic} = -T_p \int d^{p+1}\xi\, e^{-\Phi} \times \sqrt{-\det(\underline{G_{ab}} + \underline{B_{ab}} + 2\pi\alpha' \underline{F_{ab}})}$$

$$+ \underset{\sim}{\mu_p} \int \underline{e^{B+2\pi\alpha' F} \wedge \sum_q C_q^{RR}}$$

(pick $p+1$ -form)

RR q-form potential e.g. $(q = p+1)$

$$B \equiv \frac{1}{2} B_{ab}\, d\xi^a \wedge d\xi^b$$

$$C_q^{RR} = \frac{1}{q!}(C_q^{RR})_{\mu_1\cdots\mu_q}\, dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_q}$$

$$dx^\mu \equiv \frac{\partial x^\mu}{\partial \xi^a} d\xi^a$$

$$\boxed{B + 2\pi\alpha' F}$$

1-form gauge parameter

$$\delta B = d\Lambda$$

$H = dB$

$$S_{WS} \supset \frac{1}{2\pi\alpha'}\int_\Sigma \overbrace{\tfrac{1}{2}B_{\mu\nu}(x)\,dx^\mu \wedge dx^\nu}^{B} + \int_{\partial\Sigma} \underbrace{A_\mu(x)\,dx^\mu}_{A}$$

$$\delta A = -\frac{1}{2\pi\alpha'}\Lambda$$

$$F = dA\,, \quad \delta F = -\frac{1}{2\pi\alpha'} d\Lambda$$

$B + 2\pi\alpha' F$ is gauge invt!