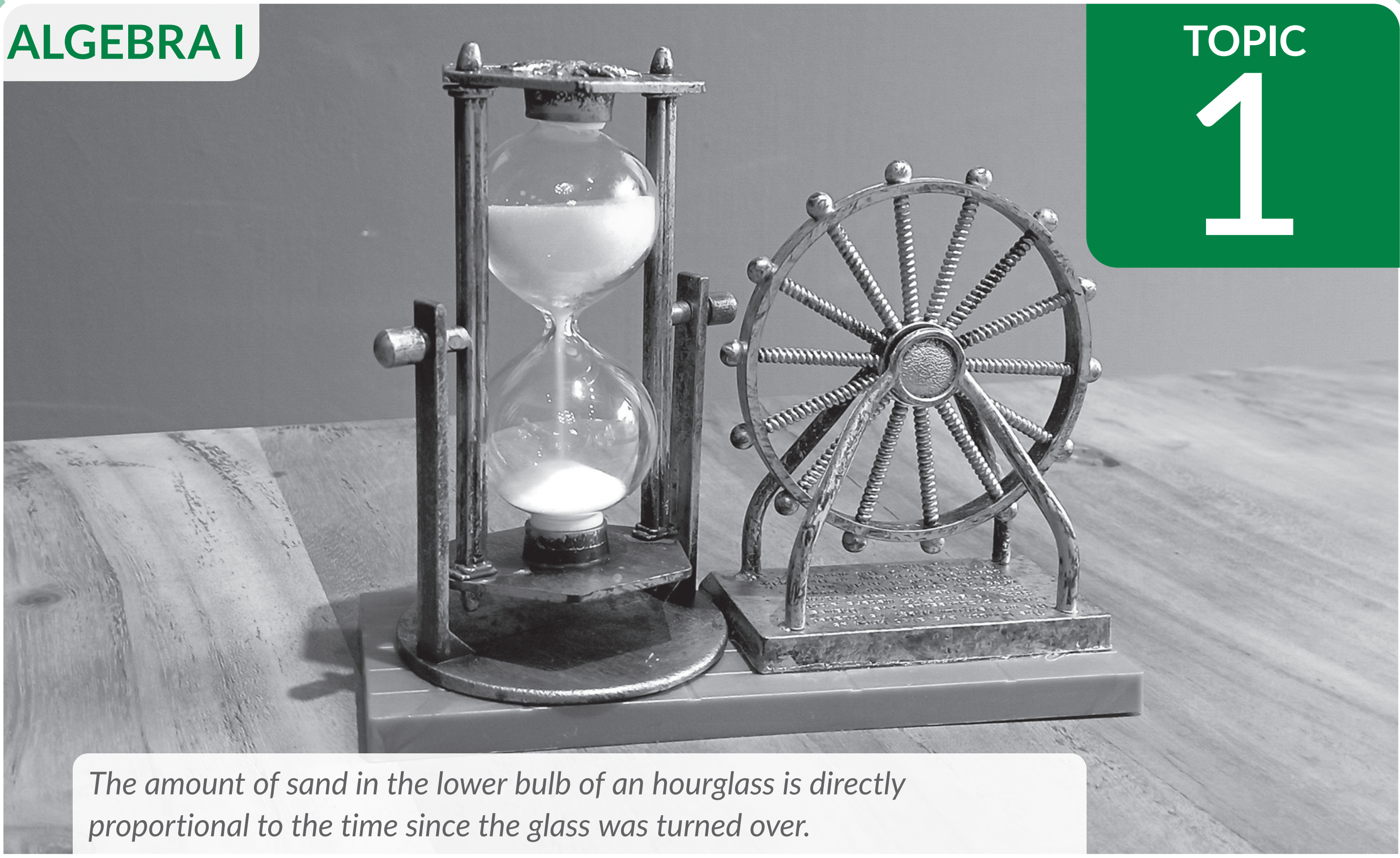
MODULE 1

Searching for Patterns

TOPIC 1



The amount of sand in the lower bulb of an hourglass is directly proportional to the time since the glass was turned over.

Quantities and Relationships

INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Apply the problem-solving model to a real-life situation.

In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Getting Started

You Already Know A Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

Ask Yourself . . .
How do your strategies change based on what you are learning and what you already know about it?

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

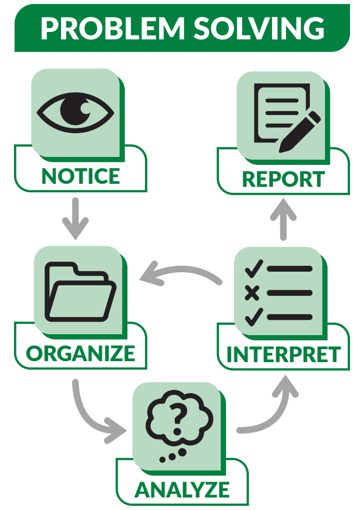
2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Think About . . .
Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

Be prepared to share your list of learning strategies with the class.

ACTIVITY 1.1

Learning Resources



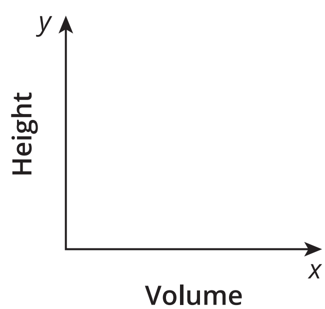
In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

Students at East High School are designing ceramic drinking cups for an art project. The students can choose from a variety of different shapes for their cups. To test the cups, hot water is poured into each cup at a constant rate. Mario chooses the cup shown.

Create a graph to represent the height of the liquid in the cup as the volume changes in Mario's cup. **Explain your reasoning**.

Mario's Cup



The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

1. Locate the phrase **Explain your reasoning** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you describe the relationship between the height of the liquid and the volume in Mario's cup?

2. What is a related word or phrase for **explain your reasoning**?

The problem-solving model provides a structure to help you become a better problem-solver.

The Academic Glossary provides definitions of terms you will see throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of new key terms in each lesson.

NOTICE

Notice and Wonder

The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the questions to ask yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about Mario's cup?

4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?

ORGANIZE

Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.

Read through the questions to ask yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy to sketch a graph that represents the height of the liquid in the cup as the volume changes in Mario's cup.

6. Why do you think the second step of the problem-solving model process is important? How can you use the questions to ask yourself to help develop a strategy to solve the problem?

Predict and Analyze

PREDICT

The third step in the modeling process involves carrying out your plan. As you carry out the plan, you will complete operations, analyze mathematical results, make predictions, and extend patterns.

Read through the questions to ask yourself for the third step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

7. Use your strategy to create a graph that represents the height of the liquid in the cup as the volume changes in Blake's cup.

8. How can the questions to ask for the third step help you communicate your mathematical thinking?

Test and Interpret

INTERPRET

The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the questions to ask yourself for the fourth step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

9. Alyssa choses a different design. Can you apply the same reasoning to create a graph to represent the height of the liquid in the cup as the volume changes in Jacquelyn's cup?

10. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?

Report

The final step in the modeling process is to share your results.

11. How does listening to others help you learn mathematics?

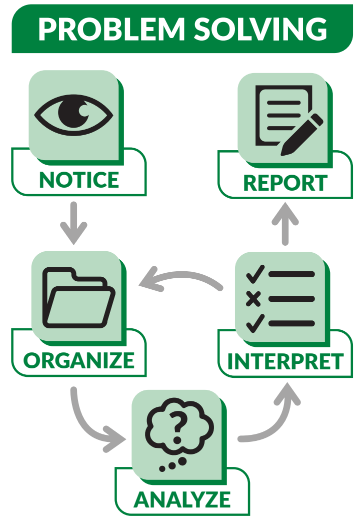
Locate the TEKS mathematical process standards in the Course Guide.

12. Which TEKS mathematical process standard(s) did you use to create a graph that represents the height of the liquid in the cup as the volume changes in Mario's cup?

Talk the Talk

The Problem-Solving Model

In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.



You will see this symbol throughout the course to remind you to use the problem-solving model.

Summarize what is involved in each phase of this problem-solving model.

Notice and Wonder

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Organize and Mathematize

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Predict and Analyze

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Test and Interpret

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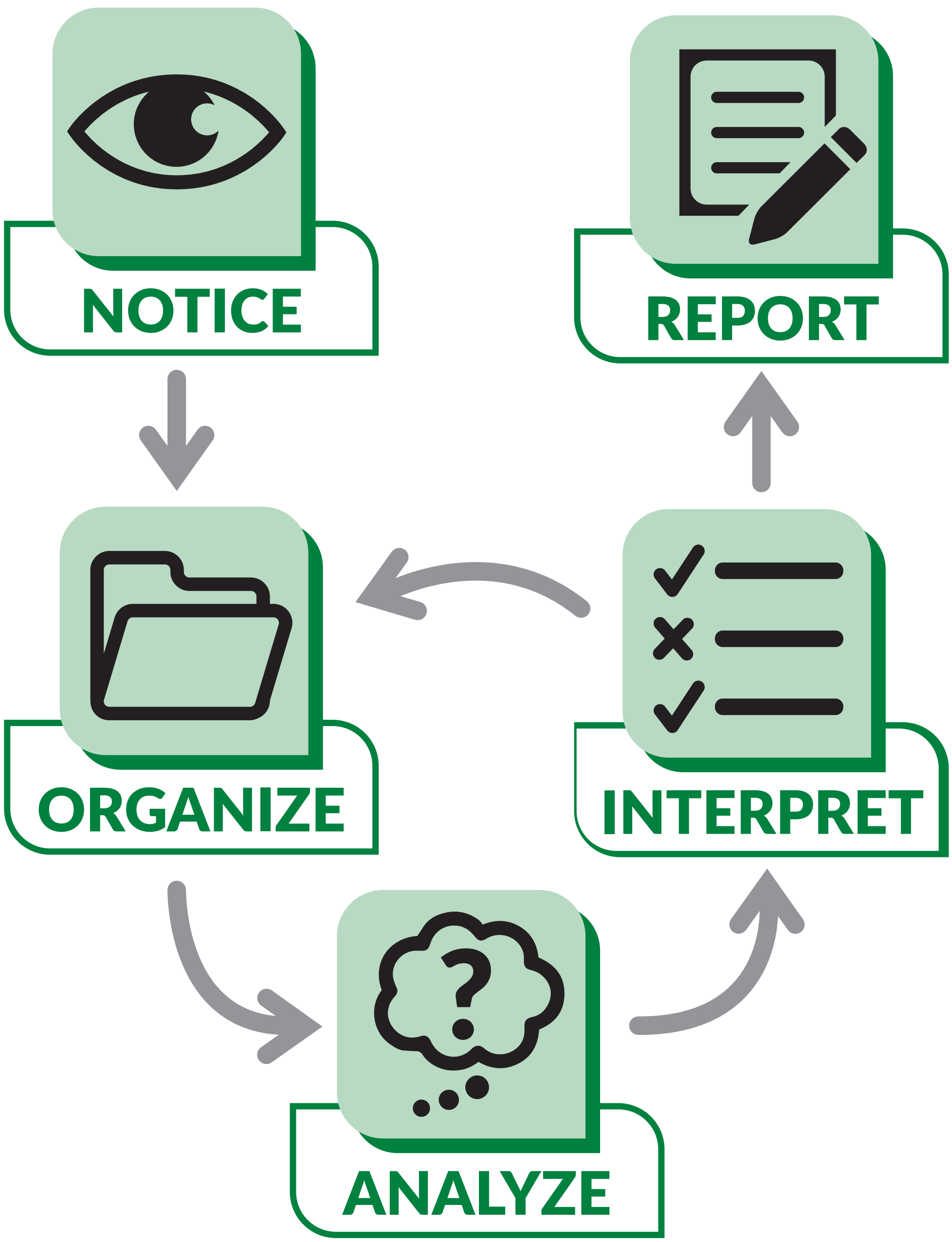
The Problem-Solving Model



1 Understanding Quantities and Their Relationships

OBJECTIVES

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.

NEW KEY TERMS

- dependent quantity
- independent quantity

You have analyzed graphs of relationships and identified important features, such as intercepts and slopes. How can the key characteristics of a graph tell a story?

Getting Started

What Comes First?

Have you ever planned a birthday party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn't make food before grocery shopping, now would you?

Consider the two quantities that are changing in each relationship.

- The number of movie tickets purchased and the total cost
- The number of eggs used and the number of cakes baked
- The number of students in attendance at school and the number of lunches served
- The number of hours driven and the number of miles to a vacation destination
- The number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool

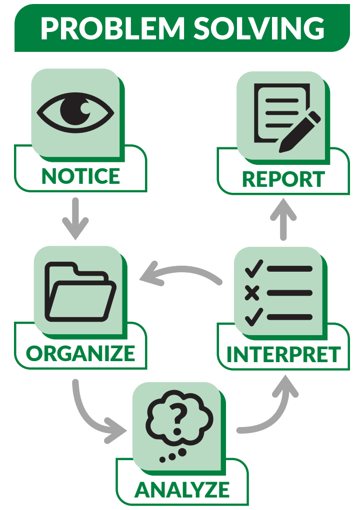
When one quantity is determined by another in a problem situation, it is called the **dependent quantity**. The quantity it is determined from is called the **independent quantity**.

1. Circle the independent quantity and underline the dependent quantity in each relationship.

2. Describe how you can determine which quantity is independent and which quantity is dependent in any problem situation.

ACTIVITY 1.1

Connecting Scenarios and Their Graphs



While a person can describe the monthly cost to operate a business or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way.

You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship with a line or a curve, it is up to you to consider the situation and interpret the meaning of the data values shown.

This activity includes six scenarios and six graphs that are located at the end of the lesson.

Ask Yourself . . .
How can graphs be used to tell a story in everyday life?

1. Read each scenario. Determine the independent and dependent quantities. Then, match each scenario to its corresponding graph. Glue the graph next to the scenario. For each graph, label the *x*- and *y*-axes with the appropriate quantity and a reasonable scale and then interpret the meaning of the origin.

   Be sure to include the appropriate units of measure for each quantity.

   Music Club
   Natalia loves music. She can lip sync almost any song at a moment's notice. She joined Songs When I Want Them, an online music store. By becoming a member, Natalia can purchase just about any song she wants. Natalia pays $1 per song.

   - Independent quantity:

   - Dependent quantity:

Ask Yourself . . .
What strategies will you use to match each graph with one of the six scenarios?

Something's Fishy

Kaya is the building manager for an office building. One of her responsibilities is cleaning the office building's 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

- Independent quantity:

- Dependent quantity:

Smart Phone, but Is It a Smart Deal?

You have your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him back with interest. He tells you that you only need to pay $1 in interest initially and then the interest will double each week after that. You consider his offer and wonder if this *really* is a good deal.

- Independent quantity:

- Dependent quantity:

It’s Magic

The Amazing Alejandro is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it.

- Independent quantity:

- Dependent quantity:

Baton Twirling

Samantha is a drum major for the high school marching band. For the finale of the halftime performance, Samantha tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- Independent quantity:

- Dependent quantity:

Skateboarding

Andrew is skateboarding on a 4-foot-tall half-pipe at the local skate park. He wants to know how far his skateboard's wheels are from the ground as he skates from the top of one side of the half-pipe to the top of the other side.

- Independent quantity:

- Dependent quantity:

ACTIVITY 1.2 Comparing and Contrasting Graphs

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?

Think About . . .
Look closely when analyzing the graphs. What do you see?

2. What differences do you notice in the graphs?

3. How did you label the independent and dependent quantities in each graph?

4. **Analyze** each graph from left to right. Describe any graphical characteristics you notice.

Review the definition of analyze in the Academic Glossary.

Think About . . .
What do the points on each graph represent?

5. Compare the graphs for each pair of scenarios given and describe any similarities and differences you notice.

   a. *Smart Phone, but Is It a Smart Deal?* and *Music Club*

   b. *Something's Fishy* and *It's Magic*

   c. *Baton Twirling* and *Skateboarding*

Talk the Talk

A Writer and a Mathematician

1. Write a scenario and sketch a graph to describe a possible trip to school.

Scenario

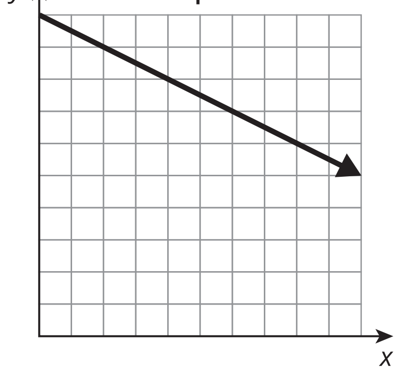
Graph

2. Describe the meaning of the points, or smooth curve, represented by your graph.

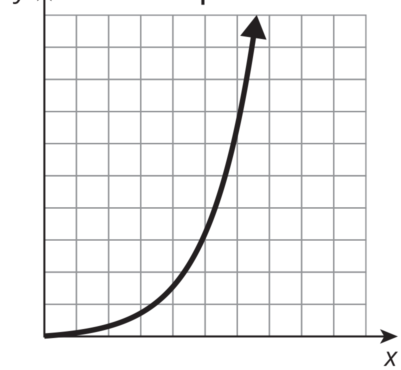
3. Compare your scenario and sketch with your classmates' scenarios and sketches. What similarities do you notice? What differences do you notice?

Graph Cutouts

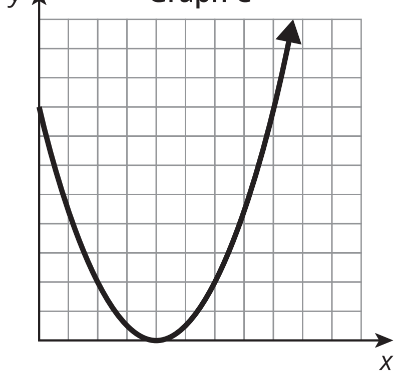
Graph A



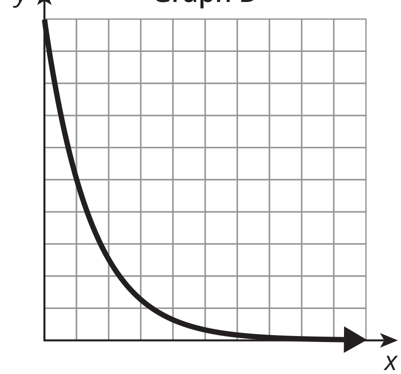
Graph B



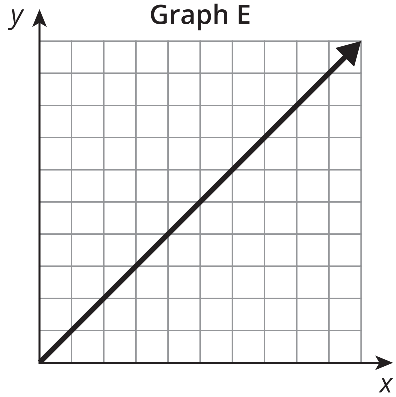
Graph C



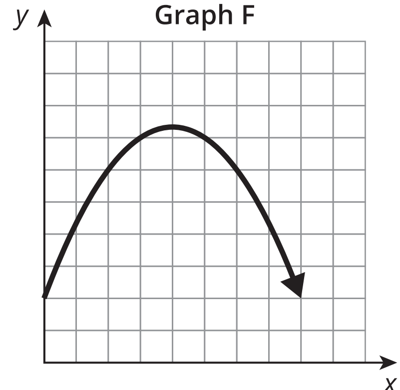
Graph D



Graph E



Graph F



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Lesson 1 Assignment

Write

Describe how you can distinguish between an independent quantity and a dependent quantity. Use an example in your description.

Remember

When one quantity is determined by another in a problem situation, it is called the *dependent quantity*. The quantity it is determined from is called the *independent quantity*. The independent quantity is represented on the *x*-axis and the dependent quantity is represented on the *y*-axis.

Practice

1. Read each scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then, analyze each graph and determine which of the provided scenarios it models. For each graph, label the *x*- and *y*-axes with the appropriate quantity and unit of measure.

   a. **Endangered Species**

   A local animal conservation organization is working with various reptile species to increase their populations. The initial population of 450 endangered turtles tripled each year for the past five years.

   b. **Video Games**

   Trung is playing video games at an arcade. Trung starts with $40 and is playing games that cost 50 cents per game.

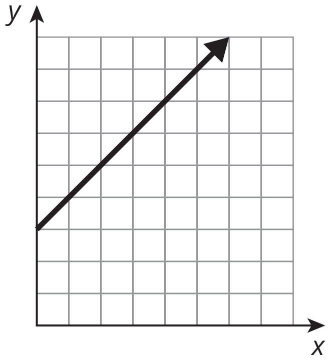
   c. **Sales Commission**

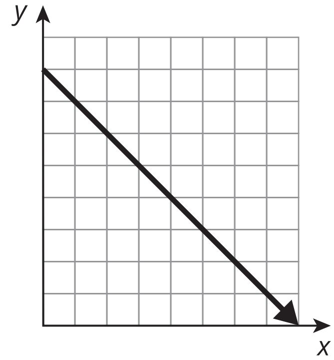
   Eduardo works as a salesman. He receives a monthly salary of $3000 as well as a 10% commission on the amount of sales.

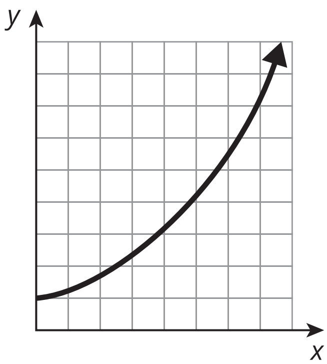
   d. **Cooling Tea**

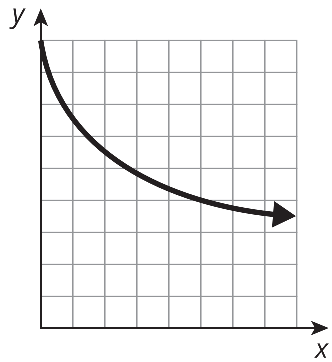
   A freshly made cup of tea is served at a temperature of about 180°F. The tea cools rapidly at first and then slows down gradually as it approaches room temperature.

Lesson 1 Assignment

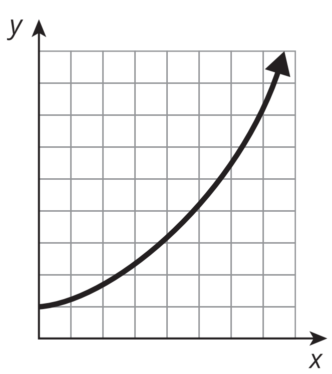
A. 

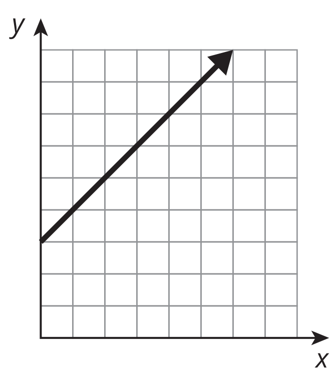
B. 

C. 

D. 

2. Compare the pair of graphs and describe any similarities and differences you notice.

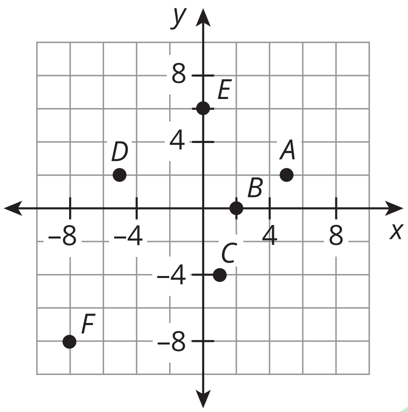




Lesson 1 Assignment

Prepare

1. Write the coordinates of each point and name the quadrant or axis where the point is located.



2 Analyzing and Sorting Graphs

OBJECTIVES

- Review and analyze graphs and graphical behavior.
- Determine similarities and differences among various graphs.
- Sort graphs and give reasons for the similarities and differences between the groups of graphs.

You have used graphs to analyze the relationship between independent and dependent quantities. Do the graphs of certain types of relationships share any characteristics?

Getting Started

Keep your graphs. You will need them in the next lesson.

Let's Sort Some Graphs

Mathematics is the science of patterns and relationships. Looking for patterns and sorting patterns into different groups based on similarities and differences can provide valuable insights. In this lesson, you will analyze many different graphs and sort them into various groups.

1. Cut out the 13 graphs at the end of the lesson. Then, analyze and sort the graphs into at least two different groups. You may group the graphs in any way you feel is appropriate.

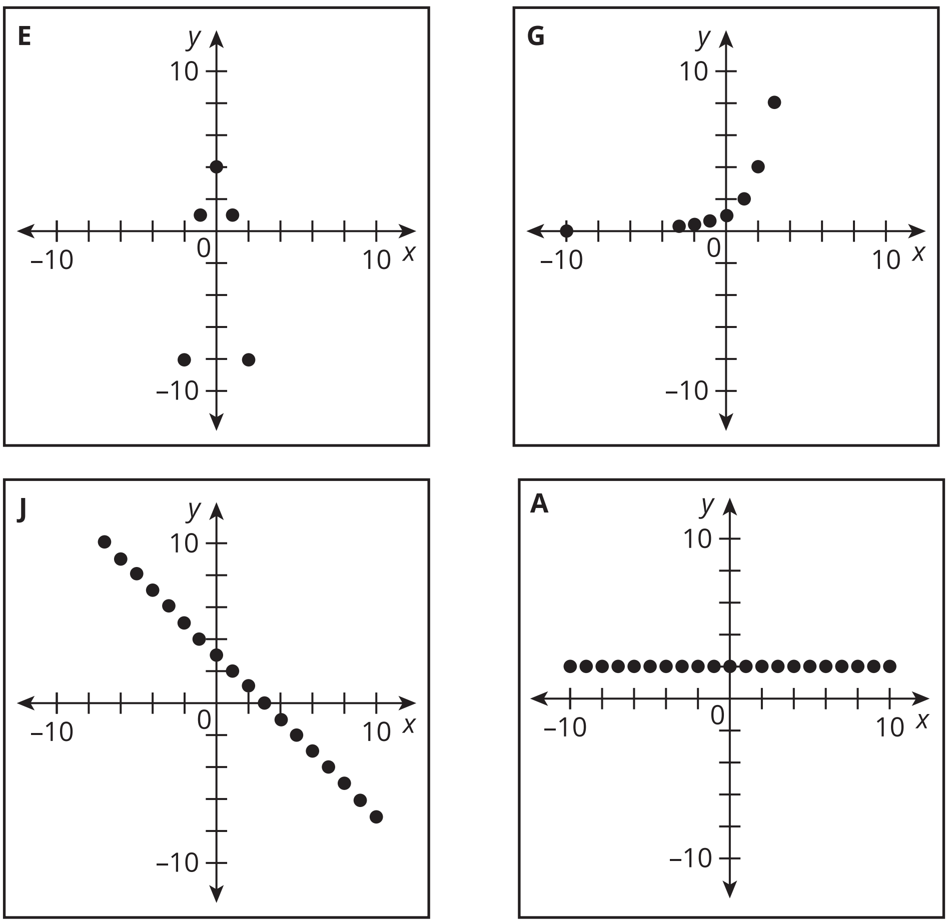
   Record the following information for each of your groups.

   - Name each group of graphs.
   - List the letters of the graphs in each group.
   - Provide a rationale for why you created each group.

ACTIVITY
2.1 Identifying Graphical Behaviors

In this activity, consider the different ways the graphs are grouped.

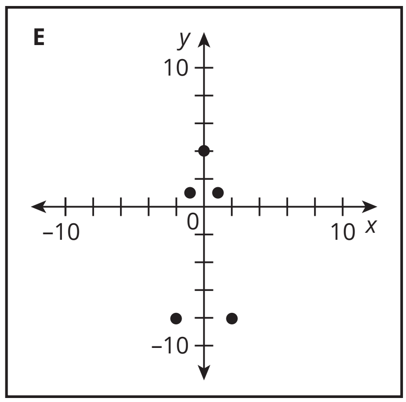
1. Chris grouped these graphs together. Why do you think Chris put these graphs in the same group?

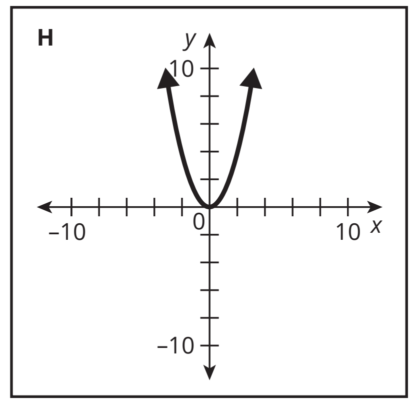


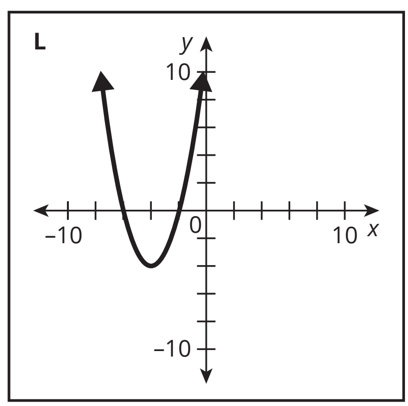
2. Consider Paola's correct grouping.

Paola

I grouped these graphs together because they all have a vertical axis of symmetry. If I draw a vertical line through the middle of the graph, the image is the same on both sides.







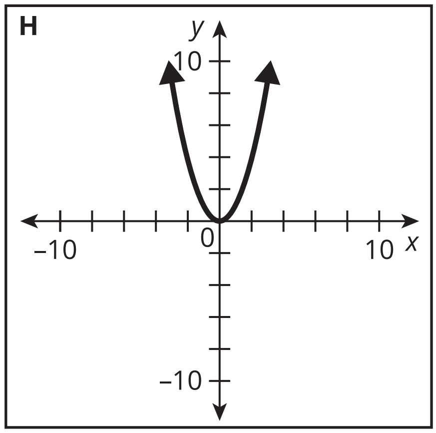
a. Show why Paola's reasoning is correct.

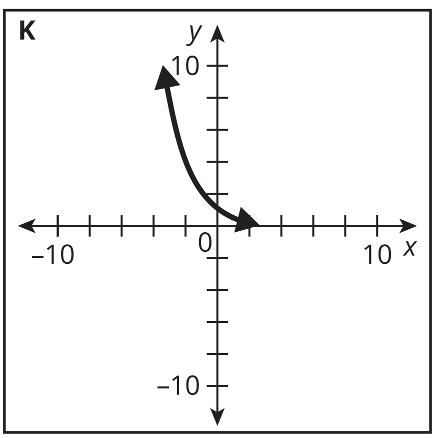
b. When possible, identify other graphs in this activity that have a vertical axis of symmetry.

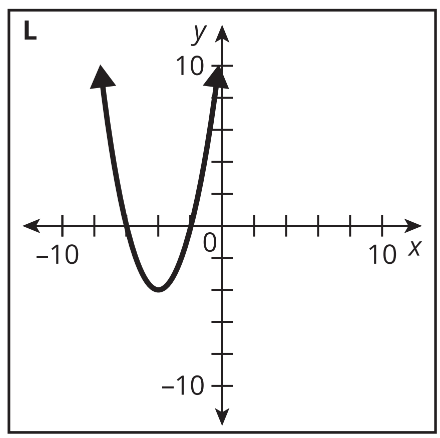
3. Consider Jorge's incorrect grouping.

Jorge

I grouped these graphs together because each graph goes through only two quadrants.



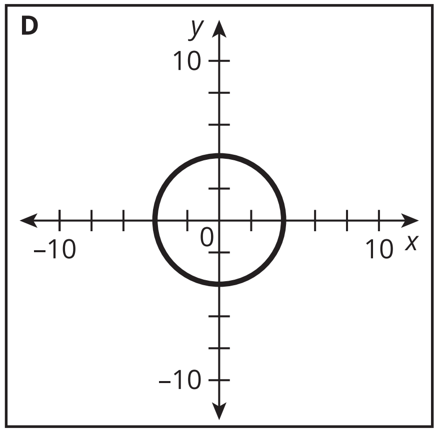


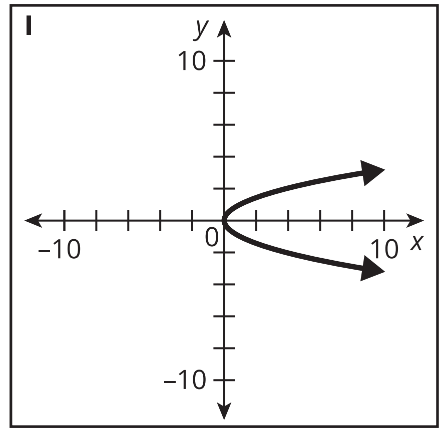


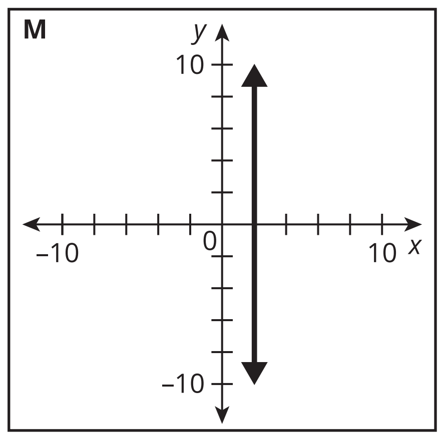
a. Explain why Jorge's reasoning is not correct.

b. When possible, identify other graphs in this activity that go through only two quadrants.

4. Elena grouped these graphs together but did not provide any rationale.







a. What do you notice about the graphs?

b. What rationale could Elena have provided?

Talk the Talk

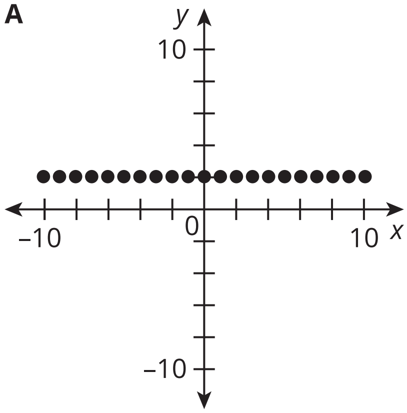
Compare and Contrast

1. Compare your groups with your classmates' groups. Create a list of the different graphical behaviors you noticed.

Ask Yourself . . .
Are any of the graphical behaviors shared among your groups? Or are they unique to each group?

Graph Cards

A



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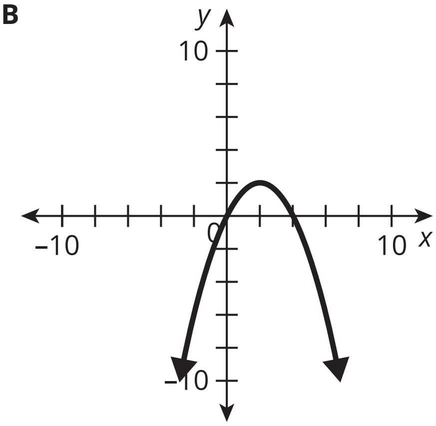

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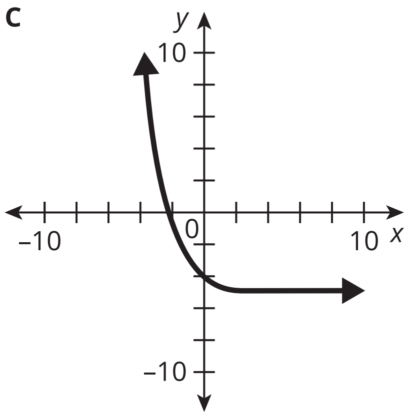

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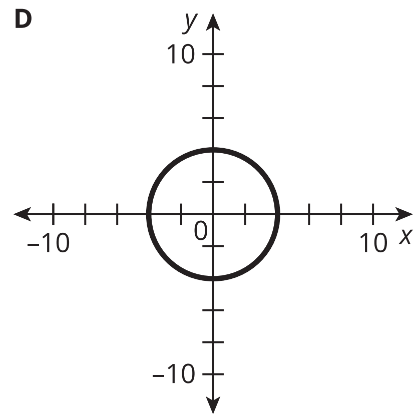

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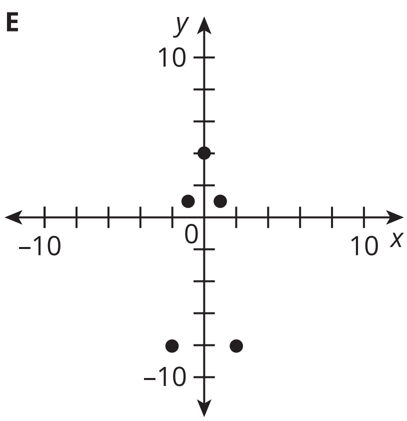

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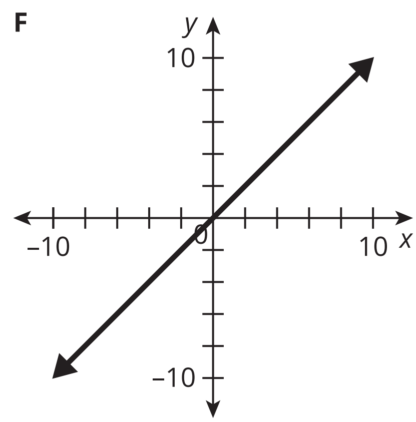

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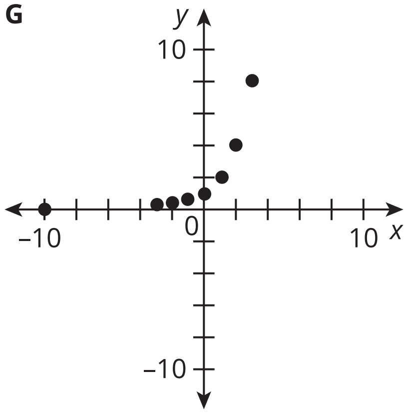

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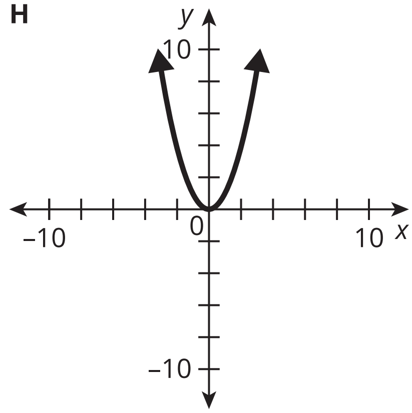

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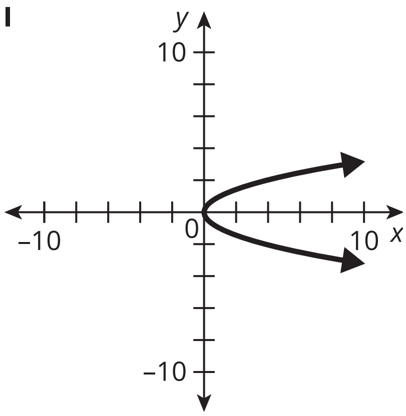

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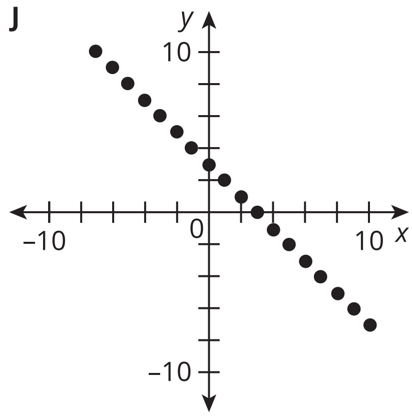

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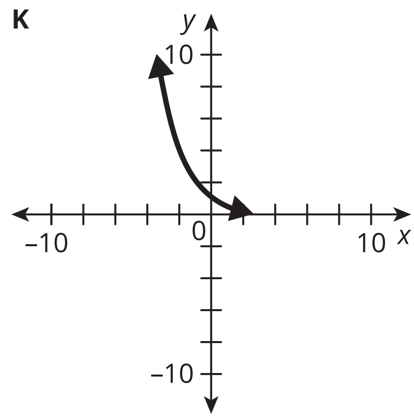
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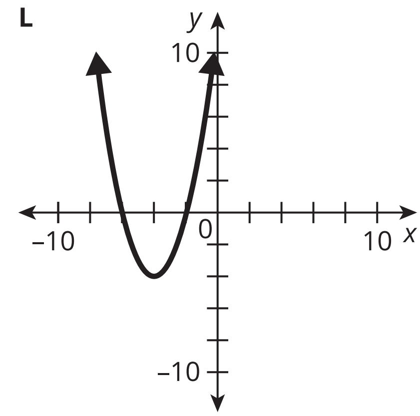
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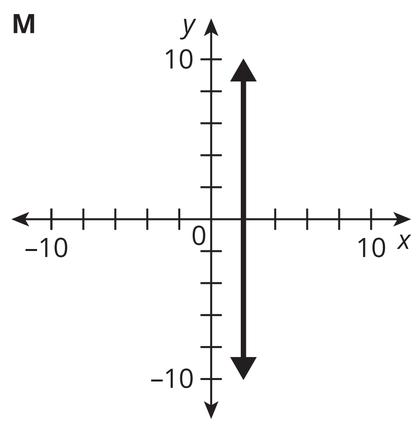
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So you can cut out the Graph Cards on the other side

Lesson 2 Assignment

Write

Describe the importance of graphical representations.

Remember

Graphs of relationships between quantities have characteristics that can give you important information about the relationship. For example, a graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing. A graph can have straight lines or smooth curves, a maximum or a minimum, or no maximum or minimum, and so on.

Practice

1. Record the letter of each graph with the given characteristic.

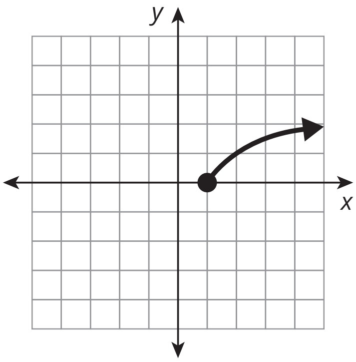
   a. Has a vertical axis of symmetry

   b. Passes through exactly 1 quadrant

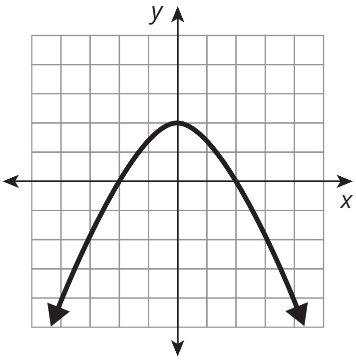
   c. Has a horizontal axis of symmetry

   d. Passes through all 4 quadrants

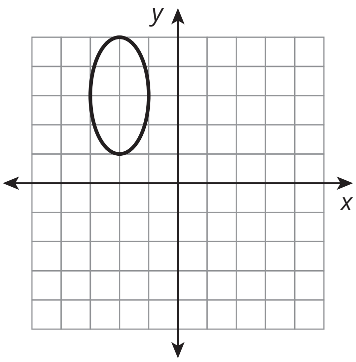
A.



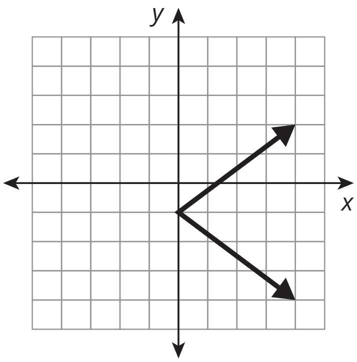
B.



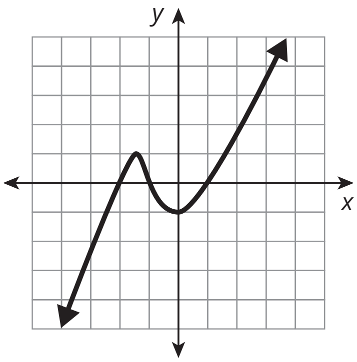
C.



D.



E.



Lesson 2 Assignment

Prepare

Create a list of the *x*- and *y*-values of the relation described by each set of ordered pairs. Write an equation using the variables *x* and *y* that could map the *x*-value to the *y*-value.

1. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)\}$

2. $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

3. $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

3 Recognizing Functions and Function Families

OBJECTIVES

- Define a *function* as a relation that assigns each element of the domain to exactly one element of the range.
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

You have sorted graphs by their graphical behaviors.
How can you describe the common characteristics of the graphs of the functions?

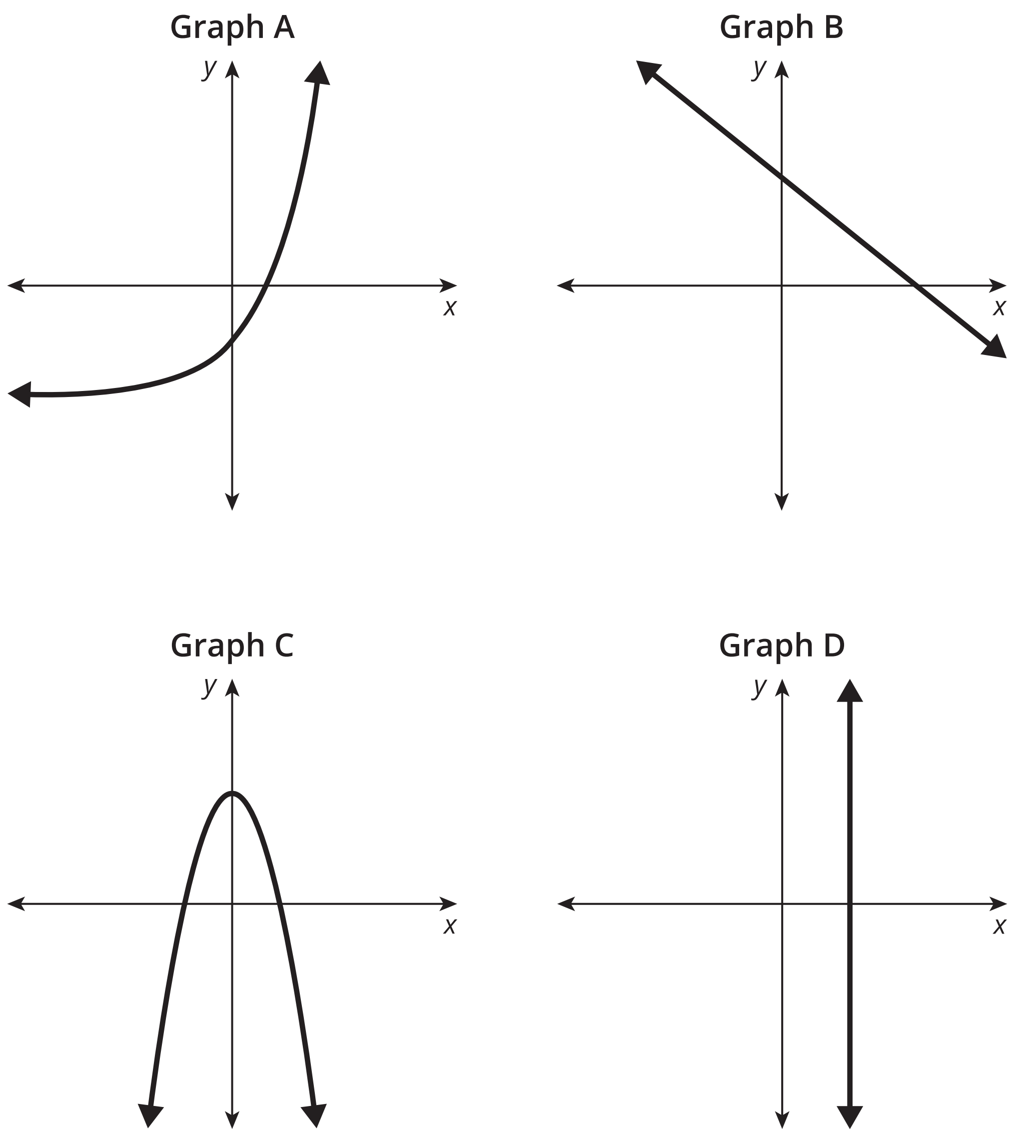
NEW KEY TERMS

- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- *x*-intercept
- *y*-intercept

Getting Started

Odd One Out

1. Which of the graphs does not belong with the others? Explain your reasoning.



ACTIVITY 3.1

Functions and Non-Functions

A relation can be represented in the following ways.

Ordered Pairs

{(−2, 2), (0, 2), (3, −4), (3, 5)}

Equation

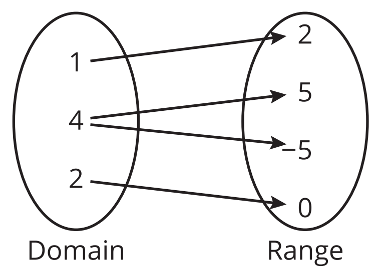
$y = \frac{2}{3}x - 1$

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

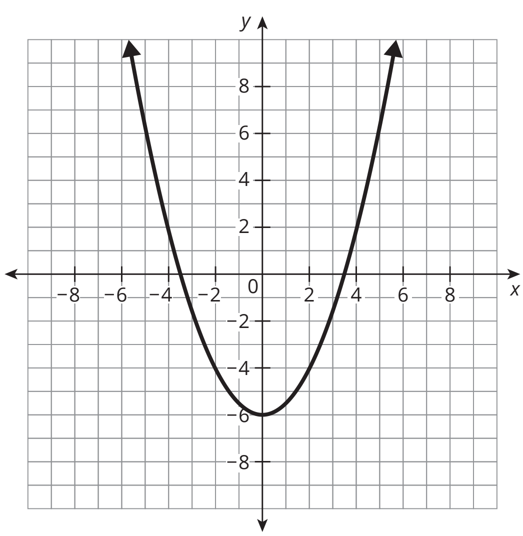
Verbal

The relation between students in your school and each student's birthday

Mapping



Graph



Table

Domain	Range
−1	1
2	0
5	−5
6	−5
7	−8

Think About . . .
So, all functions are relations, but only some relations are functions.

A **function** is a relation that assigns to each element of the domain exactly one element of the range. Functions can be represented in a number of ways.

1. Analyze the relation represented as a table. Is the relation a function? Explain your reasoning.

2. Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.

3. Analyze the relation represented verbally. Is the relation a function? Explain your reasoning.

You can write an equation representing a function using *function notation*. Let's look at the relationship between an equation and function notation.

Consider this scenario. A shirt company charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design.

The equation $y = 8x + 15$ can be written to model this situation. The independent variable x represents the number of shirts ordered, and the dependent variable y represents the total cost of the order, in dollars.

This is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it. Because this relationship is a function, you can write $y = 8x + 15$ in function notation.

$$f(x) = 8x + 15$$

name of function

independent variable

The cost, defined by f, is a function of x, defined as the number of shirts ordered.

You can write a function in a number of different ways. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as C, is a function of s, the number of shirts ordered.

4. Consider the function, $C(s) = 8s + 15$. What expression in the function equation represents:

   a. the domain of the function?

   b. the range of the function?

5. Describe the possible domain and range for this situation.

Function notation is a way of representing functions algebraically. The function notation $f(x)$ is read as "f of x" and indicates that x is the independent variable.

When f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.

WORKED EXAMPLE

You can write domain and range using set notation.

A teacher is restocking his pencil supply for this semester's tutoring group. All new students to the tutoring group receive 12 pencils. The teacher expects to have no more than 6 new students attending his group.

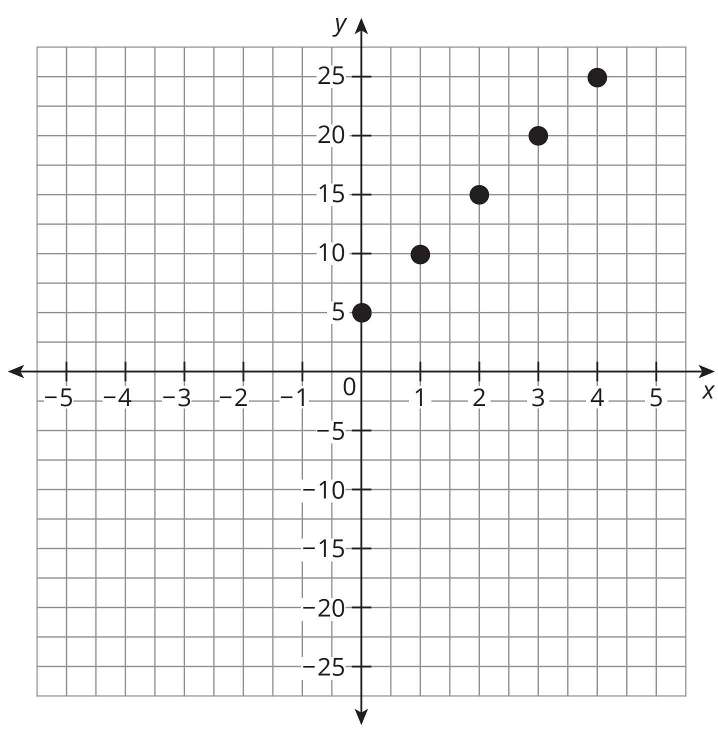
The domain for this situation is {0, 1, 2, 3, 4, 5, 6} for the possibility of having 0 to 6 new students.

The range for this situation is {0, 12, 24, 36, 48, 60, 72} for the possibility of needing between 0 and 72 pencils.

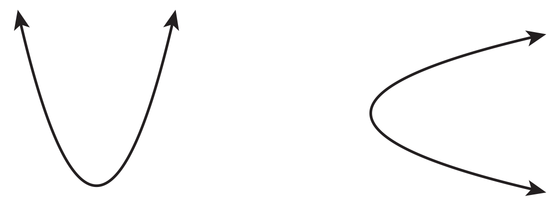
6. Write the domain and range using set notation.

   a. A neighborhood bakery makes and sells bagels every morning. The bagels are sold in sets of 6 and only the first 9 customers can purchase them.

   b.



The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.



Function Not a function

The Vertical Line Test applies for both *discrete* and *continuous graphs*.

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

7. How can you determine if a relation represented as a set of ordered pairs is a function? Explain your reasoning.

8. How can you determine if a relation represented as an equation is a function? Explain your reasoning.

9. Determine which relations represent functions. If the relation is not a function, state why not.

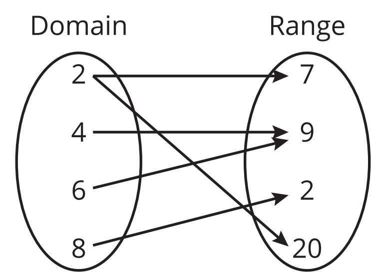
a. $y = 3^x$

b. For every house, there is one and only one street address.

c.

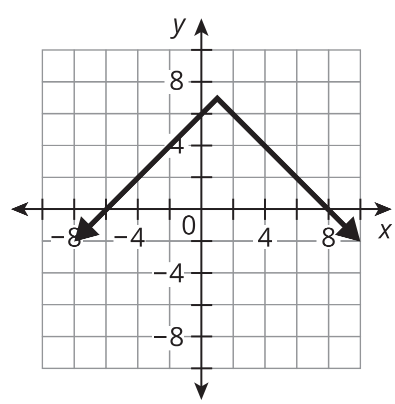
Domain	Range
-1	4
0	0
3	-2
0	4

d.



e. $\{(-7, 5), (-5, 5), (2, -2), (3, 5)\}$

f.



10. Analyze the three graphs Elena grouped together in the previous lesson, graphs *D*, *I*, and *M*. Are the graphs she grouped functions? Explain your conclusion.

11. Use the Vertical Line Test to sort the graphs in the previous lesson into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

Functions	Non-Function

ACTIVITY
3.2 Domain and Range of a Function

You have identified the domain and range of a function given its equation.

Ask Yourself . . .
How does an open or closed circle on a graph reflect the domain? The range?

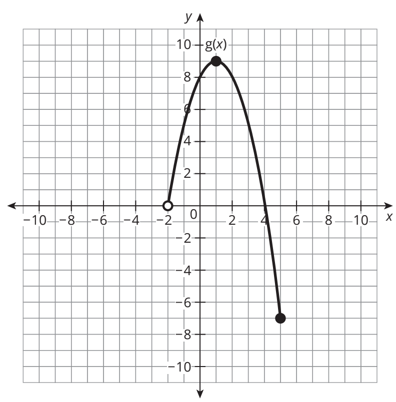
1. Explain how you can identify the domain and range of a function given:

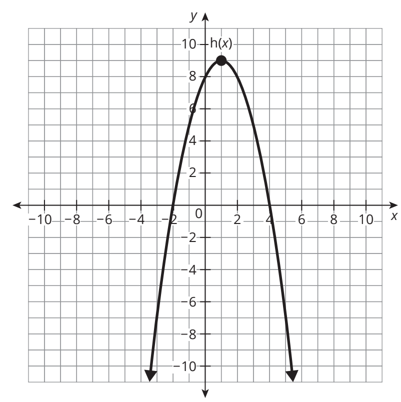
   a. a verbal statement.

   b. a graph.

WORKED EXAMPLE

There are different ways to write the domain and range of a function given its graph.





	Domain		Range	
	$g(x)$	$h(x)$	$g(x)$	$h(x)$
In Words	The domain is all real numbers greater than −2 and less than or equal to 5.	The domain is the set of all real numbers.	The range is all real numbers greater than or equal to −7 and less than or equal to 9.	The range is all real numbers less than or equal to 9.
Using Notation	$-2 < x \leq 5$	$-\infty < x < \infty$	$-7 \leq x \leq 9$	$y \leq 9$

2. Label each of the Graph Cards from the previous activity with the appropriate domain and range.

ACTIVITY 3.3

Linear, Constant, and Exponential Functions

Gather all of the graphs that you identified as functions.

A function is described as increasing when the value of the dependent variable increases as the value of the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the value of the dependent variable decreases as the value of the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the value of the dependent variable of a function remains constant over the entire domain, then the function is called a **constant function**.

When determining whether a graph is increasing or decreasing, read the graph from left to right.

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups listed.
   - Increasing function
   - Decreasing function
   - Constant function
   - A combination of increasing and decreasing

Record the function letter in the appropriate column of the table shown.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing and Decreasing

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Use graphing technology to determine the shape of its graph. Then, match the function to its corresponding graph by writing the function directly on the graph that it represents.

Think About . . .
Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions into your graphing technology.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$, where x is an integer
- $f(x) = 2$, where x is an integer
- $f(x) = 2^x$, where x is an integer
- $f(x) = -x + 3$, where x is an integer

3. Consider the six graphs and functions that are increasing functions, decreasing functions, or constant functions.

   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

   b. What is the same about all the functions in each group?

You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.

Ask Yourself . . .
What other variables have you used to represent a linear function?

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. When $f(x) = ax + b$ represents a linear function, describe the a and b values that produce a constant function.

Place these two groups of graphs off to the side. You will need them again.

ACTIVITY

3.4

Quadratic Functions

A function has an **absolute minimum** if there is a point on the graph of the function that has a *y*-coordinate that is less than the *y*-coordinate of every other point on the graph. A function has an **absolute maximum** if there is a point on the graph of the function that has a *y*-coordinate that is greater than the *y*-coordinate of every other point on the graph.

1. Sort the graphs from the combination of increasing and decreasing category in the previous activity into one of the two groups listed.

   - Those that have an absolute minimum value
   - Those that have an absolute maximum value

   Then, record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Use graphing technology to determine the shape of its graph. Then, match the function to its corresponding graph by writing the function directly on the graph that it represents. Identify the absolute maximum or minimum of each graph.

- $f(x) = x^2 + 8x + 12$
- $f(x) = -3x^2 + 4$, where x is integer
- $f(x) = x^2$

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

ACTIVITY
3.5 Function Families

You have now sorted each of the graphs and equations representing functions into one of three function families: linear, exponential, and quadratic.

1. Glue your sorted graphs and functions to the appropriate function family graphic organizer located at the end of the lesson. Write a description of the graphical behavior for each function family.

Hang on to your graphic organizers. They will be a great resource moving forward!

You've done a lot of work up to this point! You've been introduced to linear, exponential, and quadratic functions. Don't worry—you don't need to know everything there is to know about these function families right now. As you progress through this course, you will learn more about each function family.

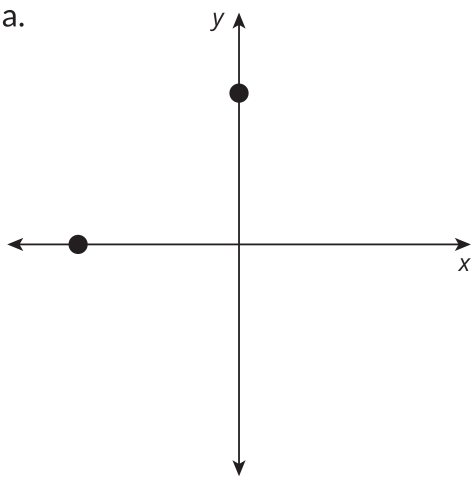
Talk the Talk

Interception!

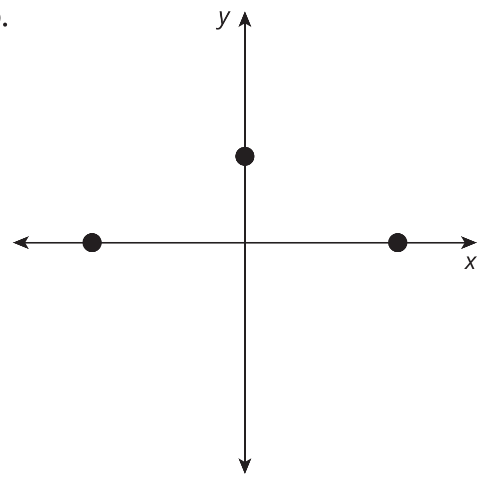
Recall that the **x-intercept** is the point where a graph crosses the *x*-axis. The ***y*-intercept** is the point where a graph crosses the *y*-axis.

1. The graphs shown represent relations with just the *x*- and *y*-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.

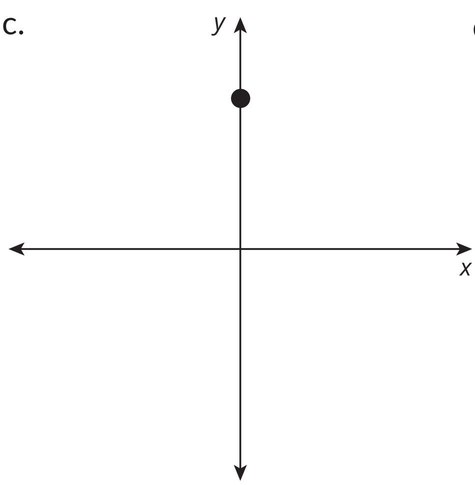
a.



b.



c.



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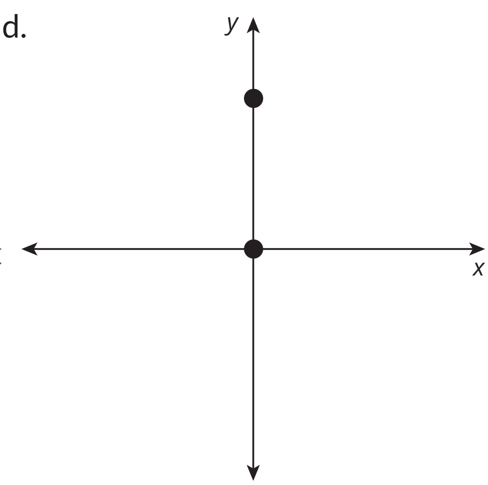


The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

Graphs

Linear Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.

Graphs

Exponential Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

Graphs

Quadratic Functions

Increasing/Decreasing/Constant:

Domain and Range:

Maximum/Minimum:

Curve/Line:

Graphical Behaviors

Lesson 3 Assignment

Write

function notation	increasing function	constant function
absolute maximum	decreasing function	absolute minimum

Choose the term that best completes each statement.

1. A way to represent equations algebraically that makes it more efficient to recognize the independent and dependent variables is called ____________.
2. When both the independent and dependent variables of a function increase across the entire domain, the function is called a(n) ____________.
3. A function has a(n) ____________ if there is a point on its graph that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph.
4. When the dependent variable of a function decreases as the independent variable increases across the entire domain, the function is called a(n) ____________.
5. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a(n) ____________.
6. A function has a(n) ____________ if there is a point on its graph that has a *y*-coordinate that is less than the *y*-coordinate of every other point on the graph.

Remember

A function is a relation that assigns to each element of the domain exactly one element of the range. The different function families include linear functions, exponential functions, and quadratic functions.

Lesson 3 Assignment

Practice

For each scenario, use graphing technology to determine the shape of its graph. Then, identify the function family, whether it is increasing, decreasing, or a combination of both, and whether it is a smooth curve or straight line. Finally, identify whether the function has an absolute minimum or absolute maximum, if there is one.

1. A fitness company is selling DVDs for one of its new cardio routines. Each DVD will sell for \$15. Due to fixed and variable costs, the profit that the company will see after selling x DVDs can be represented by the function $P(x) = 11.5x - 0.1x^2 - 150$.

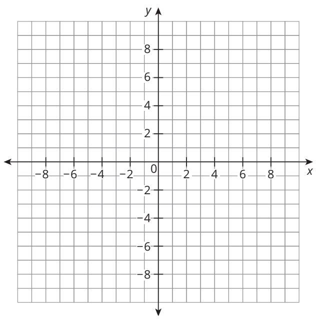
2. Mariana is going to put \$500 into an account with her bank. The bank is offering a 3% interest rate compounded annually. The amount of money that Mariana will have after x years can be represented by the function $A(x) = 500(1.03)^x$.

3. A calendar company is going to buy a new 3D printer for \$20,000. In order to plan for the future, the owners are interested in the salvage value of the printer each year. The salvage value after x years can be represented by the function $S(x) = 20{,}000 - 2000x$.

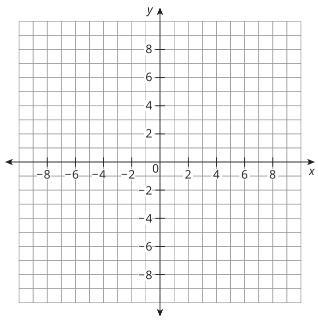
Prepare

1. Sketch a graph and write an equation for each function.

   a. Decreasing linear function



   b. Increasing exponential function



4 Recognizing Functions by Characteristics

OBJECTIVES

- Recognize similar characteristics among function families.
- Recognize different characteristics among function families.
- Determine function types given certain characteristics.

You have identified key characteristics of graphs.

How can the key characteristics help you sketch the graph of a function?

Getting Started

Name That Function!

Function Families
linear
exponential
quadratic

You have sorted graphs according to their function family. Now, consider which function families have the given characteristics.

1. Which function families can be described by the characteristic provided? Choose from the given list.

   a. The graph is a smooth curve.

   b. The graph is made up of a straight line.

   c. The graph increases or decreases over the entire domain.

   d. The graph has an absolute maximum or minimum.

2. One or more characteristics have been added to the graphical description of each function. Name the possible function families.

   a. The graph has an absolute minimum or absolute maximum and is a smooth curve.

   b. The graph either increases or decreases over the entire domain and is a straight line.

   c. The graph is a smooth curve, and either increases or decreases over the entire domain.

Each function family has certain graphical behaviors, with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can name that function!

ACTIVITY

4.1

Categorizing Scenarios into Their Function Families

You have been introduced to several function families: linear, exponential, and quadratic. Let's revisit the first lesson: *Understanding Quantities and Their Relationships*. Each of the scenarios in that lesson represents one of these function families.

Remember . . .
Each of the graphs representing the scenarios was drawn with either a continuous line or a continuous smooth curve to model the problem situation.

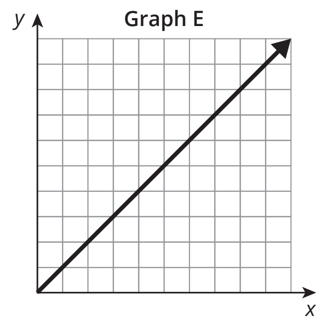
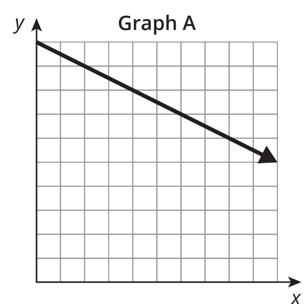
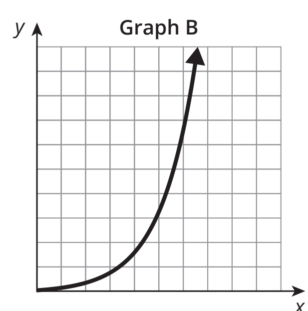
1. Describe how each scenario represents a function.

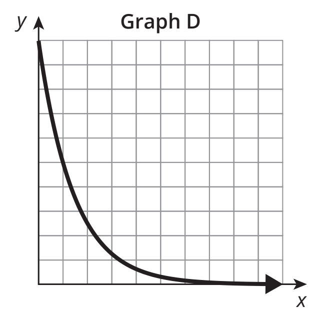
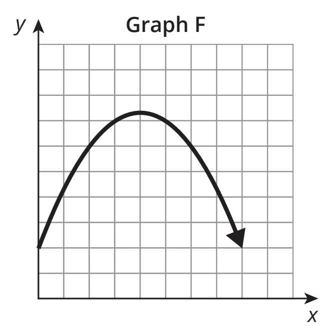
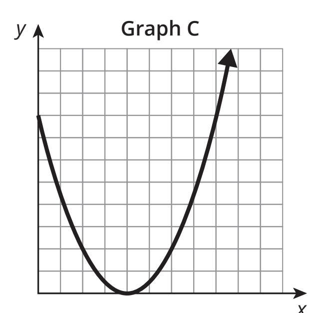
2. Complete the table on the following pages to describe each scenario.

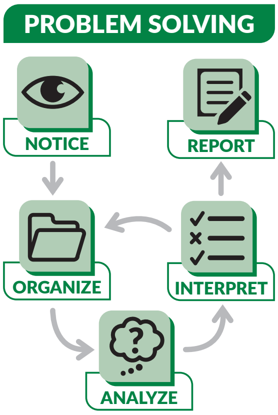
   a. Identify the appropriate function family under the scenario name.

   b. Based on the context, identify the domain as continuous or discrete.

   c. Describe the graphical behavior as increasing, decreasing, constant, or a combination.

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
Music Club			
Something's Fishy			
Smart Phone, but Is It a Smart Deal?			

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
It's Magic			
Baton Twirling			
Skateboarding			



ACTIVITY 4.2 Building Graphs from Characteristics

In this activity, you will write equations and sketch a graph based on given characteristics for the three main function families we will study this year in Algebra I: Linear Functions, Exponential Functions, and Quadratic Functions.

Use the problem-solving model whenever you see this icon.

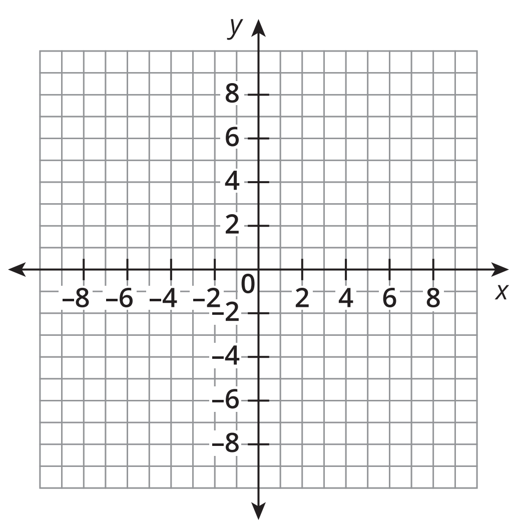
1. Use the given characteristics to create an equation and sketch a graph. Use the equations given in the box as a guide. When creating your equation, use *a*, *b*, and *c* values that are any real numbers between −3 and 3. Do not use any functions that were used previously in this topic.

> **Linear Function**
> $f(x) = ax + b$
> **Exponential Function**
> $f(x) = a \cdot b^x$
> **Quadratic Function**
> $f(x) = ax^2 + bx + c$

Think About . . .
Don't forget about the function family graphic organizers you created if you need some help.

   a. Create an equation and sketch a graph that is:
      - a function,
      - exponential,
      - continuous, and
      - decreasing.

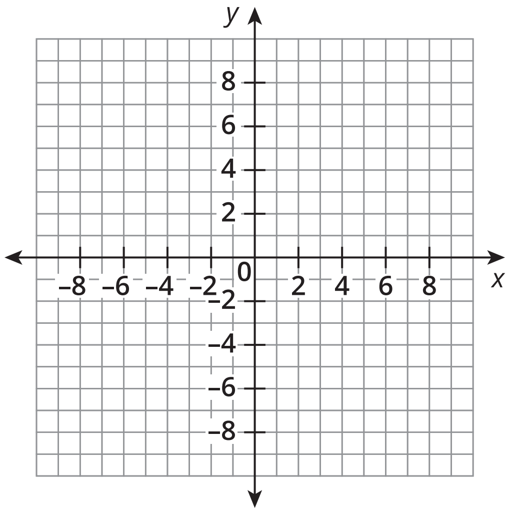
   Equation: ______________________________



b. Create an equation and sketch a graph that:

- has a minimum,
- is discrete, and
- is a quadratic function.

Equation: ______________________________

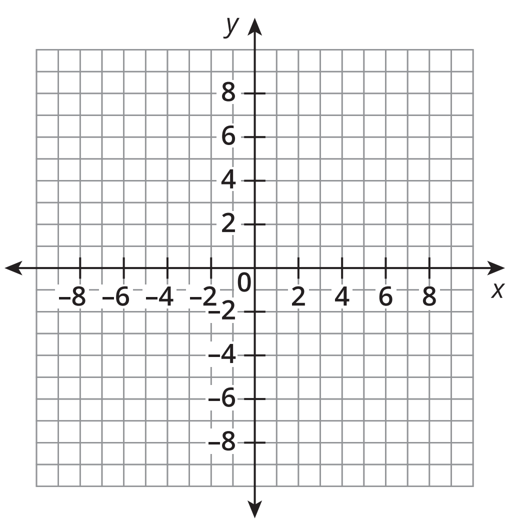


Ask Yourself . . .
Is the domain the same or different for each function?

c. Create an equation and sketch a graph that is:

- linear,
- discrete,
- increasing, and
- a function.

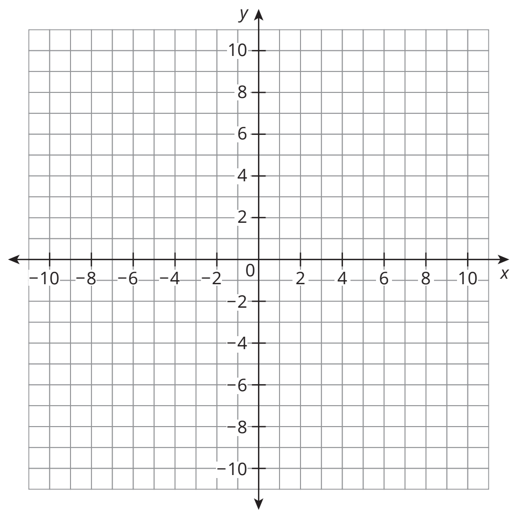
Equation: ______________________________



d. Create an equation and sketch a graph that:

- is continuous,
- has a maximum,
- is a function, and
- is quadratic.

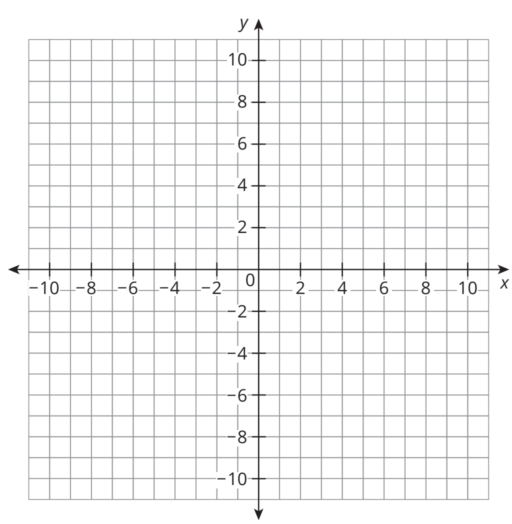
Equation: ______________________________



e. Create an equation and sketch a graph that is:

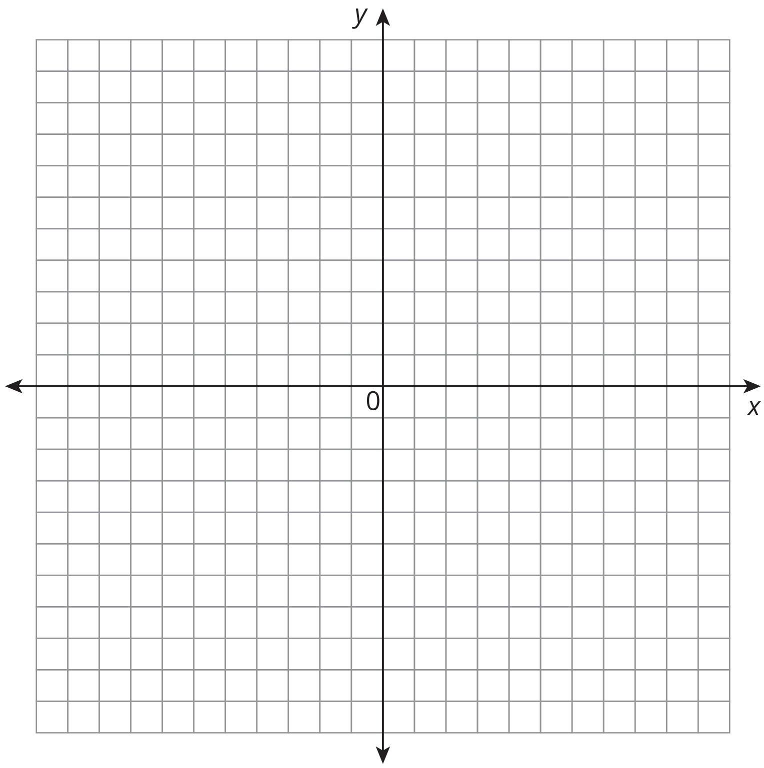
- not a function,
- continuous, and
- a straight line.

Equation: ______________________________

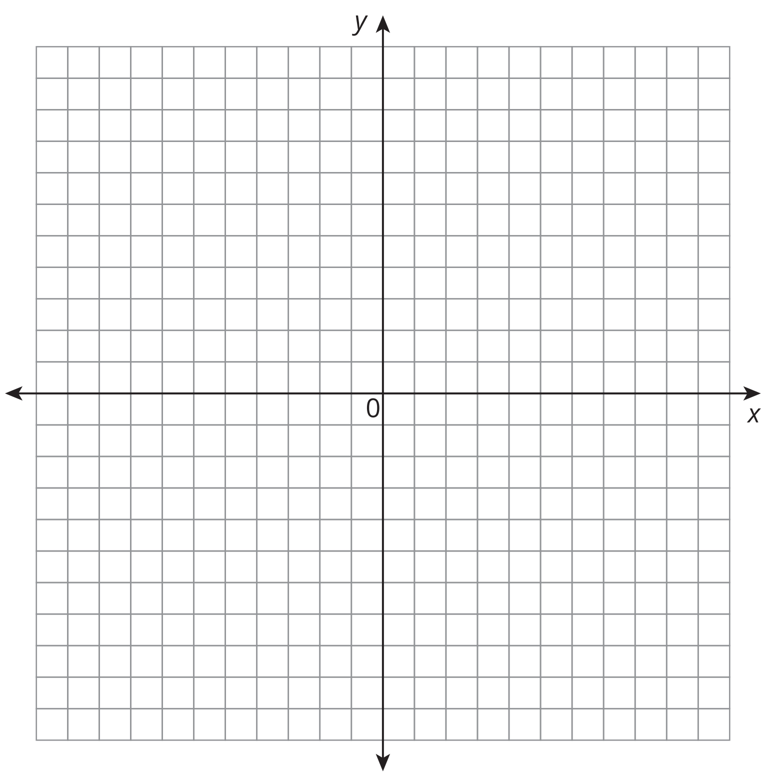


2. Create your own function. Describe certain characteristics of the function and see if your partner can sketch it. Then, sketch your partner's function based on characteristics provided.

Your Function:



Your Partner's Function:



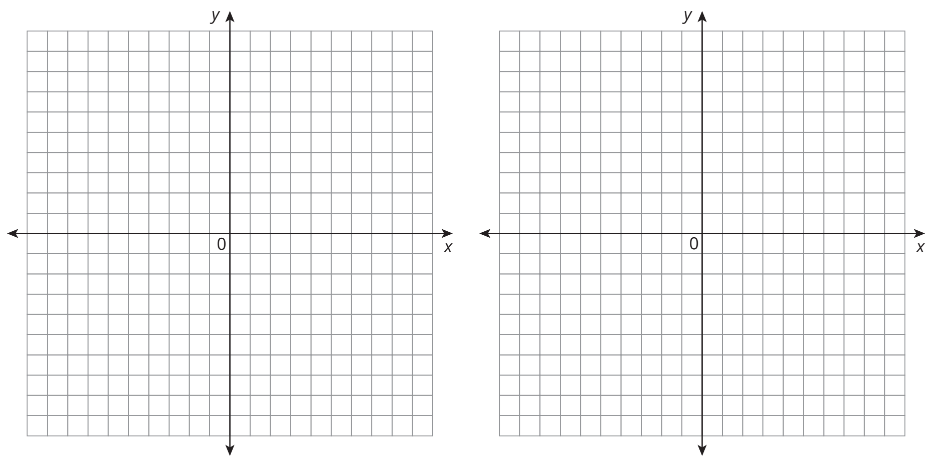
Talk the Talk

Trying to Be Unique

Throughout this lesson, you used characteristics to describe graphs.

1. Write a list of four characteristics to describe a graph.
   - ______________________________
   - ______________________________
   - ______________________________
   - ______________________________

2. Sketch two possible graphs based on your characteristics.



3. How could you modify your list of characteristics to describe a unique graph?

Lesson 4 Assignment

Write

Identify the function family or families that are described by the given characteristic(s). Choose from linear, exponential, and quadratic functions.

1. The graph of this function family has an absolute minimum.
2. The graph of this function family is decreasing over the entire domain.
3. The graph of this function family has an increasing interval and a decreasing interval and forms a U shape.
4. The graph of this function family does not have an absolute maximum or absolute minimum and is a smooth curve.
5. The graph of this function family contains straight lines and does not have an absolute maximum or absolute minimum.

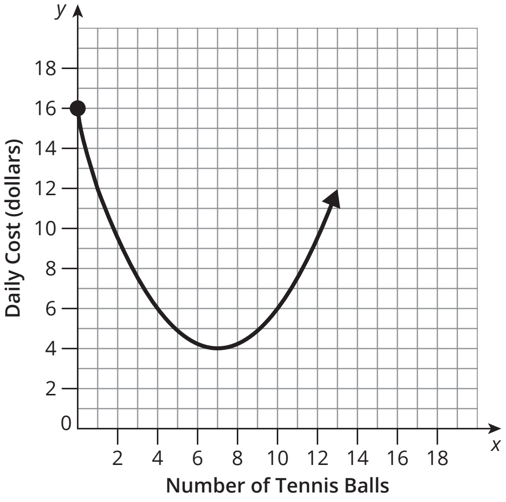
Remember

Function families have key characteristics that are common among all functions in the family. Knowing these key characteristics is useful when sketching a graph of the function.

Practice

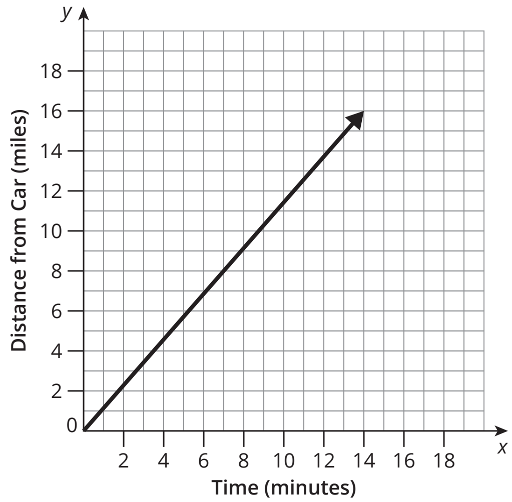
For each scenario and its graph, identify the appropriate function family. Then, based on the problem situation, identify whether the data values represented in the graph are discrete or continuous. Finally, identify the graphical behavior of the function that models the scenario based on the characteristics of its function family.

1. A manufacturing company finds that the daily costs associated with making tennis balls is high if they don't make enough balls and then becomes high again if they make too many balls. The function graphed models the daily costs of making x tennis balls.

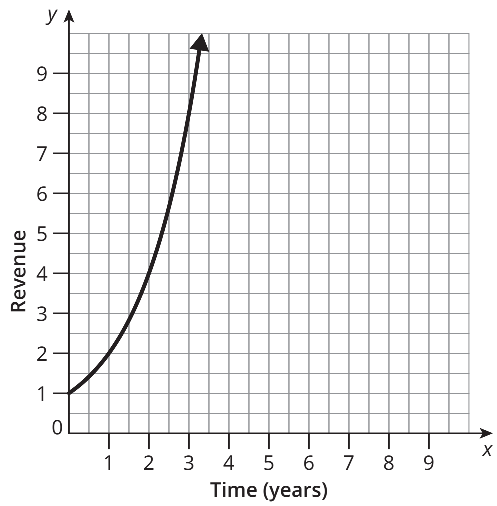


Lesson 4 Assignment

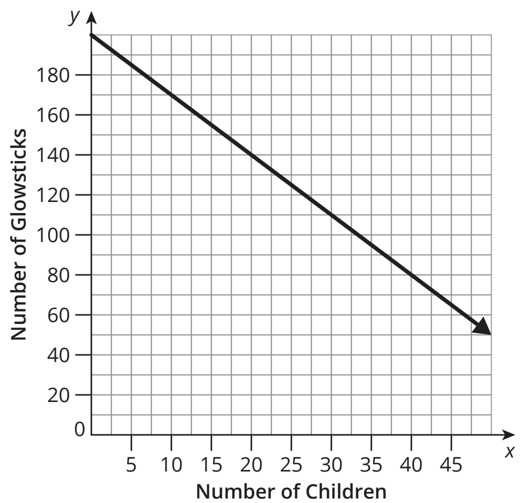
2. Jamal is training for a mountain bike race. He bikes 14 miles. If he bikes at a constant rate, the function graphed models the distance he bikes after x minutes.



3. A local television company determines that the revenue it gets from running ads doubles each year. The function graphed models the revenue from advertising after x years.



4. Mr. Patel's neighborhood is hosting a summer nighttime party in the park. They are handing out glow sticks to all the children who attend. They start with 200 glow sticks and each child receives 3 glow sticks. The function graphed models the number of glow sticks they have left after x children have entered.



Lesson 4 Assignment

Prepare

Write the next three terms in each pattern and explain how you generated each term.

1. J, F, M, A, M, J, J, A, S, ...

2. S, M, T, W, ...

3. 5, 10, 15, 20, ...

4. 100, 81, 64, 49, ...

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Quantities and Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Quantities and Relationships* topic by:

TOPIC 1: *Quantities and Relationships*	Beginning of Topic	Middle of Topic	End of Topic
choosing appropriate scale and origin for graphs.	◯	◯	◯
identifying the appropriate unit of measure for each variable or quantity.	◯	◯	◯
analyzing a graph and stating the key characteristics of the graph.	◯	◯	◯
using a problem situation to explain what the key features of a graph mean in real-world context.	◯	◯	◯
deciding whether relations represented verbally, tabularly, graphically, and symbolically define a function.	◯	◯	◯
recognizing a linear, exponential, or quadratic function by its equation or graph.	◯	◯	◯
evaluating functions, expressed in function notation, given one or more elements in their domain.	◯	◯	◯
determining the domain and range and the independent and dependent quantities in a relationship.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Quantities and Relationships* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Quantities and Relationships Summary

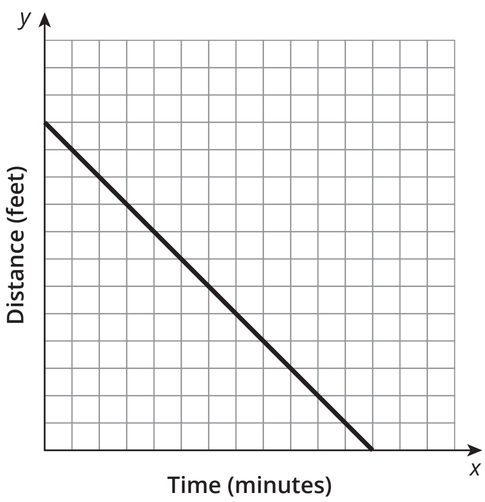
LESSON 1 Understanding Quantities and Their Relationships

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**. The quantity that the dependent quantity depends upon is called the **independent quantity**.

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal, or *x*-axis, while the dependent quantity is graphed on the vertical, or *y*-axis. Graphs can be straight lines or curves and can increase or decrease from left to right.

For example, consider the graph that models the situation where Pedro is walking home from school.

Time, in minutes, is the independent quantity and the distance Pedro is from home is the dependent quantity.



LESSON 2 Analyzing and Sorting Graphs

Looking for patterns can help when sorting and comparing graphs. Some graphs show vertical symmetry (when a vertical line is drawn through the middle of the graph, the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increases from left to right, always decreases from left to right, has straight

NEW KEY TERMS

- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/ discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]

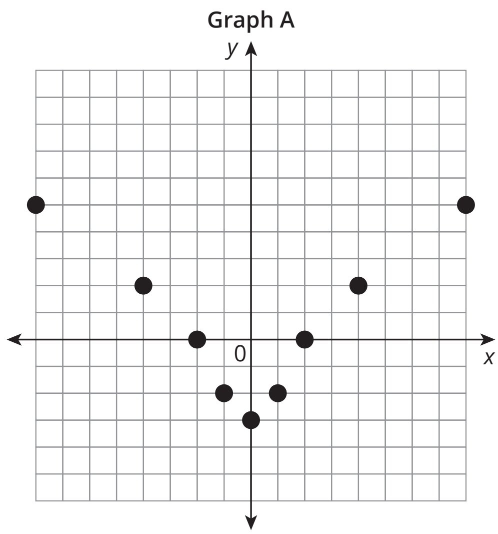
NEW KEY TERMS

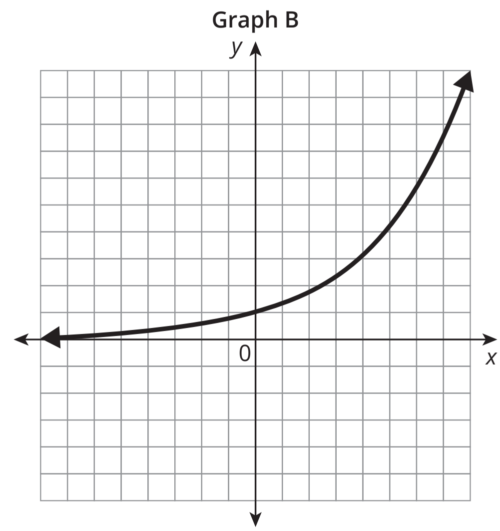
continued

- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

lines, has smooth curves, the graph goes through the origin, the graph forms a U shape, or the graph forms a V shape.

For example, Graph A has vertical symmetry. Graph B is a smooth curve that increases from left to right.



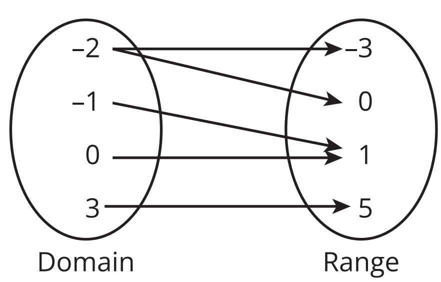


LESSON 3 Recognizing Functions and Function Families

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

A **function** is a relation between a given set of elements such that for each element in the domain, there exists exactly one element in the range. If each value in the domain has one and only one range value, then the relation is a function. If any value in the domain has more than one range value, then the relation is not a function.

The value -2 in the domain has more than one range value. The mapping does not represent a function.



Each element in the domain has exactly one element in the range. The table represents a function.

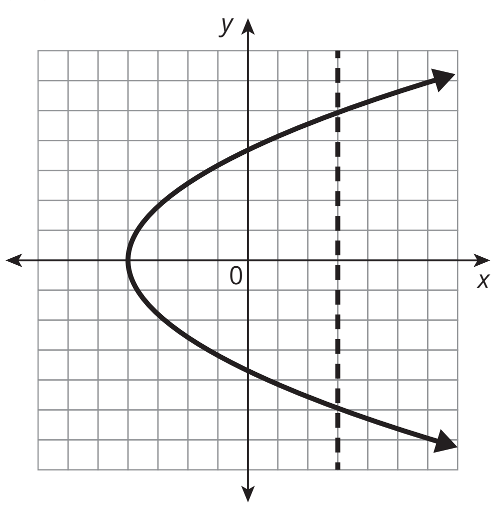
Domain	Range
2	1
6	3
10	5
14	7

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and indicates that x is the independent variable.

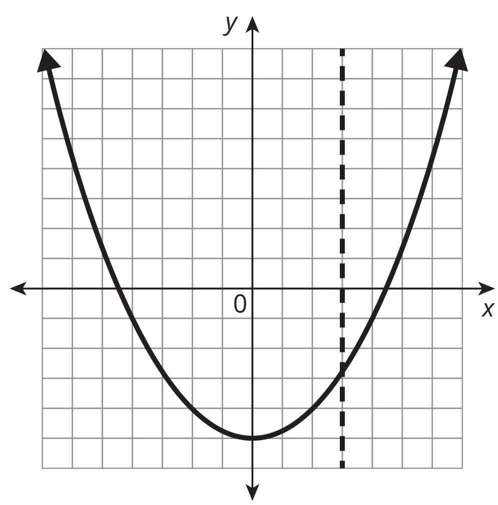
For example, consider the situation in which U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design. The equation that models the situation, $y = 8x + 15$, where x represents the number of shirts ordered and y represents the total cost of the order, can be written in function notation as $f(x) = 8x + 15$. The cost, defined by f, is a function of x, defined as the number of shirts ordered.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function. The Vertical Line Test applies to both discrete and continuous graphs. A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve with no breaks in the graph.

A line drawn vertically through the graph touches more than one point. The graph does not represent a function.



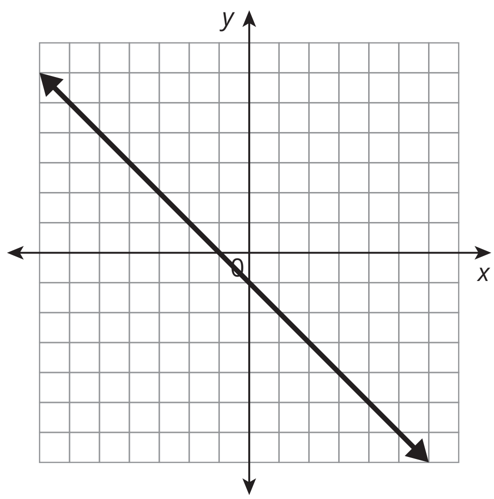
A line drawn vertically through the graph only touches one point. The graph represents a function.



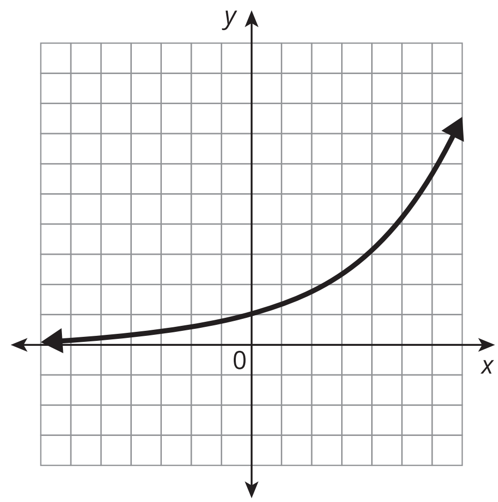
A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an **increasing function**. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.

A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

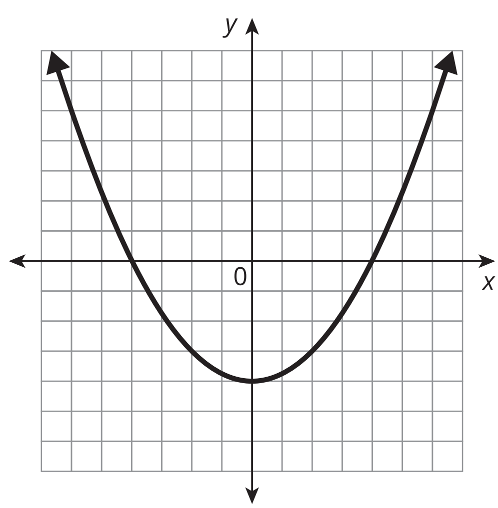


The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.



Quadratic functions have an absolute maximum or an absolute minimum. An **absolute maximum** is a point on the graph of the function that has a y-coordinate that is greater than the y-coordinate of every other point on the graph. An **absolute minimum** is a point on the graph of the function that has a y-coordinate that is less than the y-coordinate of every other point on the graph.

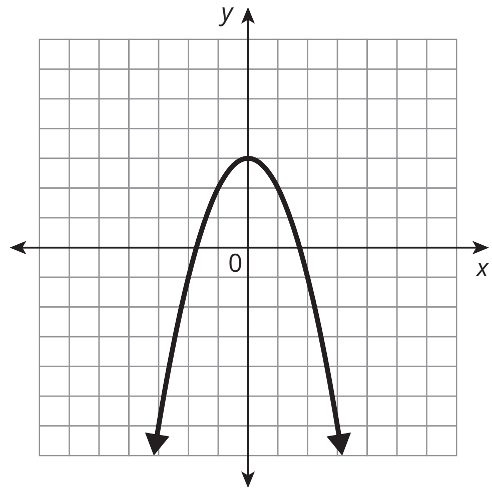
The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.



LESSON 4

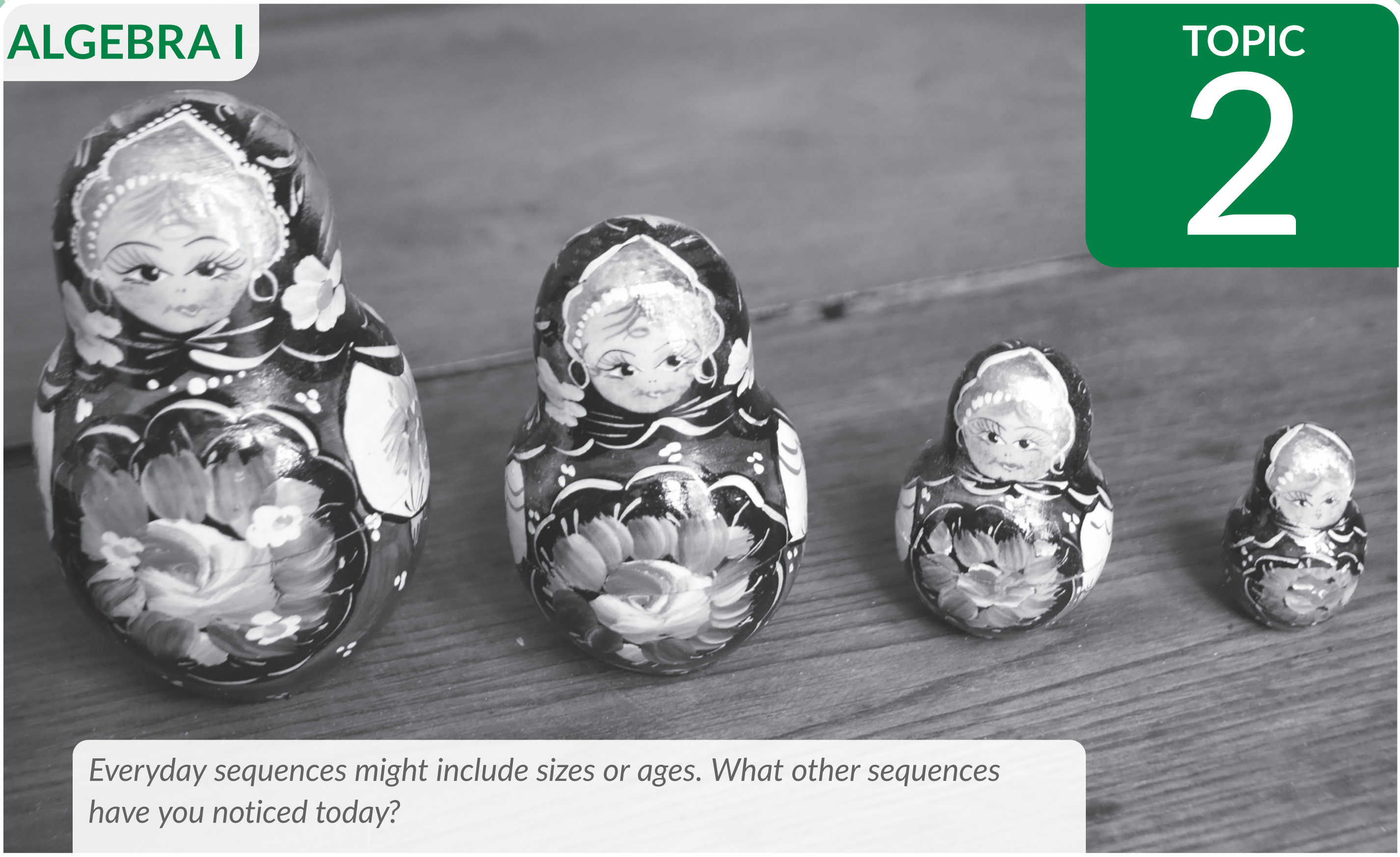
Recognizing Functions by Characteristics

Function families have key characteristics that are common among all functions in the family. Knowing these key characteristics is useful when sketching a graph of the function. For example, to sketch a graph of a continuous quadratic function with a maximum, the graph should be a smooth, U-shaped curve that increases to point and then decreases again.



ALGEBRA I

TOPIC 2



Everyday sequences might include sizes or ages. What other sequences have you noticed today?

Sequences

1 Recognizing Patterns and Sequences

OBJECTIVES

- Recognize and describe patterns.
- Represent patterns as sequences.
- Predict the next term in a sequence.
- Represent a sequence as a table of values.

NEW KEY TERMS

- sequence
- term of a sequence
- infinite sequence
- finite sequence

Since early elementary school, you have been recognizing and writing patterns involving shapes, colors, letters, and numbers.

How are patterns related to sequences and how can sequences be represented using a table of values?

Getting Started

A Pyramid of Patterns

Pascal's Triangle is a famous pattern named after the French mathematician and philosopher Blaise Pascal. A portion of the pattern is shown.

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

1. List at least two patterns that you notice.

2. Describe the pattern for the number of terms in each row.

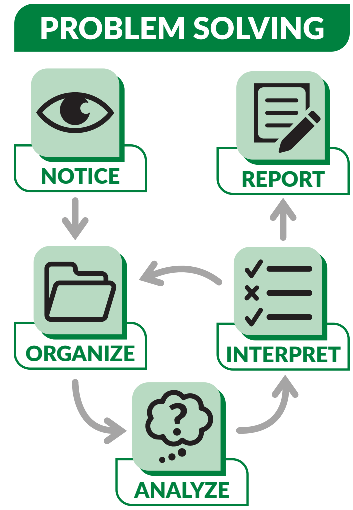
3. Describe the pattern within each row.

4. Describe the pattern that results from determining the sum of each row.

5. Determine the next two rows in Pascal's Triangle. Explain your reasoning.

ACTIVITY
1.1

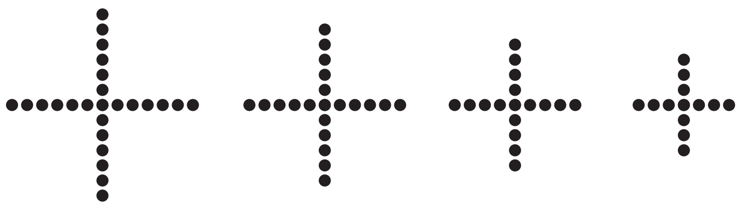
Patterns to Sequences to Tables



A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Four examples of sequences are given in this activity. For each sequence, describe the pattern, draw or describe the next terms, and represent each sequence numerically.

1. **Positive Thinking**



a. Analyze the number of dots. Describe the pattern.

b. Draw the next three figures of the pattern.

c. Represent the number of dots in each of the seven figures as a numeric sequence.

d. Represent the number of dots in each of the first seven figures as a function using a table of values.

Term Number	1	2	3	4	5	6	7
Term Value							

All numeric sequences can be represented as functions. The independent variable is the term number beginning with 1, and the dependent variable is the term value of the sequence.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

2. **Family Tree**
   Trung is investigating her family tree by researching each generation, or set, of parents. She learns all she can about the first four generations, which include her two parents, her grandparents, her great-grandparents, and her great-great-grandparents.

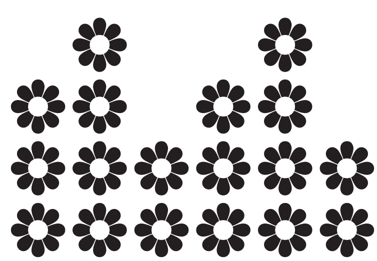
   a. Think about the number of parents. Describe the pattern.

   b. Determine the number of parents in the fifth and sixth generations.

   c. Represent the number of parents in each of the 6 generations as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6
Term Value						

3. **Paola's Pattern**
   Paola is decorating the top border of her bedroom walls with a flower pattern. She is applying decals with each column having a specific number of flowers.



   a. Think about the number of flowers in each column. Describe the pattern.

b. Determine the number of flowers in each of the next two columns.

c. Represent the number of flowers in each of the first 8 columns as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6	7	8
Term Value								

4. **Gamer Guru**
Eduardo is trying to beat his high score on his favorite video game. He unlocks some special mini-games where he earns points for each one he completes. Before he begins playing the mini-games, Eduardo has 500 points. After completing 1 mini-game, he has a total of 550 points; after completing 2 mini-games, he has a total of 600 points; and after completing 3 mini-games, he has a total of 650 points.

a. Think about the total number of points Eduardo gains from mini-games. Describe the pattern.

Ask Yourself . . .
How else can you represent this information?

b. Determine Eduardo's total points after he plays the next two mini-games.

c. Represent Eduardo's total points after completing each of the first 5 mini-games as a numeric sequence. Be sure to include the number of points he started with. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6
Term Value						

ACTIVITY 1.2

Looking at Sequences More Closely

There are many different patterns that can generate a sequence of numbers. For example, you may have noticed that some of the sequences in the previous activity were generated by performing the same operation using a constant number. In other sequences, you may have noticed a different pattern.

The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.

1. For each sequence in the previous activity, write the numeric sequence, record whether the sequence increases or decreases, and describe the sequence by stating the first term and the operation(s) used to create the sequence. The first one has been completed for you.

Problem Name	Numeric Sequence	Increases or Decreases	Sequence Description
Positive Thinking	25, 21, 17, 13, 9, 5, 1	Decreases	Begin at 25. Subtract 4 from each term.
Family Tree			
Paola's Pattern			
Gamer Guru			

2. Which sequences are similar? Explain your reasoning.

3. What do all sequences have in common?

4. Consider a sequence in which the first term is 64 and each term after that is calculated by dividing the previous term by 4. Elena says that this sequence ends at 1 because there are no whole numbers that come after 1. Chris disagrees and says that the sequence continues beyond 1. Who is correct? If Elena is correct, explain why. If Chris is correct, predict the next two terms of the sequence.

5. What is the domain of a sequence? What is the range?

When a sequence continues on forever, it is called an **infinite sequence**. If a sequence terminates, it is called a **finite sequence**.

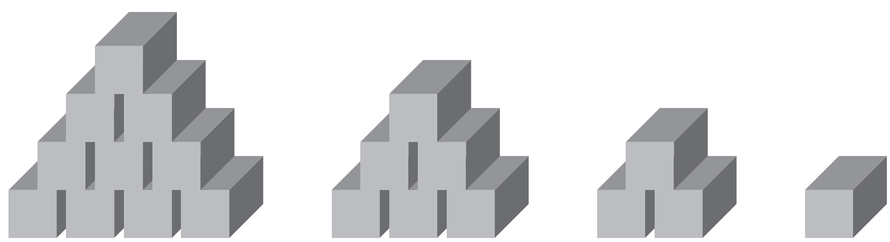
An ellipsis is three periods, which means "and so on." An infinite sequence can be represented using an ellipsis.

For example, consider an auditorium where the seats are arranged according to a specific pattern. There are 22 seats in the first row, 26 seats in the second row, 30 seats in the third row, and so on. Numerically, the sequence is 22, 26, 30, ... , which continues infinitely. However, in the context of the problem, it does not make sense for the number of seats in each row to increase infinitely. Eventually, the auditorium would run out of space! Suppose that this auditorium can hold a total of 10 rows of seats. The correct sequence for this problem situation is:

22, 26, 30, 34, 38, 42, 46, 50, 54, 58.

Therefore, because of the problem situation, the sequence is a finite sequence.

6. Does the pattern shown represent an infinite or finite sequence? Explain your reasoning.



7. For each sequence in the previous activity, write the domain and range using set notation.

    a. Positive Thinking

    b. Family Tree (6 generations):

    c. Paola's Pattern (8 columns)

    d. Gamer Guru (completes 5 mini games)

8. For each sequence in the previous activity, determine whether the sequence is a function.

    a. Positive Thinking

    b. Family Tree

    c. Paola's Pattern

    d. Gamer Guru

Talk the Talk

Searching for a Sequence

In this lesson, you have seen that many different patterns can generate a sequence of numbers.

1. Explain why the definition of a function applies to all sequences.

2. Create a sequence to fit the given criteria. Describe your sequence using figures, words, or numbers. Provide the first four terms of the sequence. Explain how you know that it is a sequence.

   a. Create a sequence that begins with a positive integer, is decreasing by multiplication, and is finite.

   b. Create a sequence that begins with a negative rational number, is increasing by addition, and is infinite.

Lesson 1 Assignment

Write

Explain why all sequences can be described as functions.

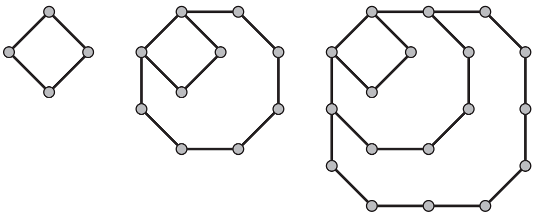
Remember

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. All numeric sequences can be represented as functions.

Practice

Consider the three sequences given. For each sequence, describe the pattern. Then, represent the sequence as a numeric sequence and as a table of values, including the first six terms.

1. **Stellar Segments**



Term Number						
Term Value						

2. **Welcome Home**
   A local construction company specializes in building and selling homes. When the company first started, they sold 1 home the first month, 3 homes the second month, 9 homes the third month, and 27 homes the fourth month.

Term Number						
Term Value						

Lesson 1 Assignment

3. **Samantha's Streaming**
   Samantha is a yoga instructor who regularly streams exercise videos on a website for her clients. One week after launching the website, she had posted a total of 6 videos. At the end of week 2, she had a total of 10 videos. At the end of week 3, she had a total of 14 videos. At the end of week 4, she had a total of 18 videos.

Term Number						
Term Value						

Prepare

Write the next three terms in each sequence. Explain how you determined each term and whether it is a function.

1. $-2, 4, -8, 16, \ldots$

2. 60, 53, 46, 39, 32, ...

3. 1, 5, 17, 53, 161, 485, ...

4. 4, 10, 16, 22, ...

2 Arithmetic and Geometric Sequences

OBJECTIVES

- Determine the next term in a sequence.
- Recognize arithmetic sequences and geometric sequences.
- Determine the common difference or common ratio for a sequence.
- Graph arithmetic and geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.
- Generate terms of a sequence when the sequence is given in function form using the recursive process.

NEW KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

You have represented patterns as sequences of numbers—a relationship between term numbers and term values.

What patterns appear when sequences are represented as graphs?

Getting Started

What Comes Next, and How Do You Know?

Review the definition of **describe** in the Academic Glossary.

Cut out Sequences *A* through *H* located at the end of the lesson.

1. Determine the unknown terms of each sequence. Then, describe the pattern under each sequence.

2. Sort the sequences into groups based on common characteristics. In the space provided, record the following information for each of your groups.
   - List the letters of the sequences in each group.
   - Provide a rationale as to why you created each group.

3. What mathematical operation(s) did you perform in order to determine the next terms of each sequence?

ACTIVITY 2.1

Defining Arithmetic and Geometric Sequences

For some sequences, you can describe the pattern as adding a constant to each term to determine the next term. For other sequences, you can describe the pattern as multiplying each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable *d*.

Remember:
When you add a negative number, it is the same as subtracting a positive number.

The common difference of a sequence is positive when the same positive number is added to each term to produce the next term. The common difference of a sequence is negative when the same negative number is added to each term to produce the next term.

WORKED EXAMPLE

Consider the sequence generated using $a_n = a_{n-1} + (-2)$, where $a_1 = 11$ and n is a whole number greater than 1.

a_1 represents the first term of the sequence and a_n represents the n^{th} term of the sequence.

Since I know the first term of the sequence, to determine the second term a_2, I add -2 to 11.

$a_2 = a_{2-1} + -2 = a_1 + -2 = 11 + -2 = 9$

I can determine the 3rd and 4th term of the sequence by continuing the pattern.

$a_3 = a_{3-1} + -2 = a_2 + -2 = 9 + -2 = 7$

$a_4 = a_{4-1} + -2 = a_3 + -2 = 7 + -2 = 5$

The sequence is 11, 9, 7, 5, ...

The pattern is to add the same negative number, -2, to each term to determine the next term.

$$11 \xrightarrow{\text{add } -2} 9 \xrightarrow{\text{add } -2} 7 \xrightarrow{\text{add } -2} 5$$

Sequence: 11, 9, 7, 5, ...

This sequence is arithmetic and the common difference *d* is -2.

a_{n-1} is the term before a_n. So, when $n = 2$, a_{n-1} is a_1.

Ask Yourself . . .
How does the function notation relate to the notation in the Worked Example?

You can also write the sequence from the Worked Example in function form. Given that $f(1) = 11$ and n is a whole number greater than 1, the function $f(n) = f(n-1) + -2$ represents the sequence.

1. Suppose a sequence has the same starting number as the sequence in the Worked Example, but is generated by $a_n = a_{n-1} + 4$.

   a. Write the first four terms of the sequence.

   b. How does the pattern change?

   c. Is the sequence still arithmetic? Why or why not?

2. Analyze the sequences you cut out in the Getting Started.

   a. List the sequences that are arithmetic.

   b. Write the common difference of each arithmetic sequence you identified.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the *common ratio*. The **common ratio** is represented by the variable *r*.

WORKED EXAMPLE

Consider the sequence generated by the function $g_n = g_{n-1} \cdot 2$ where $g_1 = 1$ and n is a whole number greater than 1.

Since I know the first term of the sequence, to determine the second term g_2, I multiply 1 by 2.

$$g_2 = g_1 \cdot 2 = 1 \cdot 2 = 2$$
$$g_3 = g_2 \cdot 2 = 2 \cdot 2 = 4$$
$$g_4 = g_3 \cdot 2 = 4 \cdot 2 = 8$$

The sequence is 1, 2, 4, 8, ...

The pattern is to multiply each term by the same number, 2, to determine the next term.

multiply by 2 multiply by 2 multiply by 2

Sequence: $\underline{\ 1\ }$, $\underline{\ 2\ }$, $\underline{\ 4\ }$, $\underline{\ 8\ }$, ...

This sequence is geometric and the common ratio *r* is 2.

Ask Yourself . . .
How does the function notation relate to the notation in the Worked Example?

You can also write the sequence from the Worked Example in function form. Given that $f(1) = 1$ and n is a whole number greater than 1, the function $f(n) = 2f(n-1)$ describes the sequence.

3. Suppose a sequence has the same starting number as the sequence in the Worked Example, but is generated by $a_n = a_{n-1} \cdot 3$.

   a. Write the first four terms of the sequence.

   b. How does the pattern change?

c. Is the sequence still geometric? Explain your reasoning.

4. Suppose a sequence has the same starting number as the sequence in the Worked Example, but its common ratio is $\frac{1}{3}$.

   a. How would the pattern change?

   b. Is the sequence still geometric? Why or why not?

   c. Write the first four terms of the new sequence.

5. Suppose a sequence has the same starting number as the sequence in the Worked Example, but it is generated by $g_n = 1 \cdot (-2)^{n-1}$.

   a. How would the pattern change?

   b. Is the sequence still geometric? Explain your reasoning.

   c. Write the first four terms of the new sequence.

6. Consider the sequence shown.

$$270, 90, 30, 10, \ldots$$

Jorge says that he can determine each term of this sequence by multiplying each term by $\frac{1}{3}$, so the common ratio is $\frac{1}{3}$. Jaylen says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.

7. Consider the sequences you cut out in the Getting Started. List the sequences that are geometric. Then, write the common ratio on each Sequence Card.

8. Consider the sequences that are neither arithmetic nor geometric. List these sequences. Explain why these sequences are neither arithmetic nor geometric.

9. Consider the first two terms of the sequence 3, 6, …

Mariana says, "This is how I wrote the sequence for the given terms."

3, 6, 9, 12, …

Ashley says, "This is the sequence I wrote."

3, 6, 12, 24, …

Who is correct? Explain your reasoning.

10. Consider the sequence 2, 2, 2, 2, 2 … Identify the type of sequence it is and describe the pattern.

11. Begin to complete the graphic organizers located at the end of the lesson to identify arithmetic and geometric sequences. Glue each arithmetic sequence and each geometric sequence to a separate graphic organizer according to its type. Discard all other sequences.

ACTIVITY

2.2 Matching Graphs and Sequences

As you have already discovered when studying functions, graphs can help you see trends of a sequence—and at times can help you predict the next term in a sequence.

1. The graphs representing the arithmetic and geometric sequences from the previous activity are located at the end of this lesson. Cut out these graphs. Match each graph to its appropriate sequence and glue it into the Graph section of its graphic organizer.

2. What strategies did you use to match the graphs to their corresponding sequences?

3. How can you use the graphs to verify that all sequences are functions?

Talk the Talk

Name That Sequence!

Write the first five terms of each sequence described and identify the sequence as arithmetic or geometric.

1. The first term of the sequence is 0 and the common difference is -6.

2. The first term of the sequence is -3 and the common ratio is $-\frac{1}{4}$.

Sequence Cards

A

$-2, -6, -18, -54,$

$\_\_\_\_\_, \_\_\_\_\_, \ldots$

B

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \_\_\_\_\_, \_\_\_\_\_, \ldots$

C

$1, -2, 3, -4, \_\_\_\_\_, \_\_\_\_\_, \ldots$

D

$-20, -16, -12, -8, -4,$

$\_\_\_\_\_, \_\_\_\_\_, \ldots$

E

$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8},$

$\_\_\_\_\_, \_\_\_\_\_, \ldots$

F

$86, 85, 83, 80, 76, \_\_\_\_\_, \_\_\_\_\_, \ldots$

G

$-16, 4, -1, \frac{1}{4}, \_\_\_\_\_, \_\_\_\_\_, \ldots$

H

$1473.2, 1452.7, 1432.2, 1411.7,$

$\_\_\_\_\_\_\_, \_\_\_\_\_\_\_, \ldots$

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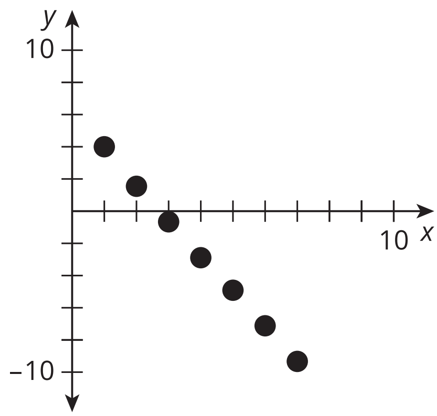
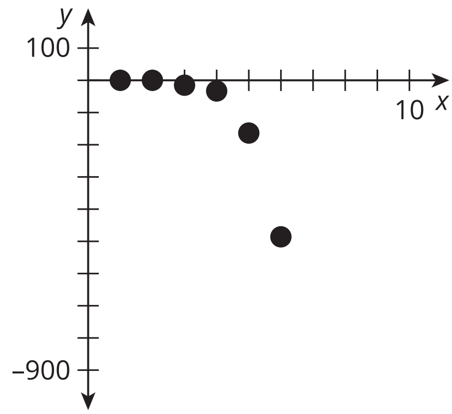
So you can cut out the Sequence Cards on the other side

Graph Cards

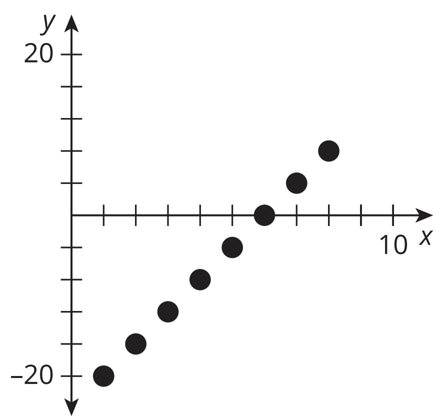
Graph 1



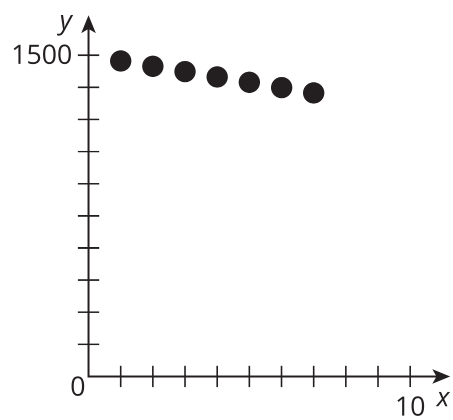
Graph 2



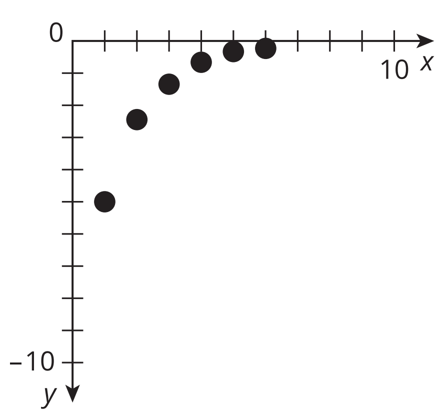
Graph 3



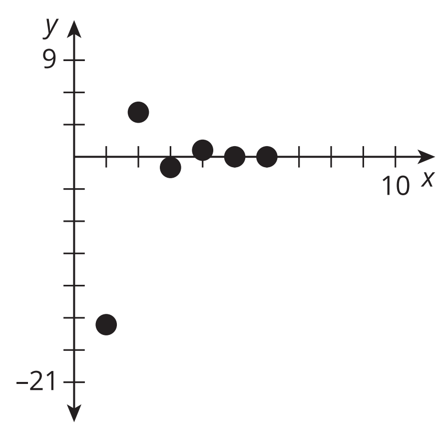
Graph 4



Graph 5



Graph 6



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Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Arithmetic Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Geometric Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Geometric Sequence

Recursive Formula

Explicit Formula

Sequence

Graph

Geometric Sequence

Recursive Formula

Explicit Formula

Lesson 2 Assignment

Write

Complete each sentence.

1. A sequence which terminates is called a(n) ______________ .
2. A(n) ______________ is an individual number, figure, or letter in a sequence.
3. A(n) ______________ is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.
4. A sequence which continues forever is called a(n) ______________ .

Remember

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant.

A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant.

Practice

Consider the first two terms of the sequence 28, 14, …

1. Determine whether the sequence is arithmetic or geometric. Explain your reasoning.

2. Suppose the sequence 28, 14, . . . is arithmetic.

   a. Determine the common difference.

   b. List the next 3 terms in the sequence. Explain your reasoning.

   c. Determine whether the sequence is finite or infinite. Explain your reasoning.

Lesson 2 Assignment

3. Suppose the sequence 28, 14, ... is geometric.

    a. Determine the common ratio.

    b. List the next 3 terms in the sequence. Explain your reasoning.

    c. Determine whether the sequence is finite or infinite. Explain your reasoning.

4. Using the first two terms 28 and 14, write the next three terms of a sequence that is neither arithmetic nor geometric.

5. A sequence is generated by the function $a_n = a_{(n-1)} + 5$ where $a_1 = 4\frac{1}{2}$ and n is a whole number greater than 1.

    a. Determine the first four terms of the sequence.

    b. Identify the sequence type and the common difference or common ratio.

Lesson 2 Assignment

6. A sequence is generated by the function $a_n = a_{n-1} \cdot \frac{1}{4}$ where $a_1 = 12$ and n is a whole number greater than 1.

   a. Determine the first four terms of the sequence.

   b. Identify the sequence type and the common difference or common ratio.

Prepare

The local bank has agreed to donate $250 to the annual turkey fund to help feed families in need. In addition, for every bank customer that donates $50, the bank will donate $25.

1. A sequence describes the relationship between the number of $50 donations and the amount of the bank's donation. Is the sequence arithmetic or geometric?

2. How can you calculate the 10th term based on the 9th term?

3. What is the 20th term?

3 Determining Recursive and Explicit Expressions from Contexts

OBJECTIVES

- Write recursive formulas for arithmetic and geometric sequences from contexts.
- Write explicit expressions for arithmetic and geometric sequences from contexts.
- Use formulas to determine unknown terms of a sequence.

NEW KEY TERMS

- recursive formula
- explicit formula

You have learned that arithmetic and geometric sequences always describe functions. How can you write equations to represent these functions?

Getting Started

Can I Get a Formula?

Notice that the first term in this sequence is the amount Jamal donates if the team hits 0 home runs.

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence?

Consider the sequence represented in this situation.

Jamal owns a sporting goods store. He has agreed to donate $125 to his old high school's baseball team for their equipment fund. In addition, he will donate $18 for every home run the team hits during the season. The sequence shown represents the possible dollar amounts that Jamal could donate for the season.

125, 143, 161, 179, ...

Number of Home Runs	Term Number (n)	Donation Amount (dollars)
0	1	
1		
2		
3		
4		
5		
6		
7		
8		
9		

1. Identify the sequence type. Describe how you know.

2. Determine the common difference or common ratio for the sequence.

3. Complete the table.

4. Explain how you can calculate the tenth term based on the ninth term.

ACTIVITY 3.1

Writing Formulas for Arithmetic Sequences

In the previous lesson you used the **recursive formula** to determine term values in a sequence. The recursive formula expresses each new term of a sequence based on the preceding term in the sequence. The recursive formula to determine the *n*th term of an arithmetic sequence is:

$$\text{nth term} \longrightarrow a_n = \underbrace{a_{n-1}}_{\text{previous term}} + d \longleftarrow \text{common difference}$$

WORKED EXAMPLE

Consider the sequence −2, −9, −16, −23, ...
You can use the recursive formula to determine the 5th term.

$$a_n = a_{n-1} + d$$
$$a_5 = a_{5-1} + -7$$

The expression a_5 represents the 5th term. The previous term is −23, and the common difference is −7.

$$a_5 = a_4 + -7$$
$$a_5 = -23 + -7$$
$$a_5 = -30$$

The 5th term of the sequence is −30.

You only need to know the previous term and the common difference to use the recursive formula.

Ask Yourself . . .
What observations can you make?

Consider the sequence showing Jamal's contribution to his old high school's baseball team in terms of the number of home runs hit.

1. Use a recursive formula to determine the 11th term in the sequence. Explain what this value means in terms of this problem situation.

Ask Yourself . . .
Did you justify your mathematical reasoning?

2. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

You can determine the 93rd term of the sequence by calculating each term before it and then adding 18 to the 92nd term, but this will probably take a while! A more efficient way to calculate any term of a sequence is to use an *explicit formula*.

An **explicit formula** of a sequence is a formula to calculate the *n*th term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:

*n*th term

common difference

$$a_n = a_1 + d\underbrace{(n - 1)}_{\text{previous term number}}$$

1st term

Remember . . .

The first term in this sequence is the amount Jamal donates when the team hits 0 home runs. So, the 93rd term represents the amount Jamal donates if the team hits 92 home runs.

WORKED EXAMPLE

You can use the explicit formula to determine the 93rd term in this problem situation.

$$a_n = a_1 + d(n - 1)$$

$$a_{93} = 125 + 18(93 - 1)$$

The expression a_{93} represents the 93rd term. The first term is 125, and the common difference is 18.

$$a_{93} = 125 + 18(92)$$

$$a_{93} = 125 + 1656$$

$$a_{93} = 1781$$

The 93rd term of the sequence is 1781.

This means Jamal will contribute a total of \$1781 when his old high school's baseball team hits 92 home runs.

3. Use the explicit formula to determine the amount of money Jamal will contribute for each number of home runs hit.

   a. 35 home runs

   b. 48 home runs

   c. 86 home runs

   d. 214 home runs

Jamal decides to increase his initial contribution and amount donated per home run hit. He decides to contribute $500 and will donate $75 for every home run the team hits.

4. Write the first five terms of the sequence representing the new contribution Jamal will donate to the the baseball team.

5. Determine Jamal's contribution for each number of home runs hit.

   a. 39 home runs

   b. 50 home runs

WORKED EXAMPLE

Use the distributive property to rewrite the explicit formula for the problem situation.

The explicit formula for the problem situation is $a_n = 125 + 18(n - 1)$.

Step 1: Use the distributive property.

$a_n = 125 + 18n - 18.$

Step 2: Combine like terms.

$a_n = 18n + 107$

6. For each arithmetic sequence, identify the first term and the common difference. Then, use the distributive property to rewrite the explicit formula.

a. $p_n = 16 - \frac{1}{2}(n - 1)$

b. $k_n = 0.6 + 0.2(n - 1)$

c. $r_n = -7(n - 1) - 2$

ACTIVITY 3.2
Writing Formulas for Geometric Sequences

When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding one too many! Then again, when it comes to cells, the more the better. Animals, plants, fungi, slime, molds, and other living creatures are composed of eukaryotic cells. During growth, generally there is a cell called a "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on.

Notice that the 1st term in this sequence is the total number of cells after 0 divisions (that is, the mother cell).

1. The sequence shown represents the growth of eukaryotic cells.

   1, 2, 4, 8, 16, ...

   a. Describe why this sequence is geometric and identify the common ratio.

   b. Complete the table of values. Use the number of cell divisions to identify the term number and the total number of cells after each division.

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	
1		
2		
3		
4		
5		
6		
7		
8		
9		

   c. Explain how you can calculate the tenth term based on the ninth term.

In the previous lesson, you used the recursive formula for a geometric sequence. The recursive formula to determine the nth term of a geometric sequence is:

nth term

common ratio

$$a_n = \underbrace{a_{n-1}}_{\text{previous term}} \cdot r$$

WORKED EXAMPLE

Consider the sequence shown.

$$4, 12, 36, 108, \ldots$$

You can use the recursive formula to determine the 5th term.

$$b_n = b_{n-1} \cdot r$$
$$b_5 = b_{5-1} \cdot (3)$$

The expression a_5 represents the 5th term. The previous term is 108, and the common ratio is 3.

$$b_5 = b_4 \cdot (3)$$
$$b_5 = 108 \cdot (3)$$
$$b_5 = 324$$

The 5th term of the sequence is 324.

Consider the sequence of cell divisions and the total number of resulting cells.

2. Write a recursive formula for the sequence and use the formula to determine the 12th term in the sequence. Explain what your result means in terms of this problem situation.

The explicit formula to determine the nth term of a geometric sequence is:

nth term

previous term number

$$a_n = a_1 \cdot r^{\overbrace{n-1}}$$

1st term

common ratio

WORKED EXAMPLE

You can use the explicit formula to determine the 20th term in this problem situation.

$$a_n = a_1 \cdot r^{n-1}$$
$$a_{20} = 1 \cdot 2^{20-1}$$

The expression a_{20} represents the 20th term. The first term is 1, and the common ratio is 2.

$$a_{20} = 1 \cdot 2^{19}$$
$$a_{20} = 1 \cdot 524{,}288$$
$$a_{20} = 524{,}288$$

The 20th term of the sequence is 524,288.

This means that after 19 cell divisions, there are a total of 524,288 cells.

Remember . . .
The first term in this sequence is the total number of cells after 0 divisions. So, the 20th term represents the total number of cells after 19 divisions.

3. Use the explicit formula to determine the total number of cells for each number of divisions.

   a. 11 divisions

   b. 14 divisions

   c. 18 divisions

   d. 22 divisions

Remember . . .
You can use parentheses or a · to represent multiplication.

Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change when each mother cell divides into 3 daughter cells.

4. Write the first five terms of the sequence for the scientist's hypothesis.

5. Determine the total number of cells in the petri dish for each number of divisions.

   a. 13 divisions

   b. 16 divisions

ACTIVITY 3.3

Writing Recursive and Explicit Formulas

In the previous lesson, you identified sequences as either arithmetic or geometric and then matched a corresponding graph.

1. Go back to the graphic organizers from the previous lesson. Write the recursive and explicit formulas for each sequence.

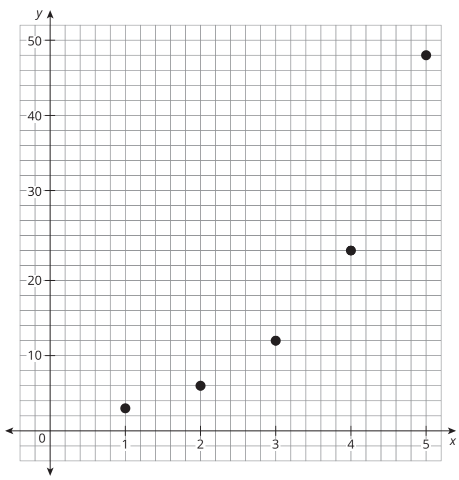
ACTIVITY 3.4

Arithmetic and Geometric Sequences

1. For each sequence, identify the sequence type and the common difference or ratio. Write an explicit formula and a recursive formula to represent the sequence.

   a. $a_1 = 2.3, a_2 = 0.3, a_3 = -1.7, a_4 = -3.7$

   b.

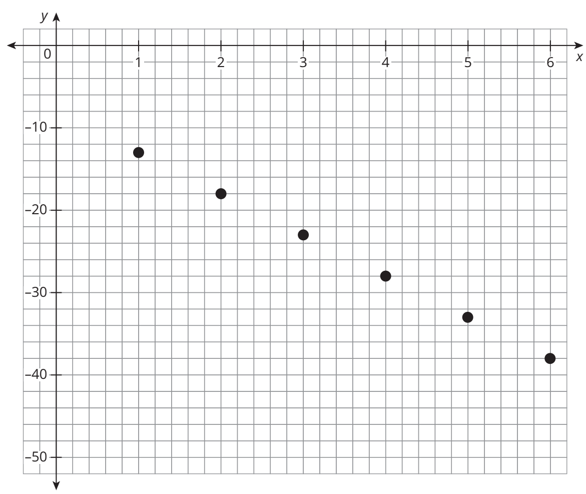


   c.

n	c_n
4	−567
5	189
6	−63
7	21
8	−7

d. $a_5 = -5.9,\ a_6 = -5.3,\ a_7 = -4.7,\ a_8 = -4.1$

e.



2. The local diner makes the best omelets in town! The chef begins each day with 150 eggs to make his famous omelets. After making 1 omelet, he has 144 eggs. After making 2 omelets, he has 138 eggs left. After making 3 omelets, he has 132 eggs left. Write a recursive and explicit formula to represent the sequence. Then, determine how many eggs the chef will have left after he makes 20 omelets.

3. Tiara is participating in a pizza-making contest. Each contestant has to bake the largest and most delicious pizza they can. Tiara's pizza has a 6-foot diameter! After the contest, she plans to cut the pizza so that she can pass the slices out to share. She begins with 1 whole pizza. Then, she cuts it in half. After that, she cuts each of those slices in half. Then she cuts each of those slices in half and so on. Write a recursive and an explicit formula for the sequence. Then, determine the size of the slice compared to the whole pizza after 10 cuts.

4. Determine the 9th term of the sequence $a_n = a_{n-1} \cdot 8$ and $a_3 = 2$.

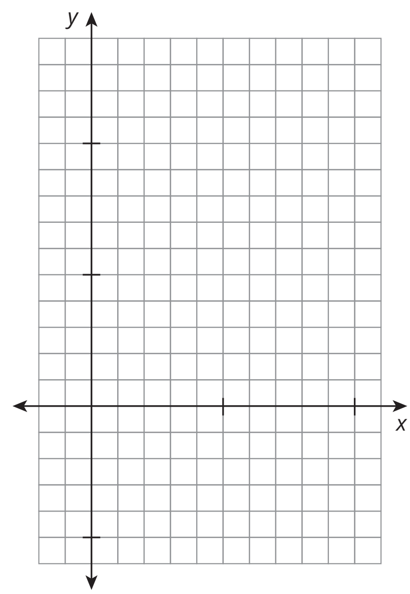
5. Determine the 12th term of the sequence $a_n = a_{n-1} + \frac{3}{4}$ and $a_9 = 6$.

ACTIVITY
3.5

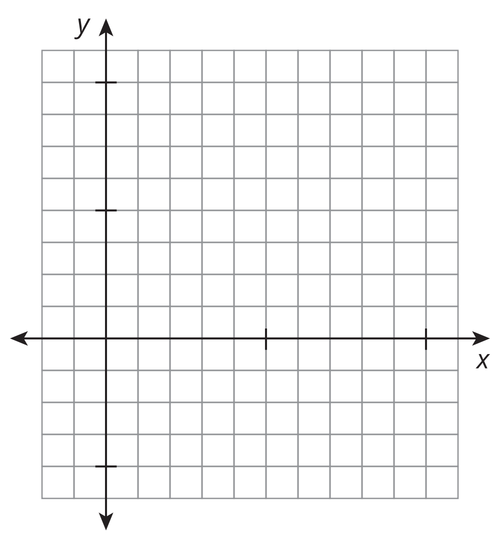
Graphing Sequences

Identify the sequence type and the common difference or ratio for each sequence. Write the explicit formula you can use to represent the terms of the sequence, then represent the sequence on the coordinate plane.

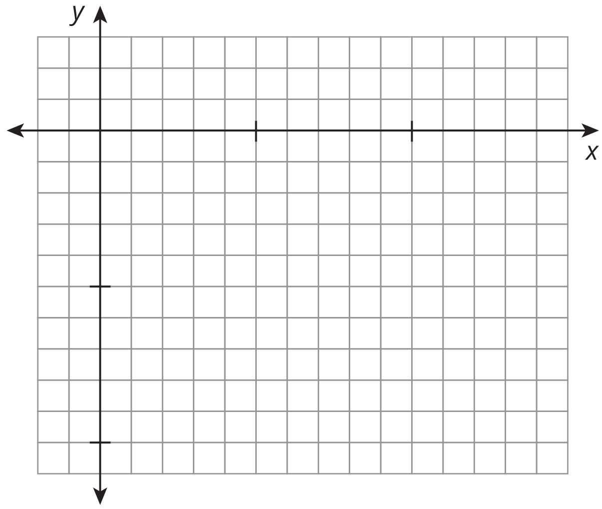
1. $a_1 = \frac{1}{3}, a_2 = -1, a_3 = 3, a_4 = -9, a_5 = 27$



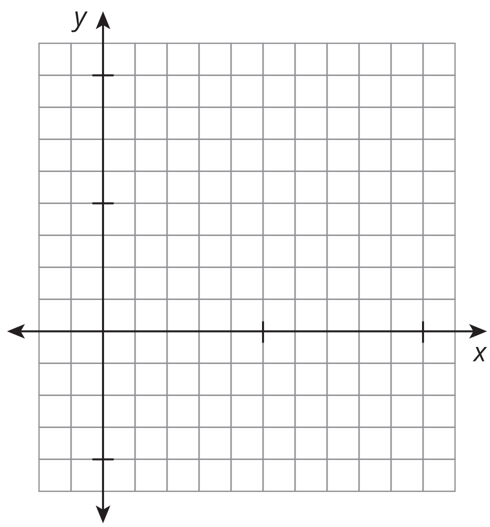
2. $h_1 = -1.2, h_2 = -1.1, h_3 = -1.0, h_4 = -0.9, h_5 = -0.8$



3. $a_3 = -12, a_4 = -14, a_5 = -16, a_6 = -18$



4. $k_3 = \frac{1}{8}, k_4 = \frac{1}{32}, k_5 = \frac{1}{128}, k_6 = \frac{1}{512}$



Talk the Talk

Pros and Cons

1. Explain the advantages and disadvantages of using a recursive formula.

2. Explain the advantages and disadvantages of using an explicit formula.

Lesson 3 Assignment

Write

Explain the difference between a recursive formula and an explicit formula in your own words.

Remember

All sequences describe functions.

The explicit formula for an arithmetic sequence is $a_n = a_1 + d(n - 1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and d is the common difference.

The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{(n - 1)}$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and r is the common ratio.

Practice

1. Lizzie must volunteer 225 hours for a community service project. She plans to volunteer for 6 hours each week. The sequence shown represents the number of volunteer hours she has left after three weeks have passed.

   225, 219, 213, 207, ...

   a. Describe this sequence.

   b. Write and simplify the explicit formula for the problem situation.

Lesson 3 Assignment

c. Use your explicit formula from part (b) to determine how many volunteer hours Lizzie has left to fulfill her requirement after 33 weeks have passed. Show your work.

d. Which formula should you use to determine how many volunteer hours Lizzie has left to fulfill her requirement after 40 weeks have passed? Explain your reasoning.

e. Calculate the number of volunteer hours Lizzie has left to fulfill her requirement after 40 weeks have passed. Explain what your answer means in terms of the problem situation.

2. The half-life of a substance is defined as the period of time it takes for the amount of the substance to decay by half. The sequence below shows the amount of a substance that will be left after a certain number of half-lives have elapsed.

   $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

   a. Describe this sequence.

Lesson 3 Assignment

b. Calculate how much of the substance will be left after 21 half-lives have elapsed. Show your work. Does your answer make sense in this problem context? Why or why not?

Prepare

Consider the first two terms of this sequence $\frac{1}{16}, -\frac{3}{16}, \ldots$

- Determine the 63rd term if this is an arithmetic sequence. Write your answer as an improper fraction in lowest terms.

- Determine the 63rd term if this is a geometric sequence. Write your answer in scientific notation.

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Sequences

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Sequences* topic by:

TOPIC 2: *Sequences*	Beginning of Topic	Middle of Topic	End of Topic
understanding that a sequence represents a relationship between term numbers (inputs) and term values (outputs).	◯	◯	◯
stating the appropriate domain for a sequence.	◯	◯	◯
distinguishing between arithmetic and geometric sequences.	◯	◯	◯
recognizing that an arithmetic sequence has a common difference between terms and a geometric sequence has a common ratio between terms.	◯	◯	◯
determining the common difference between two terms in an arithmetic sequence and the common ratio between two terms in a geometric sequence represented in tables and graphs.	◯	◯	◯
describing the graph of an arithmetic and geometric sequence.	◯	◯	◯
explaining that a recursive formula tells you how to determine the next value of a sequence from the previous value.	◯	◯	◯
explaining that an explicit formula tells you how to determine any value given the term number.	◯	◯	◯

continued on the next page

TOPIC 2: *Sequences*	Beginning of Topic	Middle of Topic	End of Topic
distinguishing between explicit and recursive formulas.			
writing recursive and explicit formulas for any sequence, including those presented as real-world scenarios.			
rewriting explicit formulas of arithmetic sequences using the distributive property.			
translating between explicit and recursive formulas.			
deciding when real-world problems model an arithmetic or geometric sequence.			

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Sequences* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Sequences Summary

LESSON 1 Recognizing Patterns and Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term in a sequence** is an individual number, figure, or letter in the sequence. Many different patterns can generate a sequence of numbers.

A sequence that continues on forever is called an **infinite sequence**. A sequence that terminates is called a **finite sequence**.

For example, consider the situation in which an album that can hold 275 baseball cards is filled with 15 baseball cards at the end of each week. A sequence to represent how many baseball cards can fit into the album after 6 weeks is 275 cards, 260 cards, 245 cards, 230 cards, 215 cards, and 200 cards. This sequence begins at 275 and decreases by 15 with each term. The pattern cannot continue forever since you cannot have a negative number of cards, so this is a finite sequence.

LESSON 2 Arithmetic and Geometric Sequences

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. This constant is called the **common difference** and is typically represented by the variable *d*. The common difference of a sequence is positive when the same positive number is added to each term to produce the next term. The common difference of a sequence is negative when the same negative number is added to each term to produce the next term.

For example, consider the sequence 14, $16\frac{1}{2}$, 19, $21\frac{1}{2}$, ... The pattern of this sequence is to add $2\frac{1}{2}$ to each term to produce the next term. This is an arithmetic sequence, and the common difference d is $2\frac{1}{2}$.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is typically represented by the variable *r*.

NEW KEY TERMS

- sequence [secuencia/sucesión]
- term of a sequence [término de una secuencia]
- infinite sequence [secuencia infinita]
- finite sequence [secuencia finita]
- arithmetic sequence [secuencia aritmética]
- common difference [diferencia común]
- geometric sequence [secuencia geométrica]
- common ratio [razón común]
- recursive formula [fórmula recursiva]
- explicit formula [fórmula explícita]
- mathematical modeling [modelado matemático]

For example, consider the sequence 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$. The pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. Therefore, this sequence is geometric and the common ratio, r, is $\frac{1}{3}$.

LESSON 3 Determining Recursive and Explicit Expressions from Context

A **recursive formula** expresses each new term of a sequence based on a preceding term of the sequence. The recursive formula to determine the nth term of an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula to determine the nth term of a geometric sequence is $a_n = a_{n-1} \cdot r$. When using the recursive formula, it is not necessary to know the first term of the sequence.

For example, consider the geometric sequence 32, 8, 2, $\frac{1}{2}$, ... with a common ratio of $\frac{1}{4}$. The 5th term of the sequence can be determined using the recursive formula.

$$a_n = a_{n-1} \cdot r$$
$$a_5 = a_4 \cdot r$$
$$a_5 = \frac{1}{2} \cdot \frac{1}{4}$$
$$a_5 = \frac{1}{8}$$

The 5th term of the sequence is $\frac{1}{8}$.

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the nth term of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. You can use the distributive property to rewrite the formula for an arithmetic sequence. The explicit formula to determine the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is currently 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

$$a_n = 3 + \frac{1}{4}(n - 1)$$
$$a_n = 3 + \frac{1}{4}n - \frac{1}{4}$$
$$a_n = \frac{1}{4}n + 2.75$$
$$a_{12} = \frac{1}{4}(12) + 2.75$$
$$a_{12} = 3 + 2.75$$
$$a_{12} = 5.75 \text{ or } 5\frac{3}{4}$$

In 12 months, the cactus will be $5\frac{3}{4}$ inches tall.

ISBN: 978-1-970197-81-5

Printed in the USA