SPH4UI

Unit 1: Mechanics

Unit 1 Overview

Lesson	Text Reference	Topic	Questions	Done
Lab 1.1		Motion of a Pendulum	Lab Write-up & Discussion	
1	1.1 & 1.2	Motion in One Dimension	Pg 19 #5, Pg 21 #3, 7 W/S – Motion in One Dimension	
2	1.3 & 1.4	Vectors	W/S – Vector Questions	
3	1.6	Relative Motion	Pg 49 #1, 4, 5, 7 & 9 W/S – Additional Relative Motion Problem	
4	1.5	Projectile Motion	Pg 43 #4, 5, 7 & 9 W/S – Projectile Problems	
Lab 1.2		Designing a Projectile Launcher	Lab Write-up & Discussion	
5		Newton's Laws in 2D	Pg 79 P#1 & 3; Pg 81 P#1, 4, 5; Pg 83 Q#1, 2, 5; Pg 89 P#1, 3; Pg 90 Q#3; W/S – Newton's Laws in Two Dimensions	
6		The Inclined Plane	Pg 82 P#2, 3; Pg 83 Q#4, 6; Pg 89 P#2; Pg 90 Q#4, 7	
7		String & Pulley Problems	Pg 82 P#6; Pg 90 Q#6, 8 W/S – Pulleys and Planes	
Lab 1.3		Determining μ Using a Mass and Pulley System	Lab Write-up & Discussion	
8	3.2	Uniform Circular Motion	Pg 119 Q#1, 3, 6, 9, 10, 11	
9	3.3 & 3.4	The Centripetal Force	Pg 124 Q#1 – 4, 6; Pg 130 Q#6, 8 W/S – UCM & Centripetal Force	
Lab 1.1		Centripetal Force	Lab Write-up & Discussion	

Lesson 1 – Motion in One Dimension

As we begin our look at motion in one dimension, we need to recall some of the most important highlights learned in grade 11. Kinematics – the study of *how* motion happens – will be our focus as we work through several typical problems.

Recall that our five kinematic equations were all developed from a general velocity-time graph.

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \quad ①$$

$$\Delta\vec{d} = \tfrac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t \quad ②$$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 \quad ③$$

$$\Delta\vec{d} = \vec{v}_2\Delta t - \tfrac{1}{2}\vec{a}\Delta t^2 \quad ④$$

$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d} \quad ⑤$$

Solving Problems

1.

2.

3.

4.

Example 1

A marathon runner accelerates uniformly at 0.20 m/s^2 from an initial velocity of 3.0 m/s. How long will it take the runner to travel a distance of 12 m?

Example 2

A Jeep accelerates from rest at a rate of 4.0 m/s^2 for 10 s. It then travels at a constant velocity for 12 s and then finally comes to rest over a displacement of 100 m. Determine the Jeep's total displacement.

Two-Body Problems

Often we will encounter questions where two bodies experience different motion and we are asked to describe the result of their motion. These questions can be tricky, but adhering to a common coordinate system and wise sign conventions will help to simplify them.

Example 3

Clyde and Delilah are at opposite ends of a 1.0 km long hallway. Clyde accelerates from rest towards Delilah at 2.0 m/s^2, while Delilah travels toward Clyde at a constant velocity of 10 m/s. How much time passes before the two meet?

Chase Problems

A special type of two-body problem, chase problems involve one object following or chasing another. Often we will be asked if the objects ever meet, and in the event that they do not, we may need to solve for the *distance of closest approach* – how close they get to one another.

Example 4

A car travelling at a constant speed of 45.0 m/s passes a police car behind a billboard. One second after the speeding car passes the billboard, the police car sets out to catch it accelerating at a constant rate of 3.00 m/s^2. How long does it take until the police car catches the speeder?

Bodies in Free Fall

These types of questions usually involve an object accelerating under the influence of gravity. Typical examples are a ball thrown in the air or dropped from a certain height. The acceleration due to gravity is 9.8 m/s^2.

<u>Example 5</u>

A baseball is thrown straight up in the air, leaving the throwers hand at an initial velocity of 8.0 m/s. How high does the ball go and how long does it take to reach this height?

<u>Example 6</u>

A rock is thrown vertically upward from the edge of a cliff at an initial velocity of 10.0 m/s. It hits the beach below the cliff 4.0 s later. How far down from the top of the cliff is the beach?

Worksheet

Motion in One Dimension

1. A physics archer shoots an arrow vertically upwards beside a building 56 m high. The arrow's initial velocity is 37 m/s. If air resistance is negligible, at what time does the arrow pass the top of the building on its way up and down?

2. A stone is dropped from rest off of a cliff of height h. At the same time, a stone is thrown upwards from the base of the cliff with an initial velocity, v_i. Assuming the second stone is thrown hard enough, derive an expression for the time the two stones meet.

3. A body moving with constant acceleration has a velocity of 12.0 cm/s in the positive x direction when it's x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is the object's acceleration?

4. A jet plane lands with a speed of 100 m/s and can accelerate with a maximum rate of -5.00 m/s² as it comes to rest.
   (a) From the instant the plane touches the runway, what is the minimum time it needs before it can come to rest?
   (b) Can this plane land at a small tropical island airport where the runway is 0.800 km long?

5. A ball starts from rest and accelerates at 0.500 m/s² while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest.
   (a) What is the speed of the ball at the bottom of the first plane?
   (b) How long does it take the ball to roll down the first plane?
   (c) What is the acceleration along the second plane?
   (d) What is the balls speed 8.00 m along the second plane?

6. Another scheme to catch the Roadrunner has failed. A safe falls from rest from the top of a 25.0 m high cliff toward Wile E. Coyote, who is standing at the base. Wile first notices the safe after it has fallen 15.0 m. How long does he have to get out of the way?

7. A physics student (who happens to enjoy rock climbing) climbs a 50.0 m cliff that overhangs a calm pool of water. He throws two stones vertically downwards, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial velocity of 2.00 m/s.
   (a) How long after the release of the first stone do the two stones hit the water?
   (b) What was the initial velocity of the second stone?
   (c) What is the velocity of both stones as they hit the water?

8. Another physics student 'borrows' a sports car for a joy ride and discovers that it can accelerate at a rate of 4.90 m/s². He decides to test the car by challenging Mr. Horn and his motorcycle. Both start from rest, but the student is so confident in his new ride that he gives Mr. Horn a 1.00 s head start. If Mr. Horn moves with a constant acceleration of 3.50 m/s² and the student maintains his acceleration of 4.90 m/s², find:
   (a) the time it takes the student to overcome Mr. Horn.
   (b) the distance he travels before he catches up with Mr. Horn.
   (c) the speed of both vehicles at the instant the student overtakes Mr. Horn.

Answers: 1. 5.5 s, 2.1 s 2. $t = h/v_i$ 3. 16.0 cm/s² 4. a) 20.0 s b) No 5. a) 3.00 m/s b) 6.00 s c) -0.300 m/s² d) 2.05 m/s 6. 0.509 s 7. a) 3.00 s b) -15.3 m/s c) 31.4 m/s [down], 34.8 m/s [down] 8. a) 6.45 s b) 73.0 m c) 26.7 m/s, 22.6 m/s

Lesson 2 – Vectors

Recall from Grade 11 that when we add vectors, we are interested in finding the **resultant** – that is the sum of a series of vectors. This could be a velocity, displacement or force.

We always draw the resultant vector from the tail of the first vector to the tip of the last.

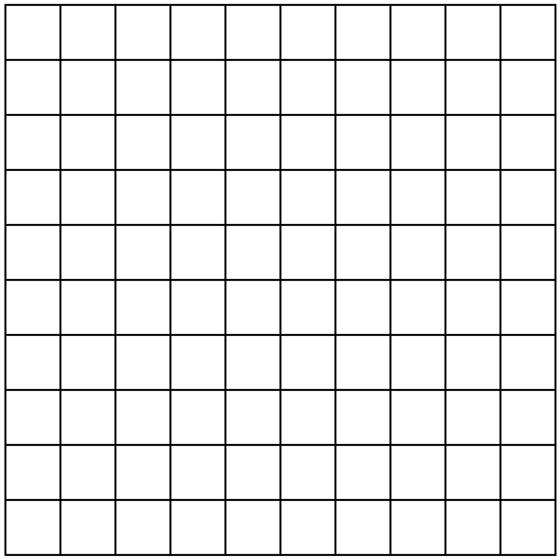
Most vector addition problems, like the one we are about to attempt, are drawn out and require a fully functional knowledge of trigonometry. We know also that we can always draw a scale diagram and use a protractor to solve the problem, but this method is often inaccurate, unreliable and time consuming.

Example 1

If $\vec{d}_1 = 0.5\,\text{km}\,[\text{N } 20^\circ \text{ E}]$, $\vec{d}_2 = 0.3\,\text{km}\,[\text{W}]$ and $\vec{d}_3 = 0.8\,\text{km}\,[\text{W } 50^\circ \text{ S}]$; find $\vec{d}_{\text{total}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$.

Algebraic Vectors

It will be to our advantage to represent vectors using algebra to simply reduce the amount of time required to solve vector problems. We will begin by investigating a general vector dropped into the Cartesian plane with its tail at the origin.



This vector can be represented in terms of a point, line segment, or an equation.

Unit Vectors

Unit vectors are unique and will become helpful if we want to discuss a general vector in terms of an equation. In two dimensions, there are two unit vectors.

Horizontal Direction		
Vertical Direction		

They both have a length of 1 and are perpendicular to each other. We can write any general vector as an equation including $\hat{i}$ and $\hat{j}$.

Good to Know
We don't have to limit our discussion of vectors to 2D. In three dimensions, the unit vector along the z-axis is called $\hat{k}$.

It is relatively easy to see, and not so tricky to prove that:

$$\left|\vec{v}\right| = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Example 2

What is the magnitude and angle of $\vec{v} = (-2,3)$? Express this vector as an equation using unit vectors.

Example 3

If $\vec{d}_1 = (2.0\hat{i} + 2.0\hat{j})$ m , $\vec{d}_2 = (2.0\hat{i} - 4.0\hat{j})$ m and $\vec{d}_3 = (-3.0\hat{i} + 5.0\hat{j})$; find $\vec{d}_{\text{total}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$.

Worksheet

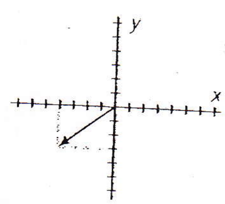
Vector Questions

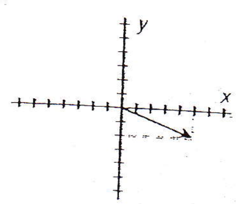
1. Find the x and y components for the following displacements:
   a. 20 km [S 30° E]
   b. 40 km [W 60° N]
   c. 10 m [N 10° E]
   d. 5 km [S 24° W]
   e. 12 km [N 45° W]

2. A student walks 20 m [N 20° E] then 120 m [N 50° W], then 150 m [W] and finally 30 m [S 75° E]. Find the student's final displacement using the geometric vector method.

3. Express the following vectors in their algebraic form.
   a) $|\vec{u}| = 12,\ \theta = 135^\circ$ b) $|\vec{v}| = 36,\ \theta = 330^\circ$
   c) $|\vec{w}| = 16,\ \theta = 190^\circ$ d) $|\vec{x}| = 12,\ \theta = 270^\circ$

4. Express the following vectors in their geometric form by stating their magnitude and direction.
   a) $\vec{u} = (-6\sqrt{3}, 6)$ b) $\vec{v} = (-4\sqrt{3}, -12)$
   c) $\vec{w} = (4, 3)$ d) $\vec{x} = (0, 8)$

5. What vector is represented in each of the following diagrams?
   a)


   b)


6. Find the magnitude and direction of the following vectors.
   a) $\vec{u} = (1, 7)$ b) $\vec{v} = (0, -6)$
   c) $\vec{w} = (-9, 12)$ d) $\vec{x} = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$
   e) $\vec{y} = (\frac{2}{\sqrt{5}}, -\frac{\sqrt{6}}{\sqrt{5}})$ f) $\vec{z} = (-\sqrt{6}, 0)$

Lesson 3 – Relative Motion

Consider the image shown. What does the image depict? Young woman or old lady?

Depending on your point of view, both answers are correct! Take another look. Each of you sees the world in a slightly different way – you all have your own point of view and this depends on your perception and past experience.

In physics, a **frame of reference** is

As it turns out all motion is relative. This is why we always must measure motion relative to a frame of reference. Consider the following rowboat example:

Both situations are mathematically and physically correct, and they serve as a perfect, simple example of one-dimensional relative motion. But it gets a bit more complex than this once we enter into two dimensions. We will use the following notation:

$\vec{v}_{\text{og}}$

$\vec{v}_{\text{mg}}$

$\vec{v}_{\text{om}}$

The best way to learn how all this fits together is to see a few examples of classic relative motion questions; the boat crossing a river, and air-navigation.

Example 1

A physics student wants to cross the Grand River. He hops into his canoe in Kitchener and paddles straight north towards Waterloo with a velocity of 5.0 km/h. If the Grand River is 5.0 km wide, how long does it take him to reach the other side?

Example 2

On another day, the student notices that there is a current flowing down the Grand River due east at 2.0 km/h towards Guelph.

a) How does the current affect the time required to cross the river?

b) How far is it from Waterloo to Guelph along the river's edge?

Example 3

On a third day of canoeing, the student decides to head to Waterloo instead of Guelph. If the conditions are the exact same conditions as in Example 2:

a) Find the direction the boat must be pointed in order to land in Waterloo.

b) What is the ground velocity of the boat?

c) How long will it take the boat to reach the other shore?

Example 4

The BAU team from Criminal Minds needs to fly in the FBI private jet due north from Washington D.C. to Belleville, ON in order to profile an UnSub. There is a wind from the west at 20.0 km/h. If the plane can fly at a velocity of 150 km/h in still air:

a) what is the plane's heading (in which direction should the pilot point the plane)?

b) What is the plane's ground velocity?

Worksheet

Additional Relative Motion Problems

1. The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground. **Ans: 147 km/h [W11.5°S]**

2. A person decides to swim across a river 84 m wide that has a current moving with a velocity of 0.40 m/s [E]. The person swims at 0.70 m/s [N] relative to the water.
   a) What is the velocity of the person with respect to the Earth? **Ans: 0.81 m/s [N 30° E]**
   b) How long will it take to cross the river? **Ans: 1.2×10^2 s**
   c) How far downstream will the person land? **Ans: 48 m**
   d) In what direction should the person swim if they want to land at a point directly north of where they started? **Ans: [N 35° W]**

3. A plane is travelling with a velocity relative to the air of 3.5×10^2 km/h [N 35° W] as it passes over Kitchener-Waterloo. The wind velocity is 62 km/h [S].
   a) Determine the velocity of the plane relative to the ground. **Ans: 3.0×10^2 km/h [N42°W]**
   b) Determine the displacement of the plane after 1.2 h. **Ans: 3.6×10^2 km [N 42° W]**

Lesson 4 – Projectile Motion

We will begin to investigate motion in two dimensions through an examination of projectile motion. Recall that a projectile is an object that moves under the influence of gravity only. Projectile motion is the combination of two motions: constant velocity motion in the horizontal direction and free-fall motion in the vertical direction.

Success Criteria for Solving Projectile Problems:

1) Draw a diagram of the situation, if necessary.
2) Obey the sign convention associated with the Cartesian coordinate system. It is often helpful to place the origin where the motion starts.
3) Resolve initial velocities into x and y components.

Example 1

A golf ball is launched from a rooftop with a velocity of 20 m/s at an angle of 30° above horizontal, from 45 m above the ground. Find the time of flight and the horizontal displacement (also called range).

<u>Example 2</u>

A Yukon rescue plane drops a package of emergency rations to a stranded party of explorers. If the plane is travelling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground, relative to where it was released?

Worksheet

Projectile Problems

1. A physics student has a pea-shooter that, when he exhales rapidly, can fire peas at an initial velocity of 18.0 m/s. If he holds the pea shooter at an angle of 24.0° above horizontal and 1.95 m above the ground, calculate:
   a) the time spent by the pea in the air.
   b) the range of the pea.
   c) the maximum height of the pea.

2. A pitcher throws an overhand fastball from an approximate height of 2.65 m and at an angle of 2.5° **below** horizontal. The catcher, catches the ball high in the strike zone, at a height of 1.02 m above the ground. If pitcher's mound and home plate are 18.5 m apart, what is the initial velocity of the pitch?

3. A cannon having a muzzle speed of 1000 m/s is used to destroy a target on a mountaintop. The target is 2000 m from the cannon horizontally and 800 m above the ground. At what angle, relative to the ground, should the cannon be fired?

4. On a field goal attempt, a kicker must kick a football from a spot 36.0 m away from the goal line and the ball must clear the cross bar 3.05 m high. When kicked, the ball leaves with an initial velocity of $(12\hat{i} + 16\hat{j})$ m/s.
   a) By how much does the ball clear or fall short of the cross bar?
   b) Does the ball approach the crossbar while still rising or while falling?

Answers:
1. a) 1.72 s b) 28.3 m c) 4.68 m 2. 45.2 m/s 3. 22.4° 4. a) Clears by 85 cm b) Falling

Lesson 5 – Newton's Laws in 2 Dimensions

Dynamics may be referred to as the *why* of motion because it attempts to explain the cause of motion rather than the effect of motion. There are certain important definitions important to our study of dynamics:

Force –

Gravity –

Free Body Diagram –

Newton's Laws, as you will recall, are a set of three rules that govern classical motion. They are:

Newton's First Law:

Newton's Second Law:

Newton's Third Law:

Armed with knowledge of these 'rules' we can describe forces such as friction, the normal force and gravity, which were the building blocks of our conceptual understanding of motion. Recall:

Normal Force –

Static and Kinetic Friction –

Coefficient of Friction –

We will now investigate a few problems involving Newton's Laws and their solutions. Make note of the fact that we will use algebraic vectors, and as always, draw free body diagrams.

Example 1
A barge is being pulled through a canal by two horses, as shown in the diagram below. If each horse applies a force of 5000 N, determine the frictional force applied by the water as the barge moves at a constant speed.

Example 2
Mr. Wharf accidentally fires two rockets on his shuttlecraft at the same time. The first rocket applies a force of 1000 N [E 25° S], and the second rocket applies a force of 1200 N [N 40° W]. If the shuttlecraft has a mass of 5.0×10^4 kg, determine the acceleration it will experience.

Example 3

Two blocks of masses 5.0 kg and 2.0 kg are placed in contact with each other on a frictionless, horizontal surface. A constant horizontal force is applied to the 5.0 kg mass. Find the acceleration of the blocks and the magnitude of the contact forces between the blocks.

Example 4

A hockey puck on a frozen pond is hit and given an initial velocity of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and the ice.

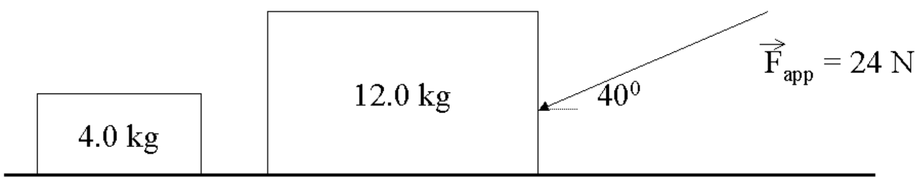
Worksheet

Newton's Laws in Two Dimensions

1. A force of 3.5 N [N 60° E] and a force of 2.8 N [S 40° W] act on the same object. Find the net force acting on the object using algebraic vectors.

   Ans: (1.23*i* – 0.39*j*) N

2. A 12.0-kg box is pushed along a horizontal surface by a 24-N force as illustrated in the diagram. The frictional force (kinetic) acting on the object is 6.0 N.



   (a) What is the acceleration of the object? **Ans: 1.0 m/s² *i***

   (b) Calculate the value of the normal force acting on the object.

   Ans: 1.3 × 10²N *j*

   (c) If the 12.0-kg object then runs into a 4.0-kg object that increases the overall friction by 3.0 N, what is the new acceleration? **Ans: 0.59 m/s² *i***

   (d) What force does the 4.0-kg object exert on the 12.0-kg object when the two are moving together? **Ans: 5.3 N**

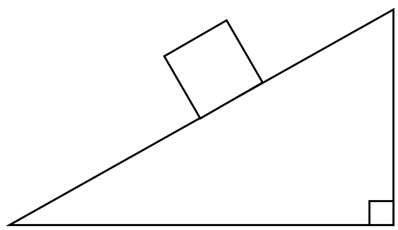
3. A spider builds its web in a window frame that is lying on the ground. It is supported by four main strands. Calculate the force of tension in strand 4 assuming the web is stable. The tensions in the other three strands are as follows:

   strand 1: 21 mN [N 20° E]
   strand 2: 16 mN [S 60° E]
   strand 3: 18 mN [S 40° W]

   Ans: (9.5*i* – 2.1*j*) mN

Lesson 6 – The Inclined Plane

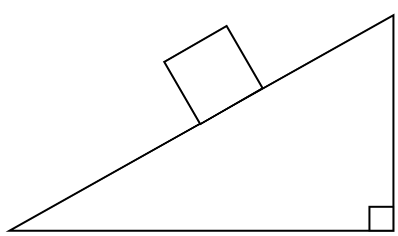
These types of problems are very common due to the fact that many real life situations involve objects sliding down a hill, ramp or some sort of incline. The key to solving them is to rotate our coordinate system so that it lines up with the acceleration of the sliding object. Lets consider, for a moment, a standard inclined plane:



The motion (or specifically, the acceleration) of the block is in the direction of the plane, along the length of the hypotenuse. For this reason, we can reorient our coordinate system to follow the motion of the block.

We need to resolve the forces in this type of problem with respect to the new coordinate system. One reason for doing so is that the normal force is no longer equal to the force of gravity.

Resolving Gravity
To do this, we'll need a diagram and some basic geometry:



It would be ideal if we could use the angle of the incline to resolve our vectors.
To do so, we need to prove that $\alpha = \beta$:

And now we can resolve the vectors:

Example 1

A student sits at the top of a frictionless, snow-covered hill. Find the student's acceleration if the angle of incline of the hill is 25°.

Example 2

A 20.0 kg crate slides down an inclined plane with a coefficient of friction of 0.30. If the hill is at an angle of 30°, what is the acceleration of the crate?

Lesson 7 – String and Pulley Problems

String and pulley problems usually consist of two or more masses, a string and one or more pulleys. They range from being very straight forward, to extremely complex.

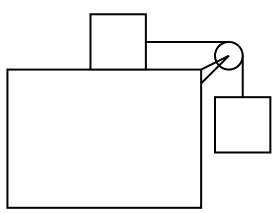
In order to solve these types of problems, we must construct free body diagrams for each object being considered, determine the direction of motion of each object and then write $\vec{F} = m\vec{a}$ for each object.

A few assumptions for all string and pulley problems in Grade 12 Physics:

- ✓ Pulleys are always frictionless and have zero mass.
- ✓ Strings are infinitely strong, have no mass and do not stretch.

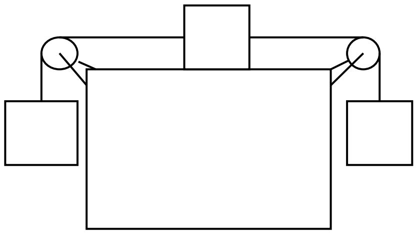
Example 1

Two 5.0 kg masses are connected as shown in the diagram below. Determine the tension in the string and the acceleration of the system if the tabletop is frictionless.



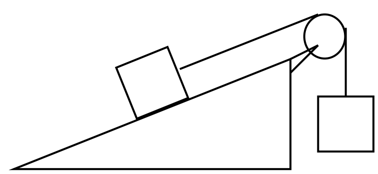
Example 2

For the system of masses and pulleys shown below, solve for the tension in each string and the acceleration of the system if $m_1 = 10$ kg, $m_2 = 5$ kg, and $m_3 = 2$ kg.



Example 3

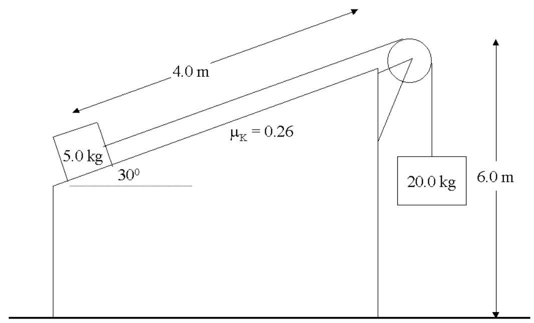
For the system of masses and pulleys shown below, solve for the tension in the string and the acceleration of the system if $\theta = 20°$, $m_1 = 2$ kg, $m_2 = 15$ kg, and the coefficient of friction between the plane and the m_1 is 0.2.



Worksheet

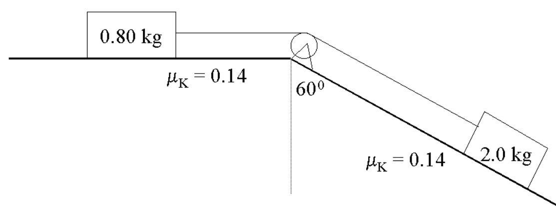
Pulleys & Planes

1. A pulley device is used to hurl projectiles from a ramp ($m_k = 0.26$) as illustrated in the diagram. The 5.0-kg mass is accelerated from rest at the bottom of the 4.0 m long ramp by a falling 20.0-kg mass suspended over a frictionless pulley. Just as the 5.0-kg mass reaches the top of the ramp, it detaches from the rope (neglect the mass of the rope) and becomes projected from the ramp.



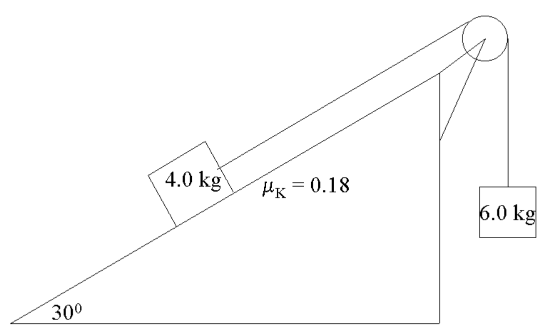
   (a) Determine the acceleration of the 5.0-kg mass along the ramp. (Provide free-body diagrams for both masses.)
   (b) Determine the tension in the rope during the acceleration of the 5.0-kg mass along the ramp.
   (c) Determine the speed of projection of the 5.0-kg mass from the top of the ramp.
   (d) Determine the horizontal range of the 5.0-kg mass from the base of the ramp.

2. A massless string connects two blocks over a frictionless pulley, as shown in the diagram.



   (a) Determine the acceleration of the blocks.
   (b) Calculate the tension in the string.
   (c) If the string broke, for what minimum value of the coefficient of static friction would the 2.0-kg block not begin to slide?

3. Two masses, 4.0 kg and 6.0 kg, are connected by a "massless" rope over a "frictionless" pulley as pictured in the diagram. The ramp is inclined at 30.0° and the coefficient of kinetic friction on the ramp is 0.18.



   (a) Draw free-body diagrams of both masses.
   (b) Determine the acceleration of the system once it begins to slide.
   (c) Determine the tension in the rope.
   (d) If the rope breaks when the 4.0-kg mass is 3.0 m from the bottom of the ramp, how long will it take for the mass to slide all the way down? Include a new free-body diagram and assume the sliding mass starts from rest.

Answers: 1. a) 6.4 m/s² b) 68 N c) 7.2 m/s d) 9.5 m 2. a) 2.3 m/s² b) 2.9 N c) 0.58 3. b) 3.3 m/s² c) 39 N d) 1.3 s

Lesson 8 – Uniform Circular Motion

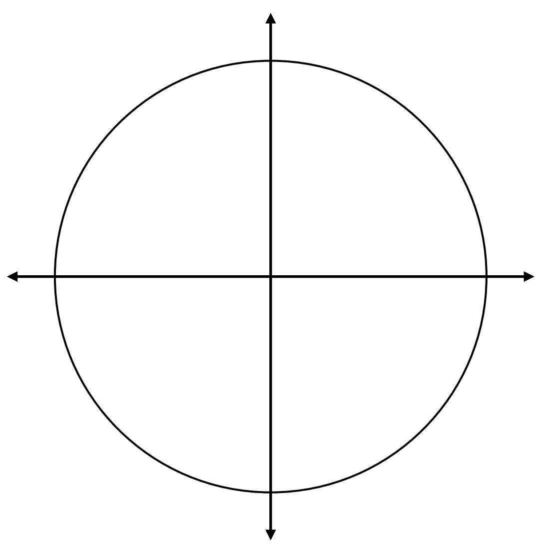
A special kind of two-dimensional motion occurs when an abject moves in a circular path.

Uniform Circular Motion

Centripetal Acceleration

As we defined above, motion in a circle involves a changing direction, which implies an acceleration. This type of acceleration is called **centripetal acceleration** and is always directed towards the middle of the circle, as we shall see.

Lets take a closer look at the path of an object in UCM:



The directions of $\vec{v}_1$ and $\vec{v}_2$ are different, which tells us that acceleration happens.

We'll start by looking at the magnitude of the velocity, the speed:

And now look at the vector notation for $\vec{v}_1$ and $\vec{v}_2$:

Substitute this into our most basic equation for acceleration:

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

The negative sign here indicates the direction of the acceleration, which for small θ would be the $-y$ direction in this case.

In fact, we could duplicate this derivation for any two velocities along the circumference of the circle and we would find something very interesting – the acceleration is always directed towards the centre of the circle!

This is the nature of centripetal acceleration. It always points towards the centre of the circular motion.

It will also be useful to derive centripetal acceleration in terms of period and frequency – we'll turn our attention to this now.

Example 2

A racecar enters a circular curve of radius 30 m at a constant speed of 25 m/s. Determine the car's centripetal acceleration.

Example 3

Space stations can produce *artificial gravity* by rotating. A space station built in the shape of a bicycle wheel has a diameter of 500 m. How many times each day should the station rotate for an astronaut to experience acceleration equal to that of Earth?

Lesson 9 – Centripetal Force

Newton's 2nd Law tells us that any object undergoing acceleration, must be experiencing an unbalanced force. The same is true in the case of uniform circular motion – whenever an object travels in a circle at a constant speed, it must have a force acting on it which is perpendicular to its velocity.

We can now write

$$\vec{F}_{\text{net}} = m\vec{a} = m\frac{v^2}{r}$$

Example 1

A 0.20 g flea sits a distance of 5.0 cm from the centre of a rotating LP record. If the record rotates at 77 rpm, what centripetal force must be provided by friction to cause the flea to maintain its uniform circular motion?

Example 2

A 0.25 kg mass is hanging from a 1.0 m massless string. If the mass is spun in a vertical circle at 5.0 m/s, determine the tension in the string at the top and bottom of the circle.

Banked Curves

When a car travels around a curve it experiences centripetal acceleration, as do all other objects. In general, the friction between the road and the tires is enough to prevent the car from slipping. (Think of the flea-on-a-record example above.)

Engineers, however, do not trust in friction. Nor should they. Instead, they play it safe and use banked curve. By banking a curve with respect to the horizontal, we can lessen the amount of friction necessary, because the normal force will help out.

Example 3

A racecar travels along a banked curve at a speed of 120 km/h (33.3 m/s). It does not depend on the force of friction to keep it on the track. If the turn is banked at an angle of 25° to the horizontal, what is the radius of rotation?

Centrifugation

In chemistry you may have needed to separate one material from another. Likely you have left a substance to stand for a long period of time – the force of gravity is doing the work here in a process called sedimentation.

This process, however, takes a long time. Often a device known as a centrifuge will be used to separate substances suspended in a liquid by spinning a sample of liquid very quickly.

Satellites

Like the force of tension in the string and mass example, Earth's gravitational force can used to keep satellites in orbit. Geosynchronous satellites (those that stay in the same spot in the sky) travel under the force of gravity and stay in uniform circular motion at tens of thousands of kilometers above Earth's surface.

Example 4

The *Anik* F1 satellite has a mass of 3021 kg. How high above the equator must the satellite be in order to maintain geosynchronous Earth orbit? Earth's period is 23 hours, 56 minutes and 4 seconds. It has a mass of 5.98×10^{24} kg and a radius of 6.38×10^{6} m.

Worksheet — Uniform Circular Motion & Centripetal Force

1. An object of mass 6.0 kg is whirled around in a vertical circle on the end of a 1.0 m long string with a constant speed of 8.0 m/s. Include a free-body diagram for each of the following questions:
   (a) Determine the maximum tension in the string, indicating the position of the object at the time the maximum tension is achieved.
   (b) What is the minimum speed the object could be rotated with and maintain a circular path?
   (c) If the object is rotated with the same speed (8.0 m/s) on a horizontal surface, what is the tension in the string if the string is parallel to the surface?

2. A 0.50-g insect rests on a compact disc at a distance of 4.0 cm from the centre. The disc's rate of rotation varies from 3.5 Hz to 8.0 Hz in order to maintain a constant data-sampling rate.
   (a) What are the insect's minimum and maximum centripetal accelerations during its rotation around the disc?
   (b) What is the minimum value of the coefficient of static friction that would prevent the insect from slipping off the disc at the slowest rotation rate?

3. A pilot of mass 75 kg takes her plane into a dive, pulling out of it along a circular arc as she nears the ground. If the plane is flying at 1.5×10^2 km/h along the arc, what is its radius such that the pilot feels four times heavier than normal? Provide an appropriate free-body diagram.

4. A rock of mass 4.0×10^2 g is tied to one end of a string that is 2.0 m in length. Holding the other end above his head, a boy swings the rock around in a circle whose plane is parallel to the ground.
   (a) If the string can withstand a maximum tension of 4.5 N before breaking, what angle to the vertical does the string reach just before breaking?
   (b) At what speed is the rock travelling just as the string breaks?

5. A flea stands on the end of a 1.0 cm long sweep second hand of a clock that rests horizontally on a table. What is the minimum coefficient of static friction that would allow the flea to stay there without slipping? Include an appropriate free-body diagram.

Answers: 1. a) 4.4×10^2 N [upward] b) 3.1 m/s c) 3.8×10^2 N 2. a) 19 m/s^2, 1.0×10^2 m/s^2 b) 2.0 3. 59 m 4. a) 29° b) 2.3 m/s 5. 1.1×10^{-5}