Lecture 5 – 9/20

Finish simple TLS calc:

- → interaction pic eqs of motion
- → particles light field + dipole interaction Ham.
- → specific cases of weak + strong excitation
- → Bloch sphere
- → light shifts
- → dressed states
- → Adiabaticity

Note: These concepts are of much wider impact than TLS !

g) Strong excitation

$\gamma \ll |\Omega_R|$ neglect decay (i.e. fast enough unitary evolution)

$$H = \begin{pmatrix} -\delta & \frac{\Omega_R}{2} \\ \frac{\Omega_R^*}{2} & 0 \end{pmatrix}$$

on resonance : $\delta = 0$

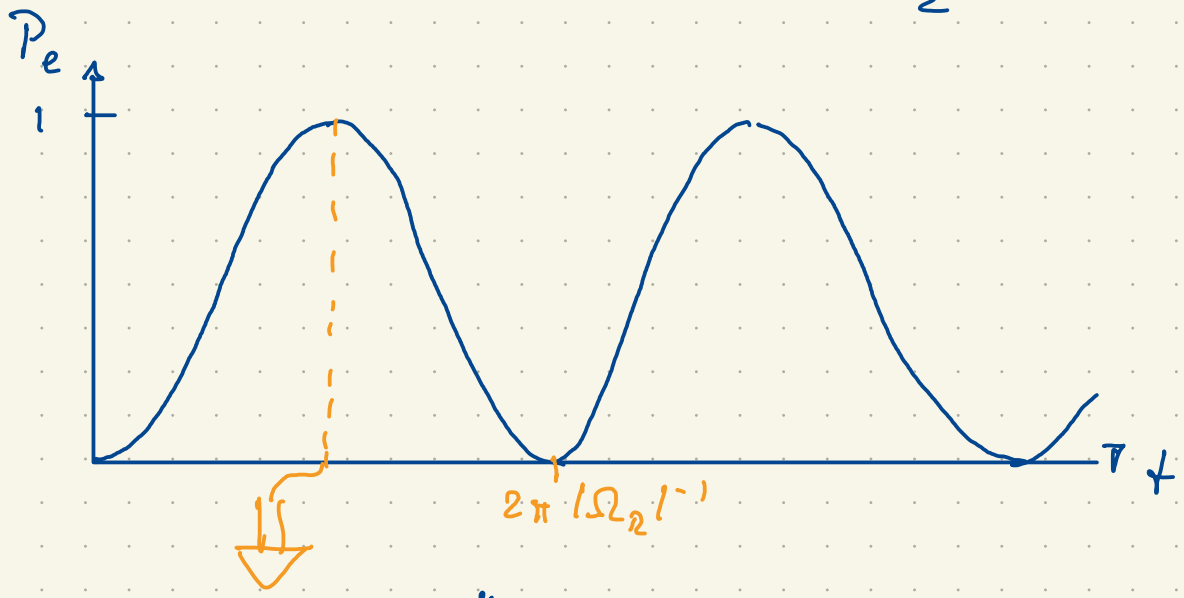
$$\ddot{\tilde{c}}_e = -i \frac{\Omega_R}{2} \dot{\tilde{c}}_g = -\frac{|\Omega_R|^2}{4} \tilde{c}_e$$

$$\dot{\tilde{c}}_g = -i \frac{\Omega_R^*}{2} \tilde{c}_e$$

harmonic oscillator

$\Rightarrow c_g(0) = 1 \quad \Rightarrow \quad P_e(t) = |c_e(t)|^2$

$$= \sin^2 \frac{|\Omega_R|}{2} t = \frac{1}{2}(1 - \cos |\Omega_R| t)$$

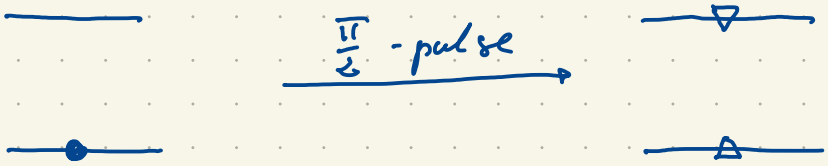


"π-pulse" : drive system w/ 100% probability from $|g\rangle$ into $|e\rangle$

π-pulse

"$\frac{\pi}{2}$ - pulse" : $|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(i|e\rangle + |g\rangle\right)$

$|\Omega_2| t = \frac{\pi}{2}$: transfer $|g\rangle$ into superposition state!



off-resonance ($\delta \neq 0$)

$$\ddot{\tilde{c}}_e + i\delta\dot{\tilde{c}}_e + \frac{|\Omega_2|^2}{4}\tilde{c}_e = 0$$

ansatz: $\tilde{c}_e \propto e^{i\alpha t} \Rightarrow \alpha^2 + \alpha\delta - \frac{|\Omega_2|^2}{4} = 0$

$$\Rightarrow \alpha = -\frac{\delta}{2} \pm i\frac{\sqrt{\delta^2 + |\Omega_2|^2}}{\tilde{\Omega}}$$

"generalized Rabi frequency"

$$\tilde{c}_e = A\, e^{-i\frac{\delta}{2}t} \sin\frac{\tilde{\Omega}}{2}t$$

(for $\tilde{c}_e(0) = 0$)

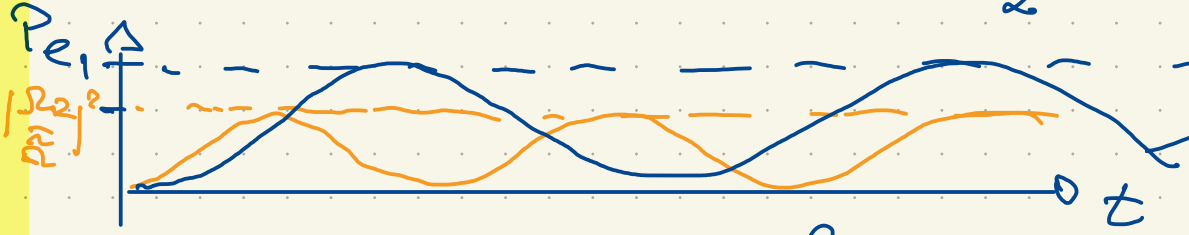
$$\dot{\tilde{c}}_e(0) = i\frac{A}{2}\left(-i(\delta - \tilde{\Omega}) + i(\delta + \tilde{\Omega})\right)$$

$$= -\frac{A}{2}\Omega \overset{!}{\equiv} -i\delta\,\tilde{c}_e(0) - i\frac{\Omega_2}{2}\tilde{c}_g(0)$$

($\tilde{c}_e(0) = 0$, $\tilde{c}_g(0) = 1$)

$$\Rightarrow A = i\frac{\Omega_2}{\tilde{\Omega}} \Rightarrow |A| \leq 1$$

$$P_e(t) = \left|\frac{\Omega_2}{\tilde{\Omega}}\right|^2 \sin^2\frac{\tilde{\Omega}}{2}t = \left|\frac{\Omega_2}{\tilde{\Omega}}\right|^2 \frac{1-\cos\tilde{\Omega}t}{2}$$



→ oscillation faster

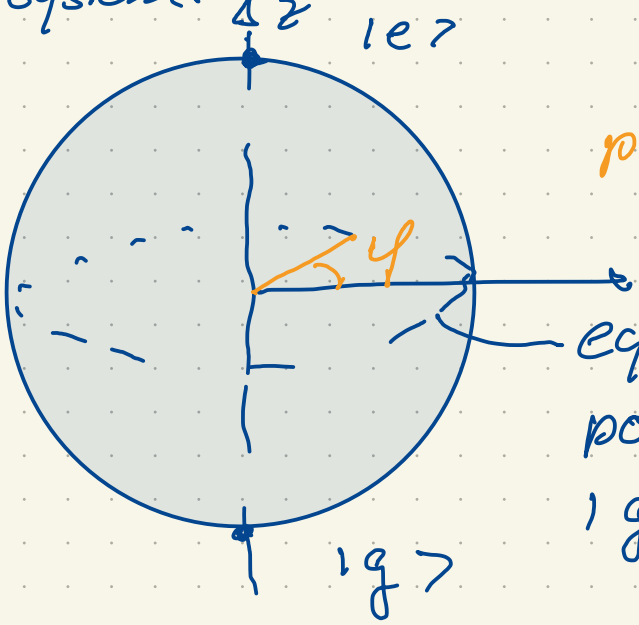
→ never full population transfer

delete for HW?

h) Bloch vector, sphere

Def: $\langle \vec{\sigma} \rangle = \langle \sigma_x \rangle \hat{x} + \langle \sigma_y \rangle \hat{y} + \langle \sigma_z \rangle \hat{z}$

most general description for any spin-$\frac{1}{2}$ (and thus, TL-) system.



$\langle \vec{\sigma} \rangle$ has tip on surface of sphere.

$$\frac{d}{dt} \langle \vec{\sigma} \rangle = \dots$$ (see HW 3)

i) Light shifts - Dressed states

- "Dressed states"

diagonalize system with respect to driving field(s)

Solve (time-indep.) Schr. Eq for

$$\frac{H}{\hbar} = \begin{pmatrix} -\delta & -\frac{\Omega}{2} \\ -\frac{\Omega^*}{2} & 0 \end{pmatrix} \Rightarrow$$

$$\lambda_\pm = -\frac{1}{2}\left(\delta \pm \sqrt{\delta^2 + |\Omega|^2}\right)$$

$$|\lambda_-\rangle = \mathcal{N}\begin{pmatrix}\sqrt{\delta^2+|\Omega|^2}-\delta\\ -\Omega^*\end{pmatrix}$$

$$|\lambda_+\rangle = \mathcal{N}\begin{pmatrix}\Omega\\ \sqrt{\delta^2+|\Omega|^2}-\delta\end{pmatrix}$$

$\begin{pmatrix}1\\0\end{pmatrix} \hat{=} |e\rangle$
$\begin{pmatrix}0\\1\end{pmatrix} \hat{=} |g\rangle$

$$\mathcal{N}^{-2} = |\Omega|^2 + \left(\sqrt{\delta^2+|\Omega|^2}-\delta\right)^2$$

$$\Omega \equiv |\Omega| e^{i\varphi}$$

Dressed states

This means:

$\delta = 0$

$$\lambda_- = \frac{|\Omega|}{2}\,; \quad |\lambda_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -e^{-i\varphi}\end{pmatrix} \overset{\Omega=|\Omega|}{=} \frac{1}{\sqrt{2}}\left(|e\rangle - |g\rangle\right)$$

$$\lambda_+ = -\frac{|\Omega|}{2} \quad |\lambda_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\varphi}\\ 1\end{pmatrix} \overset{\Omega=|\Omega|}{=} \frac{1}{\sqrt{2}}\left(|e\rangle + |g\rangle\right)$$

$|\delta| \gg |\Omega|$:

$$\sqrt{\delta^2+|\Omega|^2} \approx |\delta| + \frac{|\Omega|^2}{2|\delta|}$$

$\delta > 0$:

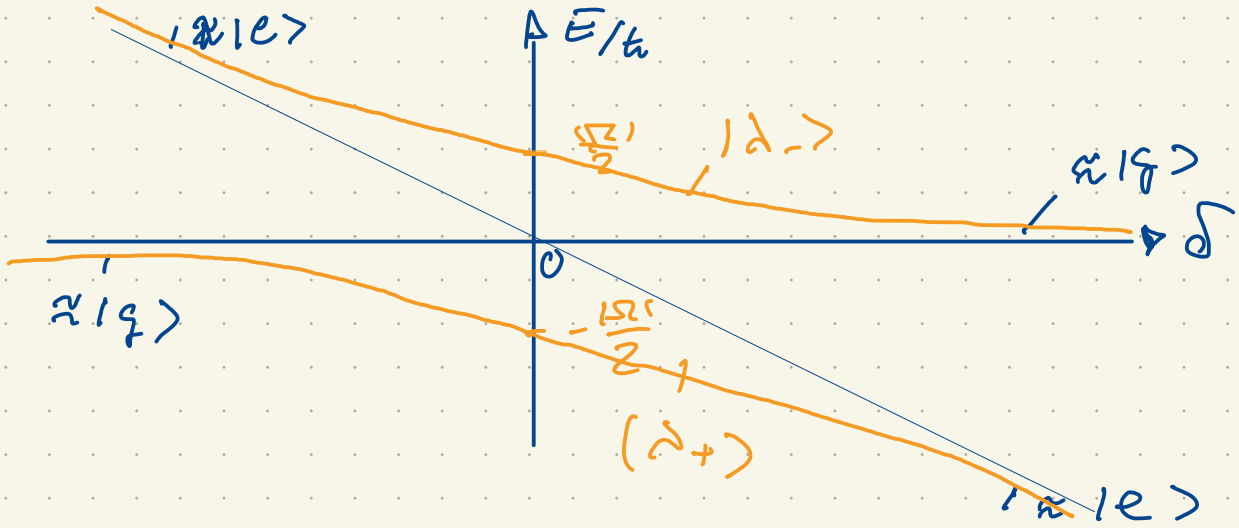
$$\lambda_- \approx \frac{|\Omega|^2}{4\delta} \qquad |\lambda_-\rangle \xrightarrow{\delta\to\infty} \begin{pmatrix}0\\1\end{pmatrix} \hat{=} |g\rangle$$

$$\lambda_+ \approx -\delta\left(-\frac{|\Omega|^2}{4\delta}\right) \qquad |\lambda_+\rangle \xrightarrow{\delta\to\infty} \begin{pmatrix}1\\0\end{pmatrix} = |e\rangle$$

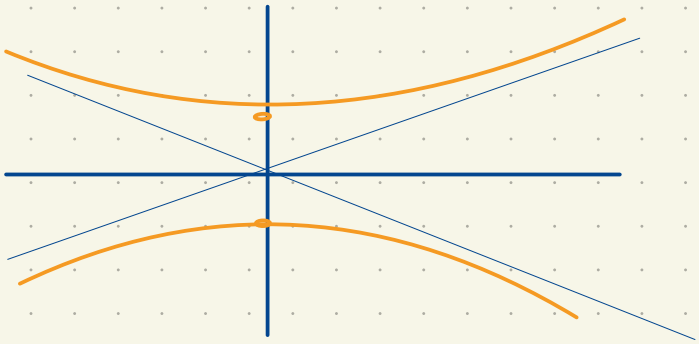
$\delta < 0$:

$$\lambda_- \approx -\delta \qquad |\lambda_-\rangle \xrightarrow{\delta\to-\infty} |e\rangle$$

$$\lambda_+ \approx \frac{|\Omega|^2}{4\delta} \qquad |\lambda_+\rangle \xrightarrow{\delta\to-\infty} |g\rangle$$



NB: $\frac{H}{\hbar} = \begin{pmatrix} \delta/2 & -\Omega/2 \\ -\Omega^*/2 & -\delta/2 \end{pmatrix}$



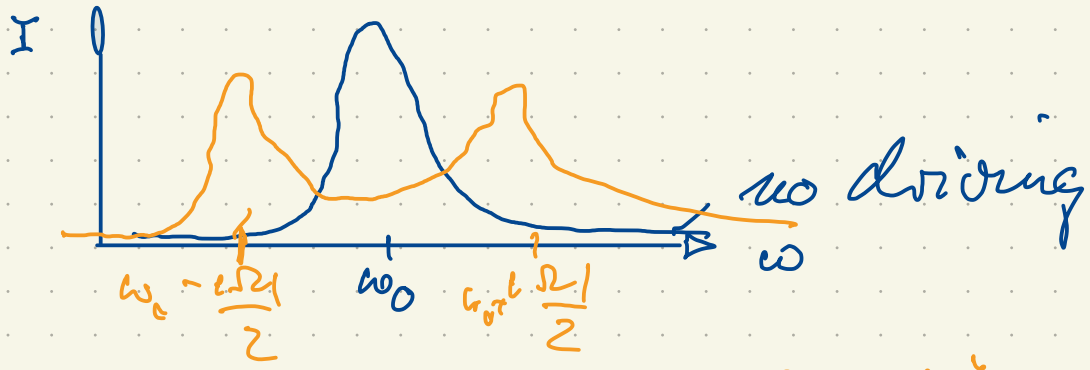
Measurement:

spectroscopy on $|g\rangle$ or $|e\rangle$ will reveal these energy shifts!

For $\delta = 0$

Both $|g\rangle, |e\rangle$ share equally in $|\lambda_\pm\rangle$ $\Rightarrow$ two equally strong lines on $\lambda_\mp = \pm \frac{|\Omega|}{2}$:

$\Rightarrow$ two lines, shifted $\pm \frac{|\Omega|}{2}$ (compared to resonance) will appear:



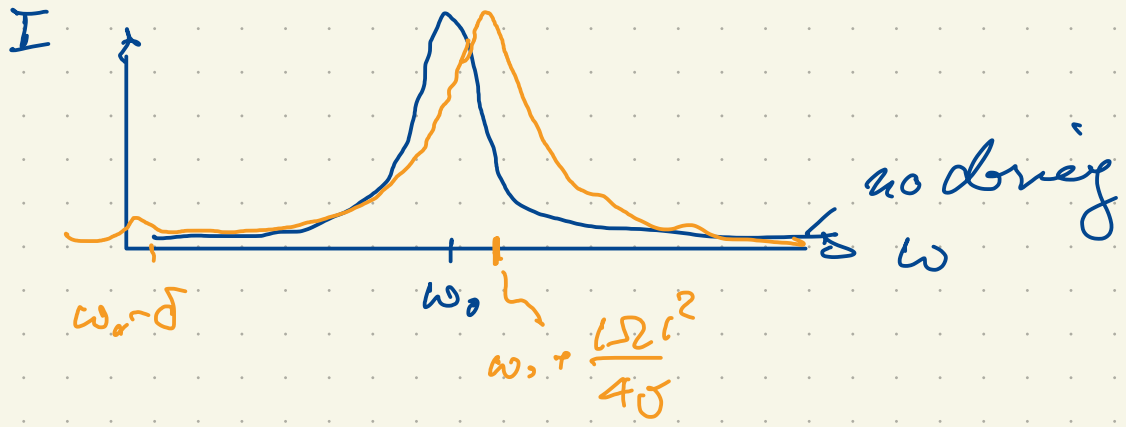
no driving

"Autler - Townes effect"

For $|\delta| \gg |\Omega|$

A measured line will appear

shifted by $\frac{|\Omega|^2}{4\delta}$. This shift is called "AC Stark" shift (or "light shift"). While the $-\delta$-shift is also visible, it belongs to the state with a much smaller contribution.)



Bloch-Siegert shift

Treat counter-rotating part of $V(t)$ like independent field:

$$\frac{H}{\hbar} \to \frac{1}{2}\begin{pmatrix} \omega+\omega_0 & \Omega \\ \Omega^* & -(\omega+\omega_0) \end{pmatrix}$$

with $\omega_0 \approx \omega \Rightarrow \omega+\omega_0 \approx 2\omega_0$

$$\boxed{\Delta E_{BS\pm} = \pm \frac{\hbar}{2} \frac{|\Omega|^2}{4\omega_0}}$$

(matters only for very strong drive!)

Remark: Since all shifts are small and are treated to lowest order in the intensity, we are justified to treat them independently.