Lecture 6 – 9/25

Finish simple TLS calc:

- → interaction pic eqs of motion
- → particular light field + dipole interaction Ham.
- → specific cases of weak + strong excitation
- → Bloch sphere
- → light shifts
- → dressed states
- → Adiabaticity
  (Adiabatic theorem, ~ passage)
- → Landau - Zener

Open systems

Note: These concepts are of much wider impact than TLS!

Topics: Adiabaticity – passage
Landau - Zener

j) Adiabatic theorem + passage

A system remains in its instantaneous eigenstate if any perturbation is acting slowly enough....

In rotating frame:

$$\frac{\partial}{\partial t}\langle\vec{\sigma}\rangle = \vec{\tilde{\Omega}} \times \langle\vec{\sigma}\rangle \qquad \text{(Bloch vector EOM)}$$

where: $\vec{\tilde{\Omega}} = -\Delta\,\hat{z} + |\Omega_R|\,\hat{x}$

in polar coordinates:

$$\langle\vec{\sigma}\rangle = \langle\sigma_r\rangle\,\hat{r} + \langle\sigma_\vartheta\rangle\,\hat{\vartheta} + \langle\sigma_\varphi\rangle\,\hat{\varphi}$$

choose: $\hat{r} \equiv \hat{\tilde{\Omega}}$

$$\left.\begin{aligned} \hat{r} &= \cos\vartheta\,\hat{z} + \sin\vartheta\,\hat{x} \\ \hat{\vartheta} &= -\sin\vartheta\,\hat{z} + \cos\vartheta\,\hat{x} \\ \hat{\varphi} &= \hat{y} \end{aligned}\right\} \qquad \begin{aligned} \cos\vartheta &\propto -\Delta \\ \sin\vartheta &\propto |\Omega_R| \end{aligned}$$

$$\dot{\hat{r}} = \dot{\vartheta}\,\hat{\vartheta}, \quad \dot{\hat{\vartheta}} = -\dot{\vartheta}\,\hat{r}, \quad \dot{\hat{\varphi}} = 0$$

$$\frac{\partial}{\partial t}\langle\vec{\sigma}\rangle = \langle\dot{\sigma}_r\rangle\,\hat{r} + \langle\dot{\sigma}_\vartheta\rangle\,\hat{\vartheta} + \langle\dot{\sigma}_\varphi\rangle\,\hat{\varphi} + \langle\sigma_r\rangle\,\overset{\propto\hat{\vartheta}}{\dot{\hat{r}}} + \langle\sigma_\vartheta\rangle\,\overset{\propto\hat{r}}{\dot{\hat{\vartheta}}} + \cancel{\langle\sigma_\varphi\rangle\,\dot{\hat{\varphi}}}^{\,0}$$

$$= \tilde{\Omega}\,\hat{r} \times \left(\langle\sigma_r\rangle\,\hat{r} + \langle\sigma_\vartheta\rangle\,\hat{\vartheta} + \langle\sigma_\varphi\rangle\,\hat{\varphi}\right)$$

$$\boxed{\begin{aligned} \hat{r}&: \quad \langle\dot{\sigma}_r\rangle - \langle\sigma_\vartheta\rangle\,\dot{\vartheta} &&= 0 \\ \hat{\vartheta}&: \quad \langle\dot{\sigma}_\vartheta\rangle + \langle\sigma_r\rangle\,\dot{\vartheta} &&= -\tilde{\Omega}\,\langle\sigma_\varphi\rangle \\ \hat{\varphi}&: \quad \langle\dot{\sigma}_\varphi\rangle &&= \tilde{\Omega}\,\langle\sigma_\vartheta\rangle \end{aligned}}$$

Assumption: $|\dot{\vartheta}| \ll \tilde{\Omega}$

zeroth order in $\dot{\vartheta}$:

$\rightarrow$ ϑ, φ components ($\perp$ to $\hat{\tilde{\Omega}}$) oscillate around $\tilde{r} \parallel \hat{\tilde{\Omega}}$ with frequency $\tilde{\Omega}$

$\rightarrow$ r-component does not change!

$\Rightarrow$ $\boxed{\langle \vec{\sigma} \rangle \text{ follows } \vec{\tilde{\Omega}}}$ (that is, the relative angle between $\langle\vec{\sigma}\rangle$ and $\vec{\tilde{\Omega}}$ does not change)

"adiabatic theorem"

Close look: (when is this a good assumption?)

$$\frac{d}{dt}\tan\vartheta = \frac{\dot{\vartheta}}{\cos^2\vartheta} = \dot{\vartheta}\,\frac{\tilde{\Omega}^2}{\Delta^2}$$

$$= \frac{d}{dt}\left(-\frac{|\Omega_2|}{\Delta}\right)_{|\Omega_2|=\Omega} - \frac{\dot{\Omega}\delta - \Omega\dot{\delta}}{\delta^2}$$

$$\Rightarrow \dot{\vartheta} = -\frac{\cancel{\Delta^2}}{\tilde{\Omega}^2}\,\frac{\dot{\Omega}\delta - \Omega\dot{\delta}}{\cancel{\Delta^2}} \quad \overset{!}{\ll} \Omega$$

we want: $|\dot{\vartheta}| \ll \tilde{\Omega}$

1) if $\underline{\dot{\Delta} \neq 0}$, $|\dot{\Omega}_2| = 0$:

$$|\dot{\Delta}| \ll \frac{\tilde{\Omega}^3}{\Omega} \quad \left(\overset{\min}{=} |\Omega_2|^2 \text{ for } \Delta = 0\right)$$

$$\boxed{|\dot{\Delta}| \ll |\Omega_2|^2}$$

(2) $\left(\dot{\Omega} \neq 0,\ \dot{\Delta} = 0\right)$

$$|\dot{\vartheta}| = \dot{\Omega}\,\frac{\Delta}{\tilde{\Omega}^2} \quad \Rightarrow |\dot{\Omega}|\,\frac{|\Delta|}{\tilde{\Omega}^2} \ll \tilde{\Omega}$$

easy to satisfy)

Example: Adiabatic passage

"rapid adiabatic passage" (RAP)

system is adiabatic $\Rightarrow$ $\langle\vec{\sigma}\rangle$ follows $\vec{\tilde{\Omega}}$!

change $\tilde{\Omega} \parallel \hat{z}$ to $\tilde{\Omega} \parallel -\hat{z}$ gives change of of population between $|e\rangle$ and $|g\rangle$.

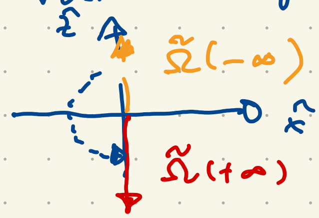
Four cases:

① $t \to -\infty$: $\Delta = -\infty$ and $\langle\vec{\sigma}(-\infty)\rangle \parallel -\hat{z}$ (i.e. $|g\rangle$)

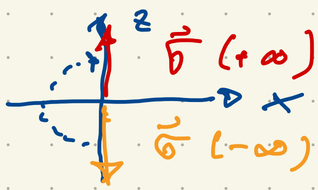
$$\Rightarrow \vec{\tilde{\Omega}}(-\infty) = -\Delta\hat{z} + |\Omega_R|\hat{x} \Rightarrow \tilde{\Omega} \parallel \hat{z}$$

$$\Rightarrow \langle\vec{\sigma}(-\infty)\rangle \parallel -\vec{\tilde{\Omega}}(-\infty) \quad \text{(anti-parallel)}$$

turn δ from $-\infty$ to $+\infty$ in time



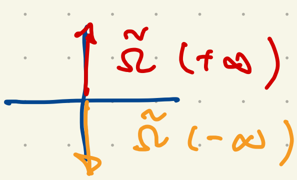
$\Rightarrow$

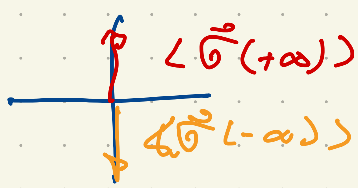


$|g\rangle \to |e\rangle$

② $t \to -\infty$: $\Delta = +\infty$, $\langle\vec{\sigma}(-\infty)\rangle \parallel -\hat{z}$ ($|g\rangle$)

$$\langle\vec{\sigma}\rangle \parallel \hat{\tilde{\Omega}} \rangle$$



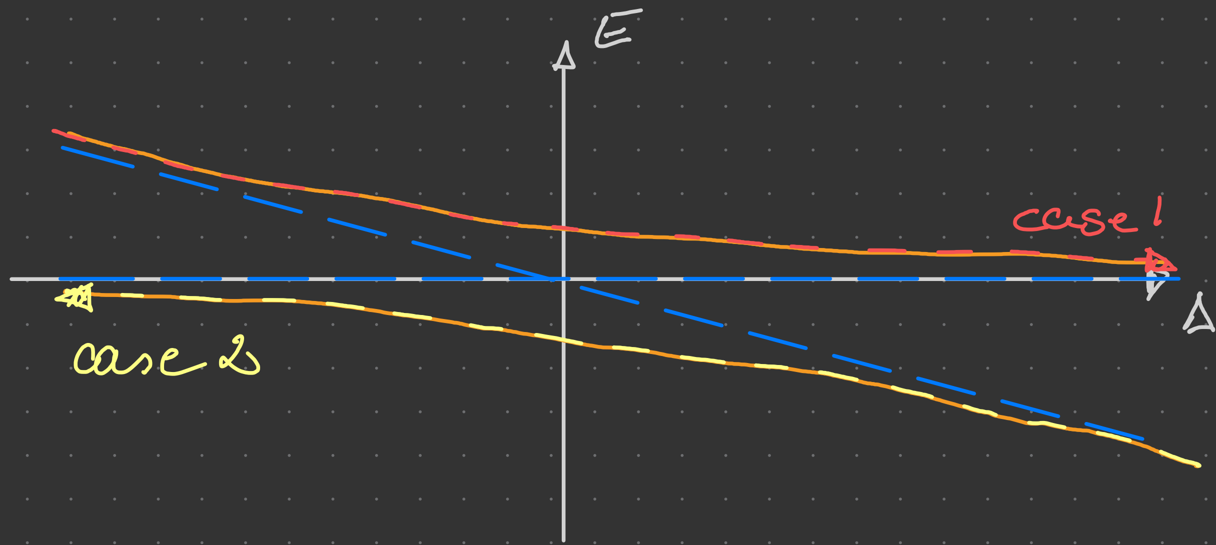


$|g\rangle \to |e\rangle$

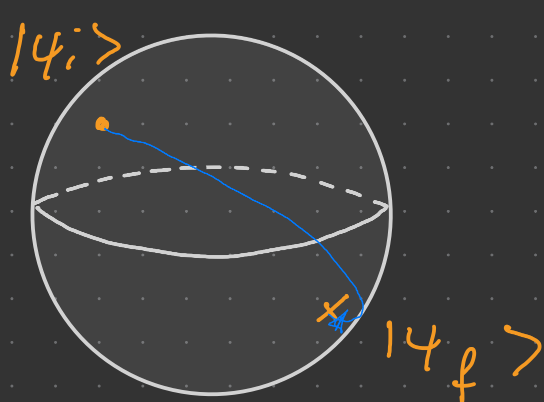
same evolution for $\langle\vec{\sigma}\rangle$!

③ + ④ $\langle\vec{\sigma}(-\infty)\rangle \parallel \hat{z}$

$\Rightarrow$ evolution $|e\rangle \to |g\rangle$

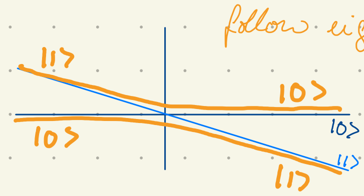


$\Rightarrow$ with a starting state <u>anywhere</u> on the Bloch sphere, RAP (i.e. sweep detuning Δ from one far detuned side to the other) leads to the state exactly opposite.



Landau - Zener crossing ← time - dependent detuning

follow eigenstate



$$i\dot{c}_e = -\frac{\Omega_R}{2} e^{-i\delta(t)t} c_g$$
$$i\dot{c}_g = -\frac{\Omega_R^*}{2} e^{i\delta(t)t} c_e$$

example: $\delta(t) = \alpha t$ (linear sweeps)

$$0 = \ddot{c}_{e/g} \pm 2i\alpha t\, \dot{c}_{e/g} + \frac{|\Omega_R|^2}{4} c_{e/g}$$

$$\tau \equiv \frac{|\Omega_R|}{2} t$$

$$\boxed{0 = c''_{e/g} \pm 2i\frac{\tau}{\Gamma} c'_{e/g} + c_{e/g}}$$

only scale is Γ

$$\boxed{\Gamma \equiv \frac{|\Omega_R|^2}{4\alpha}}$$

"Landau - Zener parameter"

(exact: Zener: Proc. Roy. Soc. London A, 137, 696 (1932).
approx: Vutha, arXiv: 1001.3322

Perturbation sol.: Γ small (i.e. change fast); $c_g(-\infty) = 1$

$$c'_{e/g} = i\, c_{g/e}\, e^{\mp i(\varphi - \frac{4\alpha}{|\Omega_R|^2}\tau^2)} \quad \left(\text{for } \frac{\Omega_R}{|\Omega_R|} = e^{i\varphi} \right)$$
$$= i\, c_{g/e}\, e^{-i(\varphi - \frac{\tau^2}{\Gamma})}$$

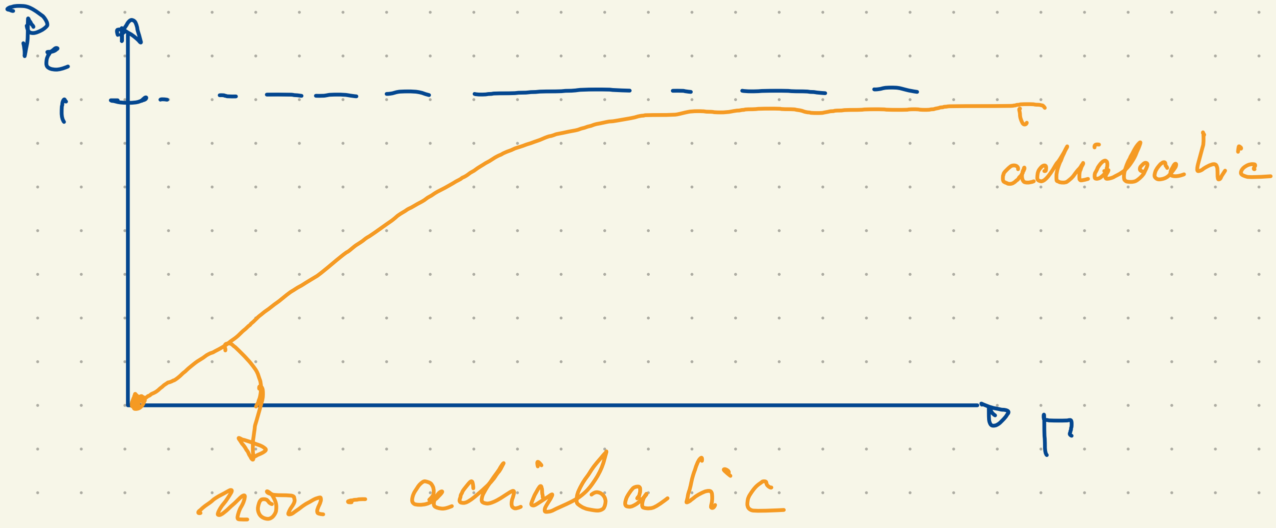
$$\Rightarrow c_e(\infty) = c_e(-\infty) + \int_{-\infty}^{\infty} d\tau \left(-i \underbrace{c_g(\tau)}_{\approx 1} e^{-i(\varphi - \frac{\tau^2}{\Gamma})} \right)$$

narrow Gaussian

$$c_e(\infty) \approx -i e^{-i\varphi_0} \sqrt{i\pi\Gamma}$$
$$P_e(\infty) = |c_e(\infty)|^2 \approx \pi\Gamma$$
$$\Rightarrow P_g(\infty) \approx 1 - \pi\Gamma$$

(educated guess: $P_g \approx e^{-\pi\Gamma}$; $\boxed{P_e \approx 1 - e^{-\pi\Gamma}}$)

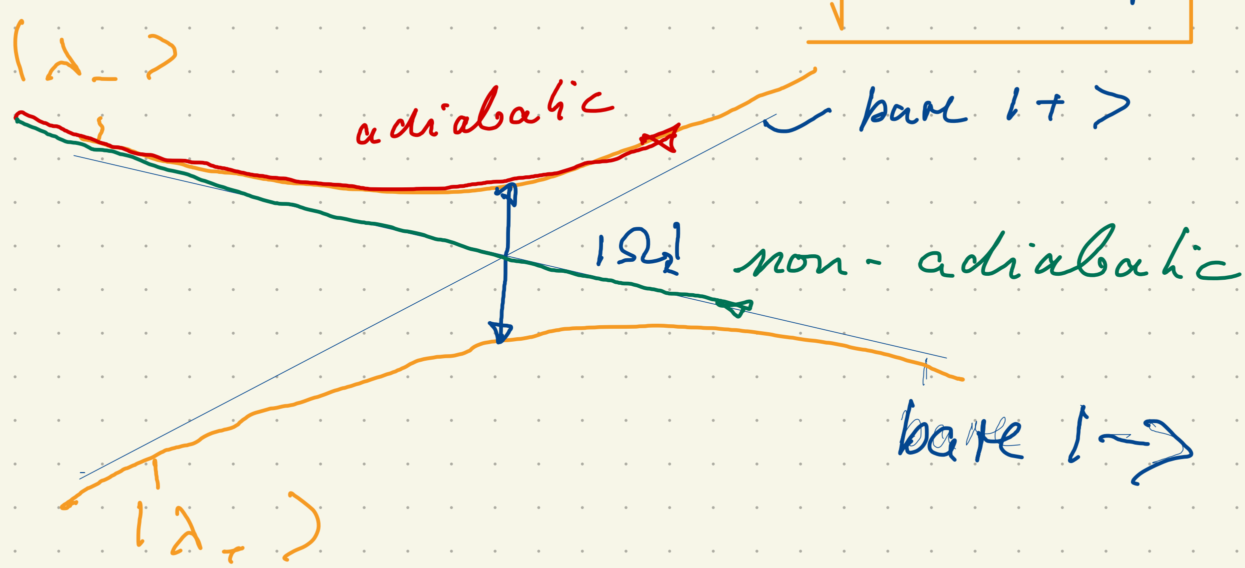
not Γ small



fast sweep (Γ small) $\Rightarrow$ no transition

slow sweep (Γ large) $\Rightarrow$ transition

$$|\dot{\delta}| = \underset{\substack{\| \\ \frac{|\Omega_R|^2}{4\Gamma}}}{|\alpha|} \ll |\Omega_R|^2 \Rightarrow \boxed{\Gamma \gg 1}$$



NB: "bare states" = eigenstates of H_0

"dressed states = eigenstates of $H = H_0 + V$