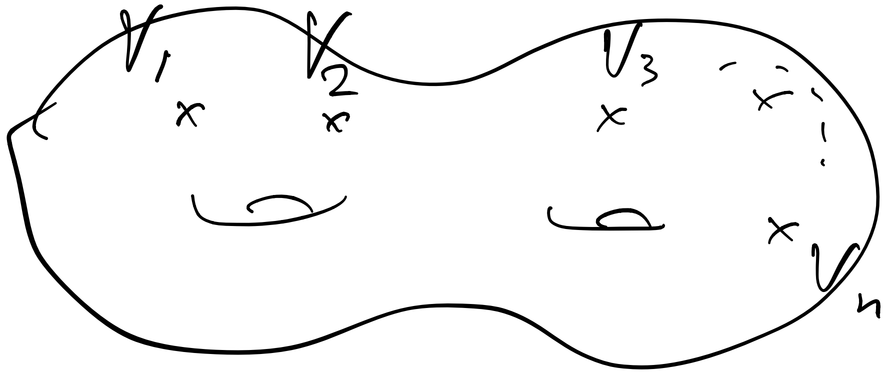


$$\int Dg_{ab}\, DX^\mu\, e^{-S_P}$$

$$\times \prod_{i=1}^{n} \int d^2\sigma_i \sqrt{g(\sigma_i)}\, V_i(\sigma_i)$$

should transform
as a scalar wrt diffeo.
wt 2, wrt Weyl

fix to conformal gauge

$$g_{ab} = \hat{g}_{ab}(t)$$

may also need to fix CKG

$$\sigma_i^{a} = \hat{\sigma}_i^{a} \qquad (i,a)\in f$$

$a = 1,2$

$i = 1,\dots,n$

$b_{ab}(\sigma)$

$$\delta\left(g_{ab} - \hat{g}_{ab}(t)\right) = -\nabla_a \delta v_b - \nabla_b \delta v_a + 2\delta\omega\, g_{ab} - \delta t^k \frac{\partial \hat{g}_{ab}(t)}{\partial t^k}.$$

c^a

ξ^k

η_i^a

$$\delta\left(\sigma_i^a - \hat{\sigma}_i^a\right) = \delta v^a(\sigma_i)$$

$$\int Dg_{ab}\, DX^\mu \prod d\sigma_i^a$$

gauge fixing

$$\rightsquigarrow \int_{\mathcal{M}_g} dt^k \int DX^\mu \prod_{(i,a)\notin f} d\sigma_i^a\, \Delta_{FP}$$

$$\Delta_{FP} = \int \overbrace{Db_{ab}\, d\eta_i^a}^{\text{"b-ghosts"}}\, \overbrace{Dc^a\, d\xi^k}^{\text{"c-ghosts"}}$$

$$\times \exp\left[-S_{bc} - \frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, b^{ab} \frac{\partial \hat{g}_{ab}(t)}{\partial t^k}\xi^k - \sum_{(i,a)\in f} \eta_i^a \cdot c^a(\sigma_i) \right]$$

$$\underline{\underline{\int \text{out } \eta, \xi}} \int Db_{ab}\, Dc^a\, e^{-S_{bc}}$$

$$\times \prod_{(i,a)\in f} c^a(\sigma_i) \prod_k \left(\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, b^{ab} \frac{\partial \hat{g}_{ab}}{\partial t^k} \right)$$

add to S_p a topological term.

genus g : $\frac{\phi_0}{4\pi}\int d^2\sigma \sqrt{g}\, R(g) = \phi_0 \chi(\Sigma)$

$\uparrow$ $2-2g$

$A_g[V_1, \dots, V_n]$

$$= e^{-\phi_0 \chi} \int_{\mathcal{M}_g} \prod_k dt^k \Big\langle \prod_k \frac{1}{4\pi} \int d^2\sigma \sqrt{\hat{g}(\sigma)}\, b^{ab}(\sigma) \overbrace{\frac{\partial \hat{g}_{ab}(\sigma)}{\partial t^k}}^{\text{"Beltrami differential"}}$$

$$\times \prod_{(i,a)\in f} c^a(\hat{\sigma}_i) \prod_{(i,a)\notin f} d\sigma_i^a \prod_{i=1}^{n} \sqrt{\hat{g}(\sigma_i)}\, V_i(\sigma_i) \Big\rangle$$

X^μ, bc CFT on $\Sigma, \hat{g}(t)$

$\# c - \# b$
$= \# \text{CKV} - \# \text{moduli} \stackrel{?}{=} 6 - 6g$ (yes!)

$g_s \equiv e^{\phi_0}$, $\qquad e^{-\phi_0 \chi} = g_s^{2g-2}$

---

bc ghost correlator.

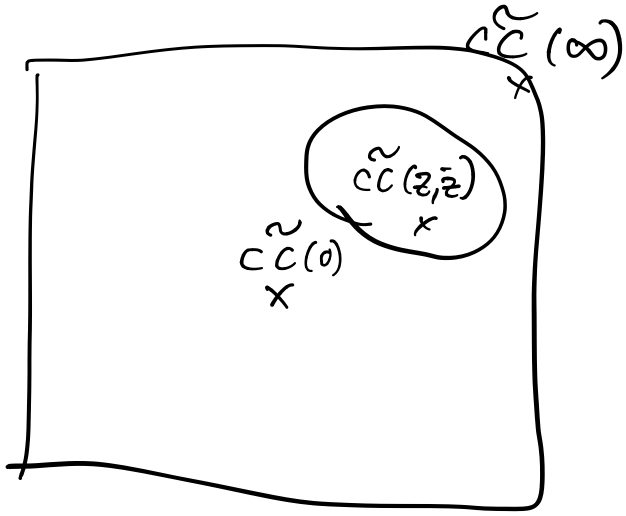
- sphere $(g=0)$

$$\Big\langle \prod_{i=1}^{3} c(z_i)\, \tilde{c}(\bar{z}_i) \Big\rangle$$

$$= - |z_{12} z_{13} z_{23}|^2$$

$$\langle\langle \downarrow, \downarrow | \tilde{c}(z) c(\bar{z}) | \downarrow, \downarrow \rangle$$

$$= + |z|^2$$



$$\updownarrow$$

$$c(z) = \sum_n \frac{c_n}{z^{n-1}}$$

$$\langle\langle \downarrow, \downarrow | \tilde{c}_0 c_0 | \downarrow, \downarrow \rangle = 1$$

$$\text{\rotatebox{90}{$\equiv$}}$$

$$\langle\langle \downarrow, \downarrow | \uparrow, \uparrow \rangle$$

$$S_{bc} \sim \int b_{ab} \nabla^a c^b$$

U(1) global sym

$c \quad +1$

$b \quad -1$

Noether current (ghost current)

$$j_{gh} = -bc.$$

$$T^{gh}(z)\, j_{gh}(0)$$

$$\sim -\frac{3}{z^3} + \frac{1}{z^2} j_{gh}(0) + \frac{1}{z}\partial j_{gh}(0)$$

$\longrightarrow$ contact term in

$$T^a{}_a(z,\bar z)\,\bar\partial j_{gh}(0)$$

$$\longrightarrow \quad \nabla_a j^a_{gh} = \frac{3}{4} R(g).$$

$\Rightarrow$ on a genus g surface,

$$\langle b \cdots c \cdots \rangle \neq 0$$

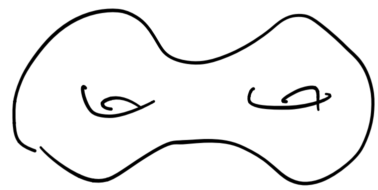
$$\# c - \# b \quad (= \#\tilde c - \#\tilde b)$$

$$= 3 - 3g$$

$$\int \mathcal{D}b_{ab}\,\mathcal{D}c^a \; e^{-S_{bc}} \cdot \; b \cdots \; c \cdots$$

$f^{ab}(\sigma)$

$$(f, f') \equiv \int d^2\sigma \sqrt{g}\, f^{ab} f'_{ab}$$



Σ

$$S_{bc} = \frac{1}{2\pi}(b, Pc) = \frac{1}{2\pi}(P^T b, c)$$

sym, traceless

$$P : \; c^a(\sigma) \rightsquigarrow (P\cdot c)_{ab}$$

$$\equiv \frac{1}{2}\left(\nabla_a c_b + \nabla_b c_a - g_{ab}\nabla_d c^d\right)$$

$$(P^T b)^a = -\nabla_b b^{ab}$$

$$P^T P, \quad P P^T \quad (\geq 0)$$

can find orthonormal basis v_n of vector fields on Σ

$$P^T P\, v_n = \lambda_n v_n, \qquad \lambda_n \geq 0.$$

some of λ_n's may be zero. $\leftarrow v_j^{(0)}$

if $\lambda_n \neq 0$ (>0)

$$u_n = \frac{1}{\sqrt{\lambda_n}} P v_n$$

easy to check: u_n's are a set of orthonormal sym, traceless tensor fields on Σ.

$$P P^T u_n = \frac{1}{\sqrt{\lambda_n}} \cdot P \underbrace{P^T P v_n}_{\lambda_n v_n}$$

$$= \lambda_n u_n .$$

there could be some additional u_n's.

$$P P^T u_n = 0 . \leftarrow u_k^{(0)}$$

$$b_{ab}(\sigma) = \sum_n \underbrace{b_n}_{\text{Fourier coefficient (Grassman variables)}} (u_n)_{ab}(\sigma)$$

$$c^a(\sigma) = \sum_n c_n \cdot (v_n)^a(\sigma)$$

$$\int D b_{ab} D c^a \, e^{-S_{bc}} \cdots$$

$$= \int \prod_n db_n \prod_m dc_m \cdot e^{-\frac{1}{2\pi}\sum_n \sqrt{\lambda_n}\, b_n c_n} \cdots$$

zero modes of c^a $(v_j^{(0)})$ $\longleftrightarrow$ CKV

zero modes of b_{ab} $(u_k^{(0)})$ $\longleftrightarrow$ moduli

[See D'Hoker & Phong for details of explicit eval. of bc-correlator on $g \geq 2$].

BRST