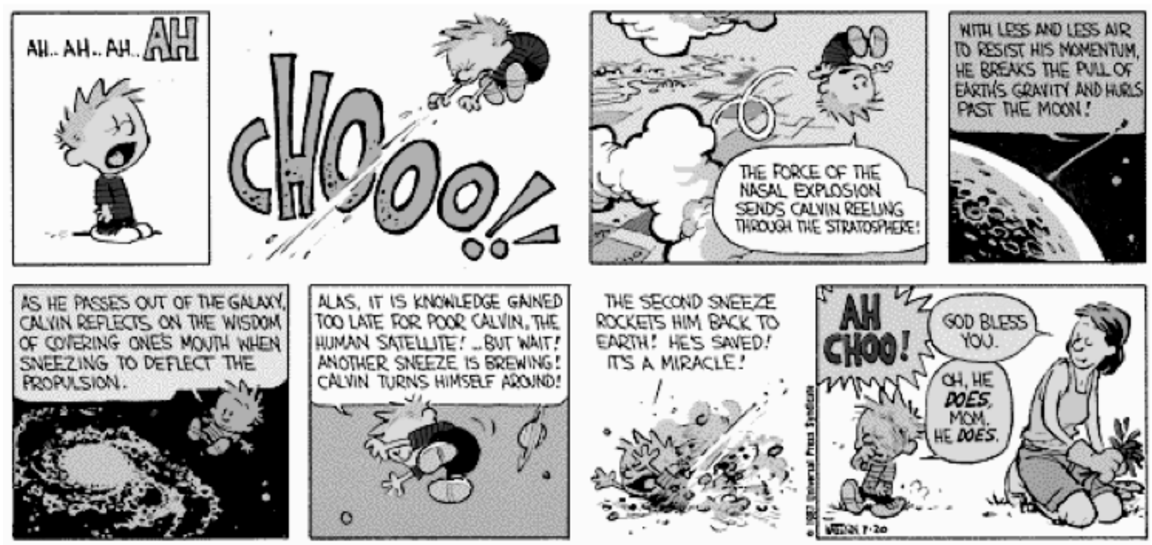

SPH4UI

Unit 2: Energy & Momentum

Unit 2 Overview

Lesson	Text Reference	Topic	Questions	Done
1	5.1	Impulse & Linear Momentum	Pg 227 Q# 4, 8, 10 – 12	
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3	5.2	Conservation of Momentum 2	W/S – Conservation of Momentum in 2 Dimensions	
Lab 2.1		Using Projectiles to Prove the Conservation of Momentum	Lab Write-up & Discussion	
4	4.1	Work & Power	Pg 169 Read Tutorial 4, P#1, 2; Pg 170 Q#3 – 6	
5	4.2, 4.3, 4.6	Types of Energy	Pg 176 Q#3 – 5, 8, 10, 12; Pg 181 Q#3, 5 – 7; Pg 200 Q#4, 5, 7	
Lab 2.2		Hooke's Law	Lab Write-up & Discussion	
6	4.5, 4.7	Conservation of Energy	Pg 191 Q#5, 6, 7; Pg 208 Q#4, 7, 8 W/S – Conservation of Energy Problems	
7	5.3, 5.4. 5.5	Collisions	Day 1: Pg 239 Q#2 – 4, 6 & Pg 248 Q#4 – 6 Day 2: Pg 253 Q#1, 6, 7 & W/S - Collisions	
Lab 2.3		1D Collisions Using a LAT	Lab Write-up & Discussion	

Lesson 1 – Linear Momentum & Impulse

The concept of linear momentum was first developed by Newton – he correctly hypothesized that a change in momentum was caused by a force.

Linear Momentum is a vector quantity that has the same direction as the velocity of the object. It is directly proportional to mass and velocity. It is given by the equation

$$\vec{p} = m\vec{v}$$

Where $\vec{p}$ is the object's linear momentum in kg·m/s, m is the mass of the object in kg, and $\vec{v}$ is the velocity in m/s.

Example 1

Calculate the momentum of a 50 g bullet traveling at $200\,\hat{i}$ m/s.

Impulse

If we want to *change* the momentum of an object, we either need to alter its mass or its velocity. But to change the velocity of an object (acceleration), there needs to be a force applied to it. Newton suggested that the rate of change of momentum is directly proportional to the force applied to it.

The change in linear momentum is called **impulse** and is given the symbol $\vec{J}$. Mathematically:

Example 2

The average accelerating force exerted on a 2.00 kg shell in a gun barrel is 1.00×10^4 N, and the muzzle velocity is 200 m/s. Calculate the impulse and the length of time it takes for the shell to exit the barrel.

Example 3

What velocity will a 300 kg snowmobile acquire if pushed from rest by a force of 6240 N [E] for 1.25 s? What average force will stop this snowmobile from moving at this speed in 1.25 s?

Example 4

A 54 kg truck tire strikes the pavement with a velocity of 25 m/s [down] and rebounds with a velocity of 20 m/s [up]. Ignoring any effects due to air resistance, determine the change in the tire's momentum.

Example 5

The impulse on a human cannon ball is 2.5×10^3 N · s. The cannon ball has a mass of 65 kg. What force does the cannon exert on the cannon ball if it takes 0.2 s for the ball to leave the cannon? How long is the barrel of the cannon if the ball leaves with a velocity of 120 km/h?

Lesson 2 – Conservation of Momentum 1

In this lesson we will consider the conservation of momentum in one dimension and next day turn our attention to the same subject matter, but in two dimensions.

As with the conservation of energy, momentum is conserved; it cannot be created or destroyed. In any collision, as we will see in a few days, the total momentum before the collision is equal to the total momentum after the collision. This concept is known as the **conservation of momentum** and it holds true for any closed system (the net external force is zero). A **system** is defined as all the objects involved in a collision.

Mathematically, we express the conservation of momentum as:

And graphically:

The conservation of momentum has numerous real-life examples:

Example 1

A shell of mass 7.0 kg leaves the muzzle of the cannon with a velocity of 490 m/s in the positive x direction. Find the recoil velocity of the cannon if its mass is 700 kg.

Example 2

An arrow flying at 60 m/s $\hat{i}$, strikes and imbeds itself in a 300 g apple at rest. After impact, the apple and arrow move horizontally at 12 m/s $\hat{i}$. What is the mass of the arrow?

Lesson 3 – Conservation of Momentum 2

In two dimensions, the conservation of momentum is no more difficult if we use an algebraic vector approach to solve the problems. This lesson we will solve two useful examples that will act as a guide for solving these types of problems.

Example 1

Two identical curling stones of mass 19.5 kg collide. The first stone hits the stationary second stone with a constant velocity of 5.0 m/s [N]. If the velocity of the first stone is 3.2 m/s [N 30° W] after the collision, find the velocity of the second stone after the collision. Assume that effects due to friction are negligible.

Example 2

A 5.0 kg bomb at rest explodes into three pieces, each of which travels parallel to the ground. The first piece, with a mass of 1.2 kg, travels at 5.5 m/s at an angle of 20° south of east. The second piece has a mass of 2.5 kg and travels 4.1 m/s at an angle of 25° north of east. Determine the velocity of the third piece.

Lesson 4 – Work

Work is the transfer of energy. Work is done when a force acts on an object causing the object to move in the direction of the force. It is a scalar quantity and is represented by the dot product of two vectors:

$$W = \vec{F} \cdot \Delta\vec{d}$$

It has units of Newton-metres, which is defined as a Joule.

Example 1

Calculate the work done by

a) applying a force of 20 N [right] to a 0.5 kg puck as it slides along a frictionless surface from rest to 10 m/s in 0.2 s.

b) lifting a 57 kg outboard motor a distance of 1.4 m from the ground up to the box of a pickup truck.

Example 2

A horse pulling a sleigh exerts a force of 100 N at an angle of 30° to the ground. The sleigh moves 30 m across a horizontal ice surface (no friction). Find the work done by the horse.

Using algebraic vectors, we note that the dot product took care of resolving Force and displacement into its proper components. And further, it only calculated the amount of force acting in the direction of displacement.

Work When Forces Act at an Angle (an alternate method)
Recall that the definition of the dot product is

$$\vec{u} \cdot \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

This definition can be readily applied to our definition of work:

$$W = \vec{F} \cdot \Delta\vec{d} = \left|\vec{F}\right|\left|\Delta\vec{d}\right|\cos\theta$$

Where $\boldsymbol{F}$ is the magnitude of the applied force, $\boldsymbol{\Delta d}$ is the distance travelled and $\boldsymbol{\theta}$ is the distance between the applied force and the distance.

Example 3
A newspaper carrier pulls a wagon with a force of 275 N at an angle of 45.0° to the horizontal. How much work is required to move the wagon 8.00 m?

Work Done By Friction
The force of friction also does work. Its action however is to remove energy from a system. For this reason, friction is known as a non-conservative force and if not considered, it will render the conservation of energy incomplete and incorrect.

Example 4
A 6.0 kg block is pulled to the right along a horizontal surface with a coefficient of friction of 0.15 by a horizontal force of 12 N for a total distance of 3.0 m. How much energy has been transferred to the block in this distance?

Power

Power is defined as the rate of energy transfer. The equation for power is given by:

$$P = \frac{W}{\Delta t}$$

Example 4
What power is required for a ski-hill chair lift that transports 500 people (average mass 65 kg) per hour to an increased elevation of 1200 m?

Lesson 5 – Types of Energy

We know that when a force F, is applied to an object over a distance d, the velocity of the object changes. We've seen this in real life and in the physics classroom. We know this is true because work is being done on the object.

Work, then, is the process of transferring energy from the force to the object, in the form of motion.

$$W = \Delta E_{\mathrm{k}}$$

This is an expression that gives us the relationship between work and kinetic energy. **Kinetic energy** on its own is expressed as:

$$E_{\mathrm{k}} = \tfrac{1}{2}mv^2$$

And is defined as the energy associated with the motion of an object.

Example 1
Calculate the kinetic energy of a 2 kg mass with a velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

Potential energy, as you are aware, is energy stored in a system that is available to do work. Potential energy takes many forms; we will concentrate on two specific types – gravitational potential energy and elastic potential energy.

Gravitational Potential Energy
Earth's gravitational field provides potential energy to objects due to it's associated force. We can derive an expression for gravitational potential energy:

Example 2

A basketball player at the free throw line shoots a 624 g ball at 7.2 m/s from 2.21 m above the ground. The hoop is 3.05 m above the ground. How much potential energy does the ball gain as is leaves the players hands and travels to the basket?

Hooke's Law

Robert Hooke developed a law for elasticity that is used when dealing with springs. Plainly, it states:

The restoring force of a spring is $\vec{F} = k\vec{x}$, where k is the spring constant.

This tells us that the force required to stretch or compress a spring is related in a linear fashion to the distance you try and stretch or compress it. Whether you pull out on an expansion spring or press in on a compression spring, the effect is always the same: the spring tries to restore itself to its original length.

It is important to note that the spring force is, as all forces are, a vector. But it is a vector that acts in different directions depending on its state. Remember that the direction of the spring force always acts to restore it's original length.

Example 3

A block of mass 4 kg is placed next to a spring with a spring constant of 200 N/m on a flat, horizontal, frictionless surface. The block is then pushed against the spring and compresses it a distance of 0.05 m. Find the acceleration of the block the instant it leaves the spring.

Elastic Potential Energy

Using a graph of Hooke's Law, $\boldsymbol{F} = k\boldsymbol{x}$, we can derive an expression for the potential energy stored in a spring:

Good to Know

Robert Hooke was an industrious scientist to say the least. Not only is he remembered as the "father of microscopy", he is credited as the person who coined the term "cell" in biology.

He worked with Robert Boyle (another extremely influential scientist) on the famous Boyle's gas law experiments and was also an important architect of his time. It was through his work in this field that he discovered his law of elasticity.

Example 4

In the following diagram, a frictionless metal block of mass 5.0 kg slides at a speed of 6.0 m/s into a fixed spring bumper with a spring constant of 720 N/m. The block eventually comes to rest. How much work does the spring do? How much does it compress?

Lesson 6 – Conservation of Energy

The conservation of energy is a well-known law in mechanics. Simply put, it states that energy cannot be created nor destroyed, but rather transferred from one form to another. We are going to investigate the conservation of energy taking both conservative and non-conservative forces into consideration.

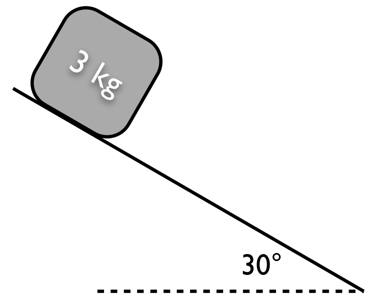
	Definition/Properties	Examples
Conservative Forces		
Non-conservative Forces		

Example 1

A ball of mass 0.500 kg is thrown downwards at a speed of 2.5 m/s from a height of 70.0 m. Neglecting air resistance, determine the velocity of the ball at a height of 22.0 m.

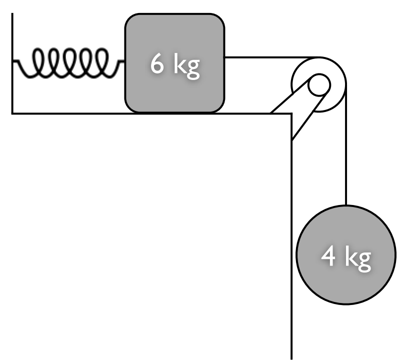
Example 2

A 3.00 kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in the diagram. The crate starts from rest at the top and experiences a constant frictional force of 5.00 N. Use energy methods to determine the speed of the crate at the bottom of the ramp.



Example 3

Two blocks are connected as shown in the diagram to the right. The block of mass 6.00 kg lies on a horizontal surface and is connected to a spring with spring constant 1.00×10^2 N/m. The system is released from rest when the spring is unstretched. If the hanging block of mass 4.00 kg falls a distance of 30.0 cm before coming to rest, calculate the coefficient of friction between the block of mass 6.00 kg and the surface.



Worksheet

Conservation of Energy

1. A toy gun fires a 94.1-g projectile disc by using a compressed spring (k = 1.72 $\times 10^3$ N/m) and a 13.1 cm long barrel. As the disc travels through the barrel, it experiences a constant frictional force of 0.13 N. If the spring is compressed 14 mm, what is the speed of the disc as it leaves the gun?

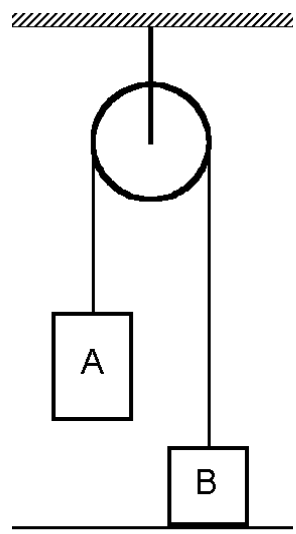
2. A 1.2-kg block is dropped from 48 cm above a spring in equilibrium. The force constant for the spring is 124 N/m. Calculate the maximum compression in the spring.

3. A 1.00-kg mass and a 2.00-kg mass are set gently on a platform mounted on an ideal spring of force constant 40.0 N/m. The 2.00-kg mass is suddenly removed. How high above its starting position does the 1.00-kg mass reach?

4. A spring is suspended from a ceiling and a 256-g mass is attached to it and pulled down to stretch the spring by 18.2 cm. The mass is released and travels through the equilibrium position with a speed of 0.746 m/s. Calculate the force constant of the spring.

5. Two boxes each of mass 12 kg are raised 1.8 m to a shelf. The first one is lifted and the second is pushed up a smooth ramp. If the applied force on the second box is 48 N, calculate the angle between the ramp and the ground.

6. Two boxes are connected over a pulley and held at rest as shown below. Box A has a mass of 15 kg and box B has a mass of 12 kg. If the bottom of box A is originally 85 cm above the floor, with what speed will it contact the floor when the system is released? Use conservation of energy and assume that friction is negligible.



Answers: 1. 1.8 m/s 2. 41 cm 3. 1.10 m 4. 31.9 N/m 5. 24° 6. 1.4 m/s

Conservation of Energy – Problems

17E. In Fig. 8-34, a runaway truck with failed brakes is moving downgrade at 80 mi/h just before the driver has the truck travel up an emergency escape ramp with an inclination of 15°. What minimum length L must the ramp have if the truck is to stop (momentarily) along it? Why are real escape ramps often covered with a thick layer of sand or gravel?

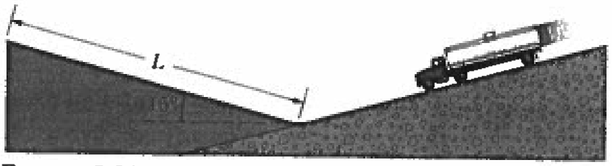


FIGURE 8-34 Exercise 17.

19P. A 1.50 kg water balloon is shot straight up with an initial speed of 3.00 m/s. (a) What is the kinetic energy of the balloon just as it is launched? (b) How much work does the weight of the balloon do on the balloon during the balloon's full ascent? (c) What is the change in the gravitational potential energy of the balloon–Earth system during the full ascent? (d) If the gravitational potential energy is taken to be zero at the launch point, what is its value when the balloon reaches its maximum height? (e) If, instead, the gravitational potential energy is taken to be zero at the maximum height, what is its value at the launch point? (f) What is the maximum height of the balloon?

23P. A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change in the gravitational potential energy of the marble–Earth system during the 20 m ascent? (b) What is the change in the elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the spring?

25P. A 2.00 kg block is placed against a spring on a frictionless 30.0° incline (Fig. 8-37). The spring, whose spring constant is 19.6 N/cm, is compressed 20.0 cm and then released. (a) What is the elastic potential energy of the compressed spring? (b) What is the change in the gravitational potential energy of the block–Earth system as the block moves from the release point to its highest point on the incline? (c) How far along the incline is the highest point from the release point?

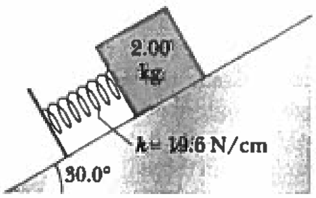


FIGURE 8-37 Problem 25.

31P. A 1.50 kg snowball is shot upward at an angle of 34.0° to the horizontal with an initial speed of 20.0 m/s. (a) What is its initial kinetic energy? (b) By how much does the gravitational potential energy of the snowball–Earth system change as the snowball moves from the launch point to the point of maximum height? (c) What is that maximum height?

33P. The string in Fig. 8-40 is L = 120 cm long, and the distance d to the fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg?

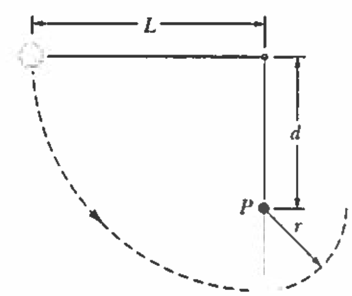


FIGURE 8-40 Problems 33 and 41.

35P. Figure 8-42 shows a pendulum of length L. Its bob (which effectively has all the mass) has speed v_0 when the cord makes an angle θ_0 with the vertical. (a) Derive an expression for the speed of the bob when it is in its lowest position. What is the least value that v_0 can have if the pendulum is to swing down and then up (b) to a horizontal position, and (c) to a vertical position with the cord remaining straight?

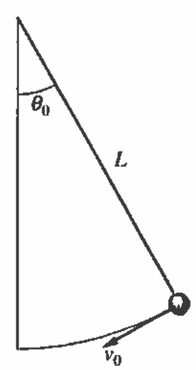


FIGURE 8-42 Problem 35.

Answers: 17. 830 ft 19. a) 6.75 J b) −6.75 J c) 6.75 J d) 6.75 J
19e) −6.75 J f) 0.459 m 23. a) 0.98 J b) −0.98 J c) 3.1 N/cm
25. a) 39.2 J b) 39.2 J c) 4.00 m 31. a) 300 J b) 93.8 J c) 6.38 m
33. a) 4.8 m/s b) 2.4 m/s 35. a) $\sqrt{v_0^2 + 2gL(1-\cos\theta_0)}$ b) $\sqrt{2gL\cos\theta_0}$
35. c) $\sqrt{gL(3+2\cos\theta_0)}$

Lesson 7 – Collisions

We have already seen a few very simple one-dimensional collisions through the examples worked in class and homework questions. We used the conservation of momentum to solve those examples. In more involved collision problems, we will also need to use the conservation of energy.

In general, there are two types of collision:

Elastic Collision –

Inelastic Collision –

The important distinction between these two types of collisions is that **momentum is constant in all collisions, but kinetic energy is constant only in elastic collisions.** So momentum is always conserved; and we can use this to our advantage, because we will be able to look at momentum *during* the collision. But we will need to take a slightly different approach when considering kinetic energy for elastic and inelastic collisions.

Example 1

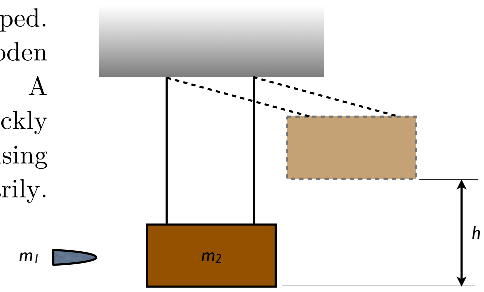
A 300 g toy train and a 600 g toy train are involved in an elastic collision on a straight section of model rail. The 300 g train, travelling at 2 m/s strikes the 600 g train at rest. Determine the velocities of both trains after the collision.

Example 2

Two metal spheres, suspended by vertical cords, initially just touch. Sphere 1 has a mass of 30 g and is pulled to the left to a height of 8.0 cm and released. It undergoes an elastic collision with sphere 2 who's mass is 75 g. Find the velocity of sphere 1 just after the collision and the height achieved by sphere 2.

Example 3

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown to the right consists of a large wooden block of mass 5.4 kg hanging from two long cords. A bullet of mass 9.5 g is fired into the block, coming quickly to rest. The block and bullet then swing upwards, rising a distance of 6.3 cm before coming to rest momentarily. Find the speed of the bullet prior to the collision.



Worksheet

Collisions

1. A 34-g bullet travelling at 120 m/s embeds itself in a wooden block on a smooth surface. The block then slides toward a spring and collides with it. The block compresses the spring (k = 99 N/m) a maximum of 12 cm. Calculate the mass of the block of wood.

2. A 38-g bullet is fired with a speed of 180 m/s into a 5.0-kg sandbag pendulum that is free to swing. To what maximum vertical height will the pendulum rise?

3. A bullet with a mass of 45 g is fired into a 8.3-kg block of wood resting on a floor against a spring. This ideal spring (k = 76 N/m) has a maximum compression of 28 cm. What was the initial speed of the bullet?

4. A 1.8-kg block, initially at rest, slides down a frictionless ramp that is angled at 35° to the horizontal. At a point 0.45 m down the slope it collides with and sticks to a stationary block of mass 1.1 kg. The blocks then continue another 0.88 m down the ramp. How long does the whole event take?

5. Two carts of mass 12 kg and 15 kg move toward each other with speeds of 2.3 m/s and 1.5 m/s respectively. If the collision between them is completely inelastic, calculate the velocity of the 15-kg cart after the collision

Answers: 1. 12 kg 2. 9.4 cm 3. 1.6×10^2 m/s 4. 0.76 s 5. 0.19 m/s