



DREAMLINE STEAM OLYMPIAD

9–10

GRADE



Grade 9–10 Mathematics Olympiad Sample Questions

1

Find the value of

$$48^2 - 47^2.$$

(A) 91**(B)** 94**(C)** 95**(D)** 97**Solution:** Use the difference of squares:

$$\begin{aligned} 48^2 - 47^2 &= (48 - 47)(48 + 47) \\ &= 1 \cdot 95 = 95. \end{aligned}$$

Answer: C) 95**2**

A mathematician has created a secret code using powers of 2. The code contains

$$\frac{2^5 \cdot 2^3}{2^4} + \frac{2^7 \cdot 2^3}{2^5} = ?$$

What number should be entered to unlock the box?

(A) 8**(B)** 16**(C)** 32**(D)** 48**Solution:** Using the laws of exponents,

$$\begin{aligned} \frac{2^5 \cdot 2^3}{2^4} &= 2^{5+3-4} = 2^4 = \boxed{16}. \\ \frac{2^7 \cdot 2^3}{2^5} &= 2^{7+3-5} = 2^5 = 32 \end{aligned}$$

Answer: D) $16 + 32 = 48$

3

Simplify:

$$3(2x - 5) - 2(x + 4).$$

A $4x - 23$

B $4x - 7$

C $8x - 23$

D $2x - 23$

Solution: Expand the brackets:

$$3(2x - 5) - 2(x + 4) = 6x - 15 - 2x - 8.$$

Therefore, $= 4x - 23$.Answer: A) $4x - 23$ **4**

Solve:

$$\frac{x - 3}{4} + 2 = 7.$$

A 17

B 19

C 21

D 23

Solution: Subtract 2:

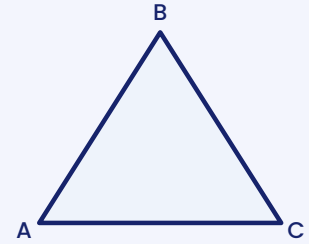
$$\frac{x - 3}{4} = 5.$$

Multiply by 4: $x - 3 = 20$.Thus, $x = 23$.

Answer: D) 23

5

Two angles of a triangle are 47° and 68° . Find the third angle.

**(A)** 55° **(B)** 65° **(C)** 75° **(D)** 85°

Solution:

The angles in a triangle add to 180° :

$$180^\circ - 47^\circ - 68^\circ = 65^\circ.$$

Answer: B) 65°

6

A password consists of:

- one letter chosen from A, B, C ;
- followed by two different digits chosen from 1, 2, 3, 4.

How many passwords are possible?

**(A)** 24**(B)** 30**(C)** 36**(D)** 48

Solution: There are 3 choices for the letter. There are 4 choices for the first digit and 3 remaining choices for the second digit:

$$3 \cdot 4 \cdot 3 = 36.$$

Answer: C) 36

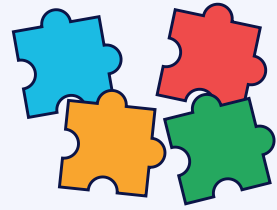
7

A puzzle master says: "I have chosen two numbers. They are the roots of

$$x^2 - 9x + 14 = 0.$$

You don't need to find the numbers. Just tell me their sum."

What is the answer?

☐ (A) 7☒ (B) 9☐ (C) 14☐ (D) 23

Solution: By Vieta's formula,

$$\alpha + \beta = 9.$$

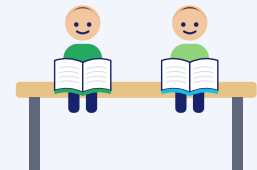
So the secret sum is .

Answer: B) 9

8

The ratio of boys to girls in a class is 3:5. After 6 boys join the class, the ratio becomes 9:10.

How many girls were originally in the class?

☐ (A) 15☒ (B) 20☐ (C) 25☐ (D) 30

Solution: Let the original numbers of boys and girls be $3k$ and $5k$.

After 6 boys join,

$$\frac{3k + 6}{5k} = \frac{9}{10}.$$

Cross-multiply: $10(3k + 6) = 45k$.

$$30k + 60 = 45k.$$

$$15k = 60, k = 4.$$

The number of girls is

$$5k = 5 \cdot 4 = 20.$$

Answer: B) 20

9

A cinema sells:

- Adult tickets for **\$12**
- Student tickets for **\$7**

A total of **34 tickets** were sold. In addition, every adult purchased **1 box of popcorn for \$3**, while every student purchased **1 drink for \$2**. The cinema collected **\$408** in total from tickets and refreshments.

How many adult tickets were sold?



(A) 14

(B) 16

(C) 17

(D) 20

Solution Adult contribution:

$$12 + 3 = 15$$

Student contribution: $7 + 2 = 9$

Let adults = a .

$$15a + 9(34 - a) = 408$$

$$15a + 306 - 9a = 408$$

$$6a = 102$$

$$a = 17$$

Thus the correct number is 17

10

Find the next number:

2, 6, 12, 20, 30, ...

(A) 40

(B) 41

(C) 42

(D) 44

Solution: The differences are

4, 6, 8, 10.

The next difference is 12, so

$$30 + 12 = 42.$$

Alternatively, the terms are

$1 \cdot 2, 2 \cdot 3, 3 \cdot 4, 4 \cdot 5, 5 \cdot 6.$

The next term is $6 \cdot 7 = 42.$

Answer: C) 42

11

A positive number x has a remarkable property: $x + \frac{1}{x} = 6$. A second clue asks for $x^2 + \frac{1}{x^2}$. Can you solve it without finding x ?

☐ (A) 30☒ (B) 34☐ (C) 36☐ (D) 38

Solution: Square the first equation:

$$\left(x + \frac{1}{x}\right)^2 = 36.$$

$$\text{Therefore, } x^2 + 2 + \frac{1}{x^2} = 36.$$

$$\text{Hence, } x^2 + \frac{1}{x^2} = 36 - 2 = \boxed{34}.$$

Answer: B) 34

12

A rectangular garden has side lengths x and y . The gardener knows that

$$x + y = 10 \text{ and } xy = 21.$$

Without finding x and y individually, can you determine

$$x^2 + y^2 = ?$$

☐ (A) 52☒ (B) 58☐ (C) 62☐ (D) 79

Solution: Use $x^2 + y^2 = (x + y)^2 - 2xy$.

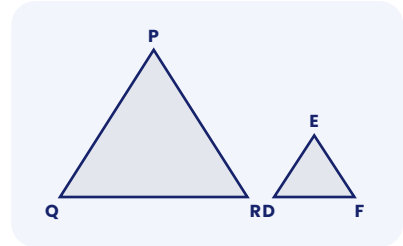
Therefore,

$$x^2 + y^2 = 10^2 - 2(21) = 100 - 42 = \boxed{58}.$$

Answer: B) 58

13

Two similar triangles have corresponding sides 6 cm and 10 cm. If another side of the smaller triangle is 9 cm, find the corresponding side of the larger triangle.

**A** 15 cm**B** 16 cm**C** 18 cm**D** 20 cm

Solution: The scale factor is $\frac{10}{6} = \frac{5}{3}$.

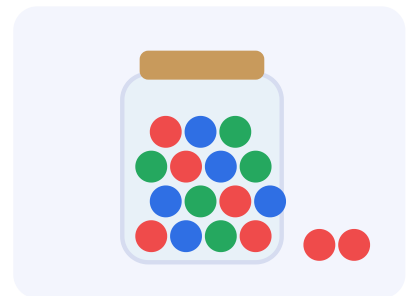
Thus the required side is

$$9 \cdot \frac{5}{3} = 15.$$

Answer: A) 15 cm

14

A bag contains 5 red balls, 4 blue balls and 3 green balls. What is the probability of choosing a ball that is not blue?

**A** $\frac{1}{3}$ **B** $\frac{5}{12}$ **C** $\frac{2}{3}$ **D** $\frac{3}{4}$

Solution: The total number of balls is

$$5 + 4 + 3 = 12.$$

The number of non-blue balls is $5 + 3 = 8$.

Therefore,

$$P(\text{not blue}) = \frac{8}{12} = \frac{2}{3}.$$

Answer: C) $\frac{2}{3}$

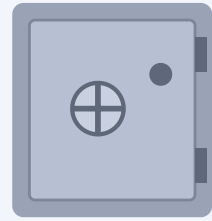
15

A high-security vault uses the number

$$7^{1000} + 7^{1001} + 7^{1002}$$

as its access code.

When the code is divided by 10, what is the remainder?



(A) 1

(B) 3

(C) 7

(D) 9

Solution: Factor out 7^{1000} :

$$7^{1000} + 7^{1001} + 7^{1002} = 7^{1000} (1 + 7 + 49) = 57 \cdot 7^{1000}$$

Since only the last digit matters, look at the pattern of powers of 7:

7, 49, 343, 2401

Last digits: 7, 9, 3, 1

The pattern repeats every 4 powers.

$$1000 \div 4 = 250$$

So 7^{1000} has the same last digit as 7^4 , which is 1.

Therefore, $57 \cdot 7^{1000}$

has the same last digit as $57 \cdot 1 = 57$.

The last digit is 7, so the remainder when divided by 10 is 7.

Answer C) 7

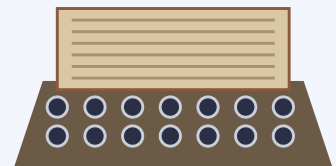
16

A mysterious machine follows the rule

$$f(x + y) = f(x) + f(y) + 2xy.$$

You are told that $f(1) = 3$.

What number will the machine produce when 3 is entered?



(A) 11

(B) 13

(C) 14

(D) 15

Solution: First find $f(2)$:

$$\begin{aligned} f(2) &= f(1 + 1) \\ &= f(1) + f(1) + 2(1)(1) \\ &= 3 + 3 + 2 = 8. \end{aligned}$$

Now, $f(3) = f(2 + 1)$

$$\begin{aligned} &= f(2) + f(1) + 2(2)(1) \\ &= 8 + 3 + 4 = \boxed{15}. \end{aligned}$$

Answer: D) 15

17

The numbers r and s are the roots of

$$2x^2 - 7x + 3 = 0.$$

and t and u are the roots of

$$y^2 - 9y + 20 = 0$$

Find $r^2 + s^2 + t^2 + u^2 = ?$

(A) $\frac{189}{4}$

(B) $\frac{197}{4}$

(C) $\frac{201}{4}$

(D) $\frac{205}{4}$

Solution:

- r and s are roots of $2x^2 - 7x + 3 = 0$
- t and u are roots of $y^2 - 9y + 20 = 0$

Find: $r^2 + s^2 + t^2 + u^2$

Step 1: Find $r^2 + s^2$

Using Vieta's formulas for

$$2x^2 - 7x + 3 = 0$$

$$r + s = \frac{7}{2}, rs = \frac{3}{2}$$

$$\text{Then } r^2 + s^2 = (r + s)^2 - 2rs$$

$$= \left(\frac{7}{2}\right)^2 - 2\left(\frac{3}{2}\right) = \frac{49}{4} - 3 = \frac{37}{4}$$

Step 2: Find $t^2 + u^2$

$$\text{For } y^2 - 9y + 20 = 0$$

$$t + u = 9, tu = 20$$

$$\begin{aligned} \text{Thus } t^2 + u^2 &= (t + u)^2 - 2tu \\ &= 9^2 - 2(20) = 81 - 40 = 41 \end{aligned}$$

Step 3: Add the results

$$\begin{aligned} r^2 + s^2 + t^2 + u^2 &= \frac{37}{4} + 41 \\ &= \frac{37 + 164}{4} = \frac{201}{4} = 50.25 \end{aligned}$$

✓ Answer $\boxed{\frac{201}{4}}$

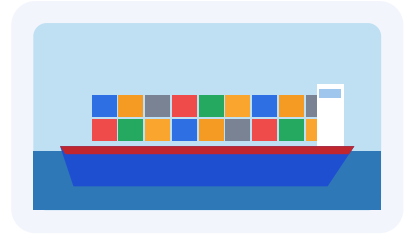
18

A cargo ship carries only two types of containers:

- Small containers weigh 3 tons each.
- Large containers weigh 5 tons each.

The total cargo weight is 31 tons.

How many different ways can the ship be loaded using at least one container of each type?



Ⓐ 1

Ⓑ 2

Ⓒ 3

Ⓓ 4

Solution: Since $3x = 31 - 5y$,
we test positive values of y .

For $y = 2$, $3x = 31 - 10 = 21 \Rightarrow x = 7$.

For $y = 5$, $3x = 31 - 25 = 6 \Rightarrow x = 2$.

The positive solutions are

$$(x, y) = (7, 2), (2, 5).$$

Therefore, there are 2 solutions.

Answer: B) 2

19

From an external point P , a tangent PT to a circle has length 12 cm. A secant through P meets the circle at A and B , where $PA = 8$ cm.

Find PB .

Ⓐ 16 cm

Ⓑ 17 cm

Ⓒ 20 cm

Ⓓ 18 cm

Solution: By the tangent-secant theorem,

$$PT^2 = PA \cdot PB.$$

Therefore, $12^2 = 8 \cdot PB$.

$$144 = 8 \cdot PB.$$

$$PB = 18.$$

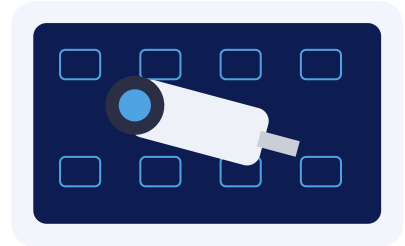
Answer: D) 18 cm

20

A security system uses **three-digit codes** made from the digits 1, 2, 3, 4, 5

Each digit can be used **at most once**. The system will unlock only if the code is **divisible by 5**.

How many different three-digit codes can unlock the system?

**(A)** 8**(B)** 12**(C)** 13**(D)** 15

Solution: A number is divisible by 5 only if its last digit is 5 (since 0 is not available).

So the code has the form: _ _ 5

For the hundreds digit, we have 4 choices:

1, 2, 3, 4

For the tens digit, 3 choices remain.

Therefore, $4 \times 3 = 12$.

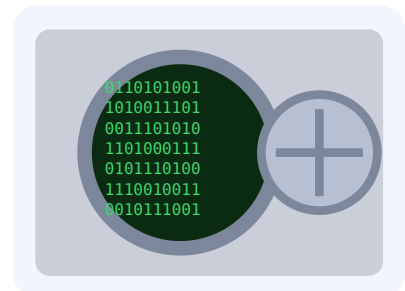
Answer: B) 12

21

A vault accepts only perfect-square divisors of

$$N = 2^4 \cdot 3^3 \cdot 5^2.$$

How many different positive perfect-square divisors does N have?

**(A)** 12**(B)** 18**(C)** 24**(D)** 30

Solution: For a divisor to be a square, every exponent must be even.

For 2^4 : 0, 2, 4

→ 3 choices.

For 3^3 : 0, 2

→ 2 choices.

For 5^2 : 0, 2

→ 2 choices.

Therefore, $3 \cdot 2 \cdot 2 = \boxed{12}$.

Answer: A) 12

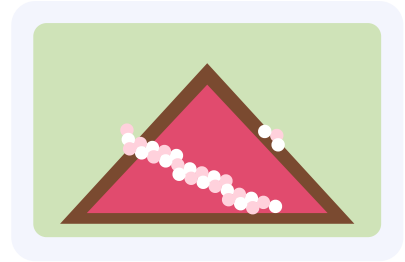
22

A triangular park is named $\triangle ABC$. Its angles are:

$$\angle A = 40^\circ, \angle B = 70^\circ, \angle C = 70^\circ.$$

The park manager marks the **incenter** I , the point where the three angle bisectors meet.

What is $\angle AIC$?

**(A)** 115° **(B)** 120° **(C)** 125° **(D)** 130°

Solution: For an incenter,

$$\angle AIC = 90^\circ + \frac{\angle B}{2}.$$

$$\begin{aligned} \text{Therefore, } \angle AIC &= 90^\circ + \frac{70^\circ}{2} \\ &= 90^\circ + 35^\circ = \boxed{125^\circ}. \end{aligned}$$

Answer: C) 125°

23

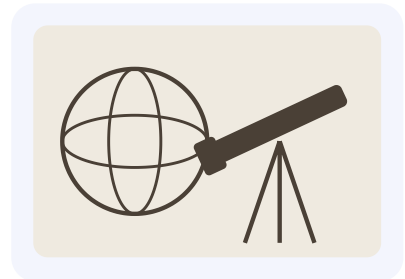
An ancient mathematician gives you only this clue:

$$\tan \theta = \frac{3}{4},$$

where θ is acute.

The final password is $\cos 2\theta$.

What is the password?

**(A)** $\frac{7}{25}$ **(B)** $\frac{9}{25}$ **(C)** $\frac{16}{25}$ **(D)** $\frac{24}{25}$

Solution: Use

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}.$$

Therefore,

$$\cos 2\theta = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{7/16}{25/16} = \boxed{\frac{7}{25}}.$$

Answer: A) $\frac{7}{25}$

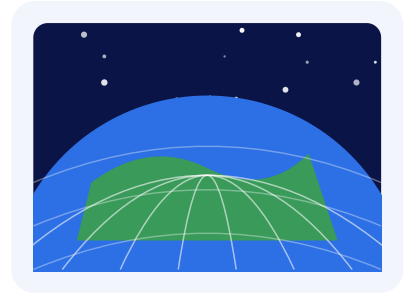
24

On a coordinate map, two landmarks are

$$A = (1, 2), B = (5, 10).$$

A straight road connects them.

How far is the origin $(0, 0)$ from this road?

**A** 0**B** $\frac{2}{\sqrt{5}}$ **C** 2**D** $2\sqrt{5}$

Solution: The slope is $m = \frac{10 - 2}{5 - 1} = 2$.

Therefore, the line is $y = 2x$,

or $2x - y = 0$.

Since $2(0) - 0 = 0$,

the origin lies **on the road**.

Hence the distance is $\boxed{0}$.

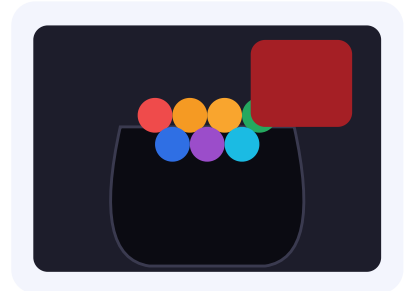
Answer: A) 0

25

A magician has a bag containing balls of 7 different colors.

Without looking, you begin taking balls from the bag.

What is the minimum number of balls you must take to guarantee that at least 4 balls have the same color?

**A** 22**B** 23**C** 24**D** 25

Solution: To avoid getting 4 of the same color, we could take at most 3 balls of each color:

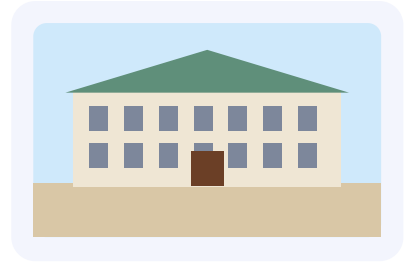
$$7 \times 3 = 21.$$

The next ball guarantees a fourth ball of some color: $21 + 1 = \boxed{22}$.

Answer: A) 22

26

A square castle courtyard has a side length of 12 m. To create four identical corner gardens, the builders remove a $4 \text{ m} \times 4 \text{ m}$ square from each corner. What is the perimeter of the remaining courtyard?

**A** 48 m**B** 56 m**C** 64 m**D** 80 m

Solution Originally, the square has perimeter $4(12) = 48 \text{ cm}$.

At each corner, removing a 4×4 square removes $4 + 4 = 8 \text{ cm}$ of the original boundary, but creates $4 + 4 = 8 \text{ cm}$ of new boundary. So the perimeter does not change. 48 cm

Answer: A) 48 cm

27

A supercomputer gives you the enormous number

$$2^{2026} + 3^{2026}.$$

You are not allowed to calculate the number itself.

The computer asks only for its remainder when divided by 5.

What should you enter?

A 1**B** 2**C** 3**D** 4

Solution: For $2^n \pmod{5}$, the pattern is

2, 4, 3, 1.

Since $2026 \equiv 2 \pmod{4}$,

$$2^{2026} \equiv 4 \pmod{5}.$$

For $3^n \pmod{5}$: 3, 4, 2, 1.

Again, $2026 \equiv 2 \pmod{4}$,

$$\text{so } 3^{2026} \equiv 4 \pmod{5}.$$

Therefore,

$$2^{2026} + 3^{2026} \equiv 4 + 4 = 8 \equiv \boxed{3} \pmod{5}.$$

Answer: C) 3

28

Solve:

$$\frac{\left(\frac{5}{6} + \frac{7}{12}\right) - \left(\frac{2}{3} - \frac{1}{4}\right)}{\frac{\left(1 - \frac{1}{3}\right)\left(1 + \frac{1}{3}\right)}{\frac{8}{9}}} + \frac{\left(\frac{4}{5} + \frac{2}{3} - \frac{7}{15}\right) \div \frac{1}{3}}{\frac{0.75 + \frac{1}{4}}{1.5 - \frac{1}{2}}} = ?$$

(A) 1**(B)** 5/2**(C)** 7/3**(D)** 4

Solution: Numerator: $\frac{5}{6} + \frac{7}{12} = \frac{10}{12} + \frac{7}{12} = \frac{17}{12}$

and $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.

Therefore, $\frac{17}{12} - \frac{5}{12} = 1$.

Denominator: $\left(1 - \frac{1}{3}\right)\left(1 + \frac{1}{3}\right) = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$.

Thus $\frac{\frac{8}{9}}{\frac{8}{9}} = 1$.

So the first part is **[1]**.

Second part: Numerator:

$$\frac{4}{5} + \frac{2}{3} - \frac{7}{15} = \frac{12}{15} + \frac{10}{15} - \frac{7}{15} = \frac{15}{15} = 1.$$

Therefore, $1 \div \frac{1}{3} = 3$.

Denominator: $0.75 + \frac{1}{4} = \frac{3}{4} + \frac{1}{4} = 1$

and $1.5 - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1$.

So $\frac{1}{1} = 1$.

Therefore the second part is $\frac{3}{1} = 3$.

Final answer $1 + 3 = \mathbf{[4]}$.

29For $x \geq 1$, solve

$$\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}} = 6.$$

(A) $\sqrt{3}$

(B) $\sqrt{12}$

(C) 8

(D) 10

Solution Observe that

$$x + 2\sqrt{x-1} = (\sqrt{x-1} + 1)^2,$$

$$\text{and } x - 2\sqrt{x-1} = (\sqrt{x-1} - 1)^2.$$

$$\text{Thus } |\sqrt{x-1} + 1| + |\sqrt{x-1} - 1| = 6.$$

Since $x \geq 1$, $\sqrt{x-1} \geq 0$. The first expression is always positive.

If $\sqrt{x-1} \geq 1$, then

$$(\sqrt{x-1} + 1) + (\sqrt{x-1} - 1) = 6,$$

$$\text{so } 2\sqrt{x-1} = 6.$$

$$\text{Hence } \sqrt{x-1} = 3$$

$$\text{and } \boxed{x = 10}$$

30

Positive integers x, y, z and t satisfy

$$x + y = 20 \text{ and } xy = 91.$$

$$\& \ z + t = 11 \text{ and } zt = 24,$$

Find $x^2 + y^2 + z^2 + t^2$

(A) 198**(B)** 238**(C)** 282**(D)** 291

Solution: Step 1: Find $x^2 + y^2$

Use $(x + y)^2 = x^2 + y^2 + 2xy$.

Therefore, $x^2 + y^2 = (x + y)^2 - 2xy$.

Substitute: $x^2 + y^2 = 20^2 - 2(91)$

$$= 400 - 182 = 218.$$

Step 2: Find $z^2 + t^2$

Similarly, $(z + t)^2 = z^2 + t^2 + 2zt$.

So, $z^2 + t^2 = (z + t)^2 - 2zt$.

Substitute: $z^2 + t^2 = 11^2 - 2(24)$

$$= 121 - 48 = 73.$$

Step 3: Add them

$$x^2 + y^2 + z^2 + t^2 = 218 + 73 = \boxed{291}$$

Answer: D) 291